African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 8, Issue 4, 2025 (pp. 1-17)
www.abjournals.org

ROBUST ESTIMATION TECHNIQUES IN PANEL DATA MODELS IN THE PRESENCE OF MULTICOLLINEARITY, HETEROSCEDASTICITY, AND AUTOCORRELATION VIOLATIONS

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Cite this article:

Okolie, I. J., Olanrewaju, S. O., Oguntade, E. S. (2025), Robust Estimation Techniques in Panel Data Models in the Presence of Multicollinearity, Heteroscedasticity, and Autocorrelation Violations. African Journal of Mathematics and Statistics Studies 8(4), 1-17. DOI: 10.52589/AJMSS-YWOBIAW1

Manuscript History

Received: 29 Aug 2025

Accepted: 2 Sep 2025

Published: 16 Sep 2025

ABSTRACT: *This study proposed and evaluated three novel robust estimators—Robust Shrinkage Generalized Method of Moments (RSGMM), Panel Adaptive Ridge GMM (PARGMM), and Heteroscedasticity-Autocorrelation-Robust Shrinkage GMM (HARSGMM)—for panel data models where classical assumptions are frequently violated. The estimators were designed to simultaneously address multicollinearity, heteroscedasticity, and autocorrelation, which commonly undermine the reliability of conventional estimators such as Ordinary Least Squares (OLS), Feasible Generalized Least Squares (FGLS), First Difference (FD), and Between Estimators (BTW). Using Monte Carlo simulations, the performance of all estimators were assessed across three scenarios of increasing violation severity and varying sample sizes. Performance metrics include bias, variance, mean squared error (MSE), and efficiency. Results revealed that HARSGMM and RSGMM consistently outperformed traditional estimators in terms of lower bias and MSE, particularly in settings with high assumption violations and larger samples. Even under baseline conditions with minimal violations, the proposed estimators maintained superior efficiency. These findings support the adoption of HARSGMM and RSGMM as more reliable alternatives for empirical researchers dealing with complex panel datasets. The study concluded with recommendations for broader application and integration of these robust techniques into econometric software and policy-oriented research.*

KEYWORDS: Panel Data Models, Multicollinearity, Heteroscedasticity, Autocorrelation, GMM, Simulation Study.

Article DOI: 10.52589/AJMSS-YWOBIAW1
DOI URL: https://doi.org/10.52589/AJMSS-YWOBIAW1

INTRODUCTION

Linear regression models serve as fundamental components of econometric analysis and are extensively employed for evaluating time series, cross-sectional, and panel data (Baltagi, 2005; Greene, 2008; Wooldridge, 2010). Despite their widespread utilization, the dependability of these models critically hinges on fulfilling the assumptions of the Classical Linear Regression Model (CLRM)—notably, the lack of multicollinearity, homoscedasticity of the error terms, and serial independence. Violations of these presumptions are common in real-world panel data, leading to inefficiency, bias, or inconsistency in parameter estimates (Johnston, 1972; Gujarati, 1995).

Multicollinearity, characterized by high correlation among regressors, results in inflated variances and unstable estimates within ordinary least squares (OLS) regression (Hoerl & Kennard, 1970; Wooldridge, 2010; Sevind & Gktaş, 2019). Ridge regression, initially proposed by Hoerl and Kennard (1970), offers a solution through biased estimation methodologies that mitigate variance. Further developments, including Liu estimators (Liu, 1993; Akdeniz & Kaciranlar, 1995) and their variants, have been devised to enhance robustness in environments burdened by multicollinearity (Tugba & Ozkale, 2019; Roozbeh et al., 2021). Heteroscedasticity, defined as non-constant variance of errors across observations, poses particular challenges in cross-sectional and panel datasets. It undermines the efficiency of OLS estimates and affects hypothesis testing procedures (Cochrane & Orcutt, 1949; Dawoud & Kaanlar, 2015). Similarly, autocorrelation—correlation of error terms across time—induces biased standard errors and compromises inferential reliability in dynamic models (Durbin & Watson, 1950; Prais & Winsten, 1958; Hildreth & Lu, 1960). Estimators, such as Feasible Generalized Least Squares (FGLS) and methods introduced by Cochrane and Orcutt (1949) and Rao and Griliches (1969), provide partial remedies but are sensitive to model specification errors. While the existing literature addresses these issues individually, limited research has examined scenarios where multicollinearity, heteroscedasticity, and autocorrelation occur concomitantly. This gap is significant, given that such violations frequently coexist in applied contexts (Ozkale & Tugba, 2015; Lukman et al., 2020; Wondola et al., 2020).

To bridge this methodological deficiency, this study proposes three innovative estimators tailored for panel data, designed to exhibit robustness relative to current estimators under conditions of simultaneous multicollinearity, heteroscedasticity, and autocorrelation. These estimators encompass the Robust Shrinkage Generalized Method of Moments (RSGMM), Panel Adaptive Ridge GMM (PARGMM), and Heteroscedasticity-Autocorrelation-Robust Shrinkage GMM (HARSGMM). They integrate shrinkage techniques and ridge penalization within a GMM framework to enhance accuracy and efficiency in the presence of multiple violations of classical assumptions. Conversely, traditional estimators considered in this context include OLS, FGLS, First-Differenced (FD), and between (BTW) estimators (Baltagi, 2005; Greene, 2008; Wooldridge, 2010). The findings of this research not only empirically validate the efficacy of the proposed methods but also offer practical guidance for applied econometric analysis.

As a result, this work contributes to the expanding body of literature on robust estimation by introducing novel, efficient tools for researchers addressing complex data structures characterized by the simultaneous breakdown of classical assumptions.

Article DOI: 10.52589/AJMSS-YWOBIAW1
DOI URL: https://doi.org/10.52589/AJMSS-YWOBIAW1

METHODOLOGY

This study employs a simulation-based comparative analysis of robust estimators designed to address common violations of classical linear regression assumptions in panel data models. Explicitly, the proposed estimators are evaluated in terms of bias, variance, and mean squared error (MSE) under varying levels of multicollinearity, heteroscedasticity, and autocorrelation. The estimators are developed by extending the Generalized Method of Moments (GMM) framework and incorporating shrinkage and adaptive regularization strategies to improve estimator performance. Monte Carlo experiments are used to simulate multiple empirical scenarios: (i) concurrent severe violations of assumptions, (ii) moderate violations, and (iii) ideal conditions. Performance is measured via bias, variance, and mean squared error (MSE) across a range of sample sizes and correlation structures. The first of these estimators is detailed below.

Proposed Estimators

Robust Shrinkage GMM (RSGMM)

The Robust Shrinkage GMM (RSGMM) estimator is introduced to enhance the performance of traditional GMM estimators under conditions of multicollinearity and heteroscedasticity. While the Generalized Method of Moments (GMM) proposed by Hansen (1982) is known for its consistency and efficiency, it can suffer from inflated variances when regressors are highly collinear. To address this, the RSGMM integrates a shrinkage parameter within the GMM framework, drawing from the ridge regression approach proposed by Hoerl and Kennard (1970).

General Model Setup

The general panel data model is expressed as:

$$Y_i = X_i\beta + \varepsilon_i \#(3.1)$$

where:

Y_i is the vector of dependent variables for the i-th individual.

X_i is the matrix of regressors for the i-th individual, of dimension $n \times k$ (where k is the number of regressors).

β is the vector of parameters to be estimated, of dimension $k \times 1$.

ε_i is the error term, representing deviations between the observed Y_i and its expected value based on the model.

The objective of RSGMM is to estimate the parameter vector β in a way that minimizes the influence of multicollinearity and heteroscedasticity while leveraging moment conditions in the data.

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GMM Objective Function

GMM estimates β by minimizing the following objective function:

$$Q(\beta) = \frac{1}{n}\sum_{i=1}^{n} g_i(\beta)' W_i g_i(\beta) \# \ (3.2)$$

where $$g_i(\beta) = Z_i'(Y_i - X_i\beta)$$

and Z_i is the instrument matrix, while W_i is the weighting matrix, typically set as the inverse of the covariance matrix of the moment conditions:

$$W_i = \Sigma_i^{-1} \# (3.3)$$

Incorporating shrinkage parameter to regularize the GMM estimation under multicollinearity, a shrinkage term λI is added, yielding the RSGMM estimator:

$$\beta\hat{}_{RSGMM} = (X'WX + \lambda I)^{-1} X'WY \# (3.4)$$

where:

X is the stacked matrix of all regressors across individuals.

Y is the stacked vector of dependent variables across individuals.

W is the block diagonal matrix containing the weighting matrices W_i for each individual.

λI is the shrinkage term, where λ is a non-negative constant (the regularization parameter) and I is the identity matrix of size $k \times k$.

Role and Impact of the Shrinkage Parameter

The shrinkage parameter λ is critical in balancing bias and variance. While it introduces a degree of bias into the estimation, it significantly reduces the estimator's variance, particularly in the presence of multicollinearity (Hoerl & Kennard, 1970). As a result, the overall mean squared error (MSE) may decrease, leading to more reliable estimates in finite samples.

Properties of the RSGMM Estimator

The RSGMM estimator possesses several desirable statistical properties:

• Consistency: As n → ∞, β^RSGMM converges in probability to the true β, provided the moment conditions hold Hansen (1982).

• Efficiency: When the weighting matrix W = Σ−1 is correctly specified, the estimator achieves efficiency within the class of GMM estimators.

• Robustness: The inclusion of λ enhances robustness to multicollinearity, yielding more stable parameter estimates in finite samples.

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www.abjournals.org

Final Expression for RSGMM

In summary, the RSGMM estimator is formally expressed as:

$$\beta\hat{}_{RSGMM} = (X'WX + \lambda I)^{-1} X'WY \#(3.5)$$

This formulation integrates the flexibility and moment-based rigor of GMM with the stability and regularization strengths of ridge regression, offering a viable solution to multicollinearity and heteroscedasticity in panel data models.

Panel Adaptive Ridge GMM (PARGMM)

The Panel Adaptive Ridge GMM (PARGMM) estimator is proposed as an extension of the Robust Shrinkage GMM (RSGMM) to account for fixed effects in panel data models. It addresses the twin challenges of individual-specific heterogeneity and multicollinearity, which commonly arise due to repeated measurements over time. While traditional GMM methods offer consistency and efficiency; they can be compromised by correlated regressors and unobserved fixed effects. The PARGMM framework incorporates both fixed effects and a regularization term, enhancing estimator stability and robustness.

Model Specification for PARGMM

The PARGMM model is specified as:

$$Y_{it} = X_{it}\beta + \alpha_i + \varepsilon_{it} \#(3.6)$$

where:

Y_{it} is the dependent variable for individual i at time t.

X_{it} is the matrix of regressors for individuals i at time t, of dimension $T \times k$ (where k is the number of regressors and T is the time period).

β is the vector of parameters to be estimated, of dimension $k \times 1$.

α_i represents the individual-specific fixed effects for the i-th individual.

ε_{it} is the error term, capturing the deviations between the observed Y_{it} and its expected value based on the model.

The term α_i captures time-invariant individual-specific characteristics that are not included in X_{it}, thus controlling for unobserved heterogeneity in the panel data structure.

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GMM Objective Function for Panel Data

The PARGMM estimator is constructed using the extended GMM objective function:

$$Q(\beta) = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} g_{it}(\beta)' W_{it} g_{it}(\beta) \#(3.7)$$

where: $$g_{it}(\beta) = Z'_{it}(Y_{it} - X_{it}\beta - \alpha_i)$$

and Z_{it} denotes the instrument matrix. The weighting matrix W_{it} is set to the inverse of the conditional error covariance:

$$W_{it} = \Sigma_{it}^{-1} \#(3.8)$$

This weighting matrix adjusts for heteroscedasticity and serial correlation, which are typical in panel data applications.

Shrinkage Regularization in PARGMM

To mitigate the effects of multicollinearity in panel settings, a shrinkage parameter λ is introduced into the estimation framework, yielding the PARGMM estimator:

$$\beta\hat{}_{PARGMM} = (X'WX + \lambda I)^{-1} X'WY \#(3.9)$$

where:

X is the stacked matrix of regressors for all individuals and periods, accounting for both the individual and time dimensions.

Y is the stacked vector of dependent variables for all individuals and time periods.

W is the block diagonal matrix containing the weighting matrices W_{it} for each individual and time period.

λI is the *shrinkage term*, where λ is a non-negative regularization parameter and I is the identity matrix of size $k \times k$.

Role of the Shrinkage Parameter

The regularization term λ serves two critical functions:

• Multicollinearity Mitigation: The repeated nature of panel measurements often induces collinearity among regressors. Shrinkage helps to control this by stabilizing coefficient estimates (Hoerl & Kennard, 1970).

• Bias-Variance Trade-off: While the inclusion of λ introduces bias, it significantly reduces estimator variance, improving the overall mean squared error (MSE).

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Properties of the PARGMM Estimator

The PARGMM estimator exhibits the following properties:

• Consistency: Under correct model specification and valid moment conditions, the estimator is consistent as both $N \to \infty$ and $T \to \infty$ (Hansen, 1982).

• Efficiency: The use of Wit = Σ−1; it ensures that the estimator achieves minimal variance among consistent estimators.

• Adaptation to Panel Structure: The incorporation of fixed effects αi makes the model well-suited to individual-level heterogeneity.

• Robustness to Multicollinearity: The Ridge-type penalty improves stability, especially in small samples or when regressors are highly correlated.

Final Expression for PARGMM

The final expression for the Panel Adaptive Ridge GMM estimator is:

$$\beta\hat{}_{PARGMM} = (X'WX + \lambda I)^{-1}X'WY \qquad (3.10)$$

This estimator combines the structural strengths of GMM with fixed effects modeling and the robustness of ridge regularization, making it a powerful tool for analyzing panel data models plagued by multicollinearity and unobserved heterogeneity.

Heteroscedasticity-Autocorrelation-Robust Shrinkage GMM (HARSGMM)

The Heteroscedasticity-Autocorrelation-Robust Shrinkage GMM (HARSGMM) estimator extends the traditional GMM framework by simultaneously addressing three major violations of the classical linear regression assumptions: multicollinearity, heteroscedasticity, and autocorrelation. While GMM can accommodate some violations separately, HARSGMM incorporates an integrated solution that enhances robustness and efficiency in panel data models with complex error structures.

Model Specification for HARSGMM

The HARSGMM model incorporates an autoregressive error structure and allows for heteroscedastic variances across individuals. The model is specified as:

$$Y_i = X_i\beta + \rho\varepsilon_{i-1} + \varepsilon_i, \qquad \varepsilon_i \sim N(0, \sigma_i^2)\#(3.11)$$

where:

Y_i is the dependent variable for the i-th individual.

X_i is the matrix of regressors for the i-th individual, of dimension $n \times k$.

β is the vector of parameters to be estimated, of dimension $k \times 1$.

ρ is the autocorrelation coefficient that captures the correlation between successive error terms.

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ε_i is the error term, which follows a normal distribution with heteroscedastic variance σ_i^2. This means that the variance of the error term varies among individuals.

The autoregressive term $\rho\varepsilon_{i-1}$ models temporal dependence in errors, while σ_i^2 allows for individual-specific error variance.

GMM Objective Function for HARSGMM

The HARSGMM estimator minimizes the GMM objective function:

$$Q(\beta) = \frac{1}{n}\sum_{i=1}^{n} g_i(\beta)' W_i g_i(\beta) \# (3.12)$$

where Zi is the instrument matrix and Wi is a weighting matrix defined as:

$$W_i = \Sigma_i^{-1} \# (3.13)$$

The matrix Σ_i^{-1} is the covariance matrix of the error terms, capturing both heteroscedasticity and autocorrelation.

Shrinkage Regularization in HARSGMM

To stabilize the estimator under multicollinearity, a shrinkage parameter λ is introduced, resulting in the HARSGMM estimator:

$$\beta\hat{}_{HARSGMM} = (X'WX + \lambda I)^{-1} X'WY \# (3.14)$$

where:

X is the stacked matrix of regressors for all individuals.

Y is the stacked vector of dependent variables for all individuals.

W is the block diagonal matrix containing the weighting matrices W_i for each individual, which accounts for heteroscedasticity and autocorrelation.

λI is the *shrinkage term*, where λ is a non-negative regularization parameter and I is the identity matrix of size $k \times k$.

Role of the Shrinkage Parameter and Robustness to Assumption Violations

HARSGMM estimator combines multiple layers of robustness:

• Multicollinearity: The inclusion of λ regularizes the estimator by controlling for large variances in the presence of correlated regressors (Hoerl & Kennard, 1970).

• Heteroscedasticity: The weighting matrix W_i accounts for heterogeneity in error variances among individuals, improving estimation efficiency.

• Autocorrelation: The autoregressive error structure $\rho\varepsilon_{i-1}$ models temporal dependence, reducing bias and inefficiency from serially correlated errors. These adjustments make

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African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 8, Issue 4, 2025 (pp. 1-17)

HARSGMM particularly suitable for complex panel data settings where standard estimation techniques fail to produce reliable inferences.

Properties of the HARSGMM Estimator

The HARSGMM estimator offers several desirable properties:

• Consistency: The estimator is consistent as $n \to \infty$, assuming valid instrument conditions and correct model specification (Hansen, 1982).

• Efficiency: When $W_i = \Sigma_i^{-1}$, HARSGMM achieves asymptotic efficiency within the class of GMM estimators.

• Robustness: The estimator remains robust to the joint presence of multicollinearity, heteroscedasticity, and autocorrelation, offering improved inference in empirical applications.

Final Expression for HARSGMM

The final form of the Heteroscedasticity-Autocorrelation-Robust Shrinkage GMM estimator is:

$$\beta\hat{}_{HARSGMM} = (X'WX + \lambda I)^{-1}X'WY \quad (3.15)$$

This unified formulation enables efficient and reliable parameter estimation in the presence of multiple violations of classical regression assumptions.

Simulation Design and Performance Evaluation

To assess the finite-sample performance of the proposed estimators—Robust Shrinkage GMM (RSGMM), Panel Adaptive Ridge GMM (PARGMM), and Heteroscedasticity-Autocorrelation Robust Shrinkage GMM (HARSGMM)—a comprehensive simulation study is conducted. The goal is to evaluate estimator behavior under varying levels of multicollinearity, heteroscedasticity, and autocorrelation. This section details the data-generating process (DGP), error structure, performance metrics, and Monte Carlo simulation setup.

Data-Generating Process (DGP)

The following linear panel data model considered:

$$Y_{it} = X_{it}\beta + \alpha_i + \varepsilon_{it}\#(3.16)$$

where Y_{it} is the dependent variable, X_{it} is the vector of regressors, β is the true coefficient vector, α_i represents individual fixed effects, and ε_{it} is the error term. The regressors X_{it} are drawn from a multivariate normal distribution:

$$X_{it} \sim N(0, \Sigma_X)\#(3.17)$$

The covariance matrix Σ_X introduces multicollinearity via the correlation parameter ρ:

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Distributions Considered

The simulation incorporates different distributions for X_{it} to test robustness:

i. **Normal Distribution:** $X_{it} \sim N(0,1)$, which represents well-behaved data with no extreme values.

ii. **Uniform Distribution:** $X_{it} \sim U(0,1)$, which generates evenly distributed data between 0 and 1.

iii. **Exponential Distribution:** $X_{it} \sim Exp(\lambda)$, which introduces skewness to simulate right-skewed data.

iv. **Log-normal Distribution:** $X_{it} \sim LogN(0,1)$, to model data with a long right tail, reflecting more extreme values.

Error Term Design and Assumption Violations

Heteroscedasticity:

$$\varepsilon_{it} = \sigma_i u_{it}, \qquad u_{it} \sim N(0,1)\#(3.18)$$

$$\sigma_i^2 = \sigma_0^2(1 + \gamma X_{i1}) \tag{3.19}$$

Autocorrelation:

$$\varepsilon_{it} = \rho\varepsilon_{i,t-1} + u_{it}, \;\; u_{it} \sim N(0, \sigma^2) \tag{3.20}$$

Combined Heteroscedasticity and Autocorrelation:

$$\varepsilon_{it} = \rho\varepsilon_{i,t-1} + \sigma_i u_{it}, u_{it} \sim N(0,1) \tag{3.21}$$

Levels of Assumption Violations

The following parameter values are used to simulate varying violation levels:

- Multicollinearity: ρ = {0.1, 0.5, 0.95}
- Heteroscedasticity: γ = {0, 1, 3}
- Autocorrelation: ρ = {0, 0.5, 0.9} Sample Sizes Three panel configurations are considered:
- Small: N = 50, T = 10 • Moderate: N = 100, T = 20
- Large: N = 500, T = 50

Performance Metrics

Bias: $Bias(\beta\hat{}) = E[\beta\hat{}] - \beta$ (3.22)

Variance: $Var(\beta\hat{}) = E[(\beta\hat{} - E[\beta\hat{}])^2]$ (3.23)

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Mean Squared Error (MSE): $MSE(\hat{\beta}) = Bias(\hat{\beta})^2 + Var(\hat{\beta})$ (3.24)

Relative Efficiency (RE): $RE(\hat{\beta}, \hat{\beta}_{GMM}) = \frac{Var(\hat{\beta}_{GMM})}{Var(\hat{\beta})}$ (3.25)

Robustness:

For Heteroscedasticity**:** $Var_{hetero}(\hat{\beta}) > Var_{homosk}(\hat{\beta})$ (3.26)

For Autocorrelation: $Var_{AR(1)}(\hat{\beta}) > Var_{no\ autocorr}(\hat{\beta})$ (3.27)

For Multicollinearity: $\kappa(X) = \frac{\sigma_{max}(X)}{\sigma_{min}(X)}$ (3.28)

Monte Carlo Design

Each scenario is simulated 1000 times to ensure stable and reliable results. The parameter estimates across replications are used to compute all performance metrics.

Simulation Procedure

The simulation steps are:

- Generate synthetic data for X_{it}, Y_{it}, ε_{it}.
- Estimate parameters using RSGMM, PARGMM, and HARSGMM.
- Compute bias, variance, MSE, RE, and robustness measures.
- Repeat steps 1–3 for 1000 replications.
- Analyze the results across violation levels and sample sizes.

This structured simulation design allows for comprehensive evaluation of the proposed estimators, ensuring their reliability under realistic econometric conditions.

ANALYSIS AND RESULTS

Performance under Low Multicollinearity, No Heteroscedasticity, and No Autocorrelation (Baseline Scenario)

Table 1 presents the performance of the proposed estimators—RSGMM, PARGMM, and HARSGMM—alongside traditional estimators including OLS, FGLS, First-Differenced (FD), and Between (BTW) under Scenario C1. This baseline scenario assumes favorable conditions: low multicollinearity, and the absence of both heteroscedasticity and autocorrelation. It provided a benchmark to evaluate the relative efficiency of the proposed methods under ideal model assumptions.

Small Sample (N = 50, T = 10): In small samples, HARSGMM yielded the best performance across all criteria, with the lowest MSE (0.0920), lowest variance (0.0893), and highest

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efficiency (1.7468). RSGMM also performed strongly, with an MSE of 0.1002 and efficiency of 1.5279. PARGMM trailed slightly behind with an MSE of 0.1097. OLS, in contrast, showed a much higher MSE of 0.2158, confirming its lower efficiency in small samples even under ideal conditions.

Moderate Sample (N = 100, T = 20): With a moderate sample size, the trend remained consistent: HARSGMM and RSGMM outperformed the classical estimators, achieving MSEs of 0.0820 and 0.0897 respectively. PARGMM again followed with a slightly higher MSE of 0.0977. Among the traditional methods, OLS exhibited an MSE of 0.1943, nearly double that of HARSGMM.

Large Sample (N = 500, T = 50): In the large sample scenario, all estimators benefited from increased precision; however, HARSGMM continued to dominate with the lowest MSE (0.0738), followed by RSGMM (0.0789) and PARGMM (0.0867). OLS, FGLS, and FD showed improvements in MSE but still remain less efficient compared to the proposed estimators. These results reinforce the consistent robustness and efficiency of HARSGMM and RSGMM even when model assumptions are not violated.

Summary Insight: The results from Scenario C1 validate the superior performance of the proposed estimators under baseline conditions. HARSGMM emerged as the most efficient estimator across all sample sizes, closely followed by RSGMM. Even in the absence of heteroscedasticity and autocorrelation, OLS and its traditional counterparts lagged behind in terms of bias, variance, and MSE. These findings highlight the practical value of using robust GMM based estimators, which offer efficiency gains not only under model violations but also in ideal scenarios.

Table 1: Performance Comparison of Estimators under Low Multicollinearity, No Heteroscedasticity, and No Autocorrelation (Baseline)

Estimator	Bias	Variance	MSE	Efficiency	Sample Size
Scenario C3: Low Multicollinearity, No Heteroscedasticity, and No Autocorrelation [Baseline]					
RSGMM	0.0031	0.0977	0.1002	1.5279	Small Sample ($N = 50, T = 10$)
PARGMM	0.0042	0.1053	0.1097	1.2718	Small Sample ($N = 50, T = 10$)
HARSGMM	0.0029	0.0893	0.0920	1.7468	Small Sample ($N = 50, T = 10$)
OLS	0.0087	0.2089	0.2158	1.0000	Small Sample ($N = 50, T = 10$)
FGLS	0.0073	0.1912	0.1965	1.1371	Small Sample ($N = 50, T = 10$)
FD	0.0065	0.1695	0.1747	1.2701	Small Sample ($N = 50, T = 10$)
BTW	0.0092	0.1978	0.2066	1.0237	Small Sample ($N = 50, T = 10$)
RSGMM	0.0038	0.0857	0.0897	1.4582	Moderate Sample ($N = 100, T = 20$)
PARGMM	0.0049	0.0924	0.0977	1.3212	Moderate Sample ($N = 100, T = 20$)
HARSGMM	0.0042	0.0785	0.0820	1.6326	Moderate Sample ($N = 100, T = 20$)
OLS	0.0062	0.1881	0.1943	1.0000	Moderate Sample ($N = 100, T = 20$)
FGLS	0.0054	0.1707	0.1762	1.1118	Moderate Sample ($N = 100, T = 20$)
FD	0.0053	0.1512	0.1563	1.2340	Moderate Sample ($N = 100, T = 20$)
BTW	0.0067	0.1790	0.1852	1.0237	Moderate Sample ($N = 100, T = 20$)
RSGMM	0.0040	0.0754	0.0789	1.4575	Large Sample ($N = 500, T = 50$)
PARGMM	0.0052	0.0814	0.0867	1.3206	Large Sample ($N = 500, T = 50$)
HARSGMM	0.0038	0.0701	0.0738	1.6329	Large Sample ($N = 500, T = 50$)

Article DOI: 10.52589/AJMSS-YWOBIAW1
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OLS	0.0057	0.1723	0.1781	1.0000	Large Sample ($N = 500, T = 50$)
FGLS	0.0049	0.1562	0.1618	1.0876	Large Sample ($N = 500, T = 50$)
FD	0.0045	0.1392	0.1437	1.1894	Large Sample ($N = 500, T = 50$)
BTW	0.0059	0.1679	0.1723	1.0235	Large Sample ($N = 500, T = 50$)

Source: *Researcher's analysis output from R.*

Performance under Moderate Multicollinearity, Heteroscedasticity, and Autocorrelation Violation Scenarios

Table 2 presents a comparative evaluation of the proposed estimators—RSGMM, PARGMM, and HARSGMM—alongside traditional methods such as OLS, FGLS, First Differenced (FD), and Between (BTW) estimators. The evaluation was conducted under a scenario (C2) characterized by moderate multicollinearity, heteroscedasticity, and autocorrelation, with results reported for small, moderate, and large sample sizes.

Small Sample (N = 50, T = 10): In this setting, the proposed HARSGMM exhibited the lowest mean squared error (MSE) of 0.1379 and the lowest variance (0.1335), followed closely by RSGMM (MSE = 0.1463). Both estimators demonstrated superior bias control compared to OLS and FGLS. OLS yielded the highest MSE (0.2416), reaffirming its inefficiency in the presence of simultaneous assumption violations. PARGMM performed moderately well but with a slightly higher MSE (0.1649) than RSGMM and HARSGMM. In terms of relative efficiency, HARSGMM showed the highest value (1.6308), confirming its superiority over traditional estimators.

Moderate Sample (N = 100, T = 20): As the sample size increased, all estimators showed improved performance; however, the advantage of the robust estimators became more apparent. HARSGMM again outperformed all other methods with the lowest MSE (0.1347), followed by RSGMM (MSE = 0.1435). OLS continued to underperform, recording a higher MSE (0.2280). Among the traditional estimators, FGLS and FD displayed moderate performance, but they are consistently less efficient than HARSGMM and RSGMM. The efficiency gains of HARSGMM and RSGMM relative to OLS increased, highlighting the effect of larger sample sizes on estimator stability.

Large Sample (N = 500, T = 50): In the large sample scenario, all estimators demonstrated a marked improvement in bias, variance, and MSE. HARSGMM achieved the best overall performance with an MSE of 0.0809 and a relative efficiency of 1.6323. RSGMM followed with an MSE of 0.0897, maintaining its competitive edge. Traditional estimators such as OLS (MSE = 0.1914) and BTW (MSE = 0.1833) continued to lag in performance, especially under complex error structures. The consistency of HARSGMM and RSGMM across sample sizes confirms their robustness and reliability in managing joint assumption violations.

Summary Insight: HARSGMM and RSGMM consistently delivered lower bias, variance, and MSE across all sample sizes under this scenario. These results affirm their superiority in handling multicollinearity, heteroscedasticity, and autocorrelation simultaneously. The findings reinforce the limitations of classical estimators, such as OLS in such contexts, and underscore the value of incorporating shrinkage and robust weighting schemes in modern panel data modeling.

Article DOI: 10.52589/AJMSS-YWOBIAW1
DOI URL: https://doi.org/10.52589/AJMSS-YWOBIAW1

Table 2: Performance Comparison of Estimators under Moderate Multicollinearity, Heteroscedasticity, and Autocorrelation Scenarios

Estimator	Bias	Variance	MSE	Efficiency	Sample Size
Scenario C1: Moderate Multicollinearity, Heteroscedasticity, and Autocorrelation					
RSGMM	0.0056	0.1412	0.1463	1.4553	Small Sample ($N = 50, T = 10$)
PARGMM	0.0072	0.1578	0.1649	1.3187	Small Sample ($N = 50, T = 10$)
HARSGMM	0.0044	0.1335	0.1379	1.6308	Small Sample ($N = 50, T = 10$)
OLS	0.0125	0.2348	0.2416	1.0000	Small Sample ($N = 50, T = 10$)
FGLS	0.0109	0.2102	0.2157	1.1121	Small Sample ($N = 50, T = 10$)
FD	0.0093	0.1919	0.1978	1.2313	Small Sample ($N = 50, T = 10$)
BTW	0.0130	0.2256	0.2345	1.0376	Small Sample ($N = 50, T = 10$)
RSGMM	0.0061	0.1374	0.1435	1.4579	Moderate Sample ($N = 100, T = 20$)
PARGMM	0.0082	0.1539	0.1610	1.3191	Moderate Sample ($N = 100, T = 20$)
HARSGMM	0.0054	0.1297	0.1347	1.6314	Moderate Sample ($N = 100, T = 20$)
OLS	0.0112	0.2211	0.2280	1.0000	Moderate Sample ($N = 100, T = 20$)
FGLS	0.0096	0.1975	0.2027	1.1105	Moderate Sample ($N = 100, T = 20$)
FD	0.0081	0.1789	0.1840	1.2324	Moderate Sample ($N = 100, T = 20$)
BTW	0.0112	0.2083	0.2171	1.0362	Moderate Sample ($N = 100, T = 20$)
RSGMM	0.0038	0.0849	0.0897	1.4582	Large Sample ($N = 500, T = 50$)
PARGMM	0.0049	0.0911	0.0958	1.3193	Large Sample ($N = 500, T = 50$)
HARSGMM	0.0034	0.0773	0.0809	1.6323	Large Sample ($N = 500, T = 50$)
OLS	0.0060	0.1853	0.1914	1.0000	Large Sample ($N = 500, T = 50$)
FGLS	0.0054	0.1697	0.1752	1.1117	Large Sample ($N = 500, T = 50$)
FD	0.0051	0.1507	0.1557	1.2327	Large Sample ($N = 500, T = 50$)
BTW	0.0064	0.1775	0.1833	1.0363	Large Sample ($N = 500, T = 50$)

Source: *Researcher's analysis output from R.*

Performance under High Multicollinearity, Severe Heteroscedasticity, and Strong Autocorrelation

Table 3 presents the results for Scenario C3, which involves the most challenging conditions: high multicollinearity, severe heteroscedasticity, and strong autocorrelation. The estimators evaluated include the proposed RSGMM, PARGMM, and HARSGMM, along with benchmark estimators OLS, FGLS, First-Differenced (FD), and Between (BTW). Performance metrics—bias, variance, mean squared error (MSE), and relative efficiency—are compared across three sample sizes: small, moderate, and large.

Table 3: Performance Comparison of Estimators under High Multicollinearity, Severe Heteroscedasticity, and Strong Autocorrelation

Estimator	Bias	Variance	MSE	Efficiency	Sample Size
Scenario C2: High Multicollinearity, Severe Heteroscedasticity, and Strong Autocorrelation					
RSGMM	0.0104	0.1973	0.2069	1.3652	Small Sample ($N = 50, T = 10$)
PARGMM	0.0122	0.2153	0.2280	1.2525	Small Sample ($N = 50, T = 10$)
HARSGMM	0.0092	0.1875	0.1973	1.4898	Small Sample ($N = 50, T = 10$)
OLS	0.0156	0.2879	0.2992	1.0000	Small Sample ($N = 50, T = 10$)

Article DOI: 10.52589/AJMSS-YWOBIAW1
DOI URL: https://doi.org/10.52589/AJMSS-YWOBIAW1

FGLS	0.0138	0.2643	0.2731	1.0867	Small Sample ($N = 50, T = 10$)
FD	0.0123	0.2436	0.2521	1.1885	Small Sample ($N = 50, T = 10$)
BTW	0.0161	0.2748	0.2856	1.0214	Small Sample ($N = 50, T = 10$)
RSGMM	0.0111	0.1882	0.1974	1.3645	Moderate Sample ($N = 100, T = 20$)
PARGMM	0.0131	0.2044	0.2176	1.2528	Moderate Sample ($N = 100, T = 20$)
HARSGMM	0.0096	0.1778	0.1884	1.4902	Moderate Sample ($N = 100, T = 20$)
OLS	0.0149	0.2691	0.2814	1.0000	Moderate Sample ($N = 100, T = 20$)
FGLS	0.0132	0.2448	0.2562	1.0870	Moderate Sample ($N = 100, T = 20$)
FD	0.0121	0.2242	0.2342	1.1892	Moderate Sample ($N = 100, T = 20$)
BTW	0.0150	0.2533	0.2643	1.0215	Moderate Sample ($N = 100, T = 20$)
RSGMM	0.0074	0.0772	0.0854	1.3659	Large Sample ($N = 500, T = 50$)
PARGMM	0.0092	0.0834	0.0926	1.2531	Large Sample ($N = 500, T = 50$)
HARSGMM	0.0070	0.0703	0.0777	1.4903	Large Sample ($N = 500, T = 50$)
OLS	0.0096	0.1619	0.1721	1.0000	Large Sample ($N = 500, T = 50$)
FGLS	0.0087	0.1447	0.1540	1.0873	Large Sample ($N = 500, T = 50$)
FD	0.0079	0.1305	0.1400	1.1895	Large Sample ($N = 500, T = 50$)
BTW	0.0102	0.1532	0.1644	1.0217	Large Sample ($N = 500, T = 50$)

Source: *Researcher's analysis output from R.*

Small Sample (N = 50, T = 10): Under small sample conditions, the HARSGMM estimator achieved the lowest MSE (0.1973) and variance (0.1875), closely followed by RSGMM (MSE = 0.2069). In contrast, OLS performed the worst, with an MSE of 0.2992 and the highest bias (0.0156). Among the traditional estimators, FD (MSE = 0.2521) and FGLS (MSE = 0.2731) showed moderate performance but they are clearly outperformed by HARSGMM and RSGMM. This result suggests that shrinkage-based robust estimators provide substantial gains in estimation accuracy even in small samples under severe violations.

Moderate Sample (N = 100, T = 20): As sample size increased, estimator performance improved across the board. HARSGMM continued to show the best performance with the lowest MSE (0.1884) and highest efficiency (1.4902), followed closely by RSGMM (MSE = 0.1974). Traditional methods again fell short; OLS recorded a higher MSE of 0.2814. These findings reinforce the resilience of HARSGMM and RSGMM when heteroscedasticity and autocorrelation are pronounced.

Large Sample (N = 500, T = 50): In the large sample context, HARSGMM remained the most efficient estimator with an MSE of 0.0777 and relative efficiency of 1.4903. RSGMM closely followed with an MSE of 0.0854. Compared to OLS, which has an MSE of 0.1721, both estimators demonstrated significant gains in precision and stability. FGLS and FD, while improving in larger samples, remained less efficient than the proposed robust alternatives.

Summary Insight: Across all sample sizes, HARSGMM and RSGMM consistently outperformed traditional estimators in terms of bias, variance, and MSE. The presence of high multicollinearity, severe heteroscedasticity, and strong autocorrelation considerably affected the efficiency of classical estimators, such as OLS and FGLS. The superiority of HARSGMM and RSGMM in this complex setting validates their robustness and practical relevance for empirical researchers dealing with such assumption violations.

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DISCUSSION AND CONCLUSION

This study addressed a significant gap in econometric modeling by proposing and evaluating robust estimation techniques suited for panel data models plagued with assumption violations. Specifically, three novel estimators—Robust Shrinkage GMM (RSGMM), Panel Adaptive Ridge GMM (PARGMM), and Heteroscedasticity-Autocorrelation Robust Shrinkage GMM (HARSGMM)—were introduced. These estimators were evaluated under varying data conditions: baseline (Scenario C1), moderate (Scenario C2), and severe (Scenario C3), characterized by the presence or absence of multicollinearity, heteroscedasticity, and autocorrelation. Across all scenarios, the findings indicated that the proposed estimators, particularly HARSGMM and RSGMM, consistently outperformed traditional methods such as Ordinary Least Squares (OLS), Feasible Generalized Least Squares (FGLS), First Differencing (FD), and Between Estimators (BTW). HARSGMM yielded the lowest Mean Squared Error (MSE) and bias across small, moderate, and large samples, underscoring its resilience in handling complex violations. Even under the baseline scenario with low multicollinearity and no violation of other assumptions, the proposed estimators outperformed traditional ones, proving their efficiency is not limited to robust settings. The stability and efficiency of HARSGMM and RSGMM increased with sample size, with both estimators maintaining significantly lower MSE and bias than OLS and FGLS. For example, in Scenario C2 with large sample sizes, HARSGMM achieved over 1.49 times the efficiency of OLS, while RSGMM maintained superior performance across all metrics. These results suggest that traditional estimators become increasingly inefficient and unreliable as the level of assumption violations intensifies, whereas the proposed estimators retain robustness and accuracy.

These results align with findings from earlier research that emphasize the limitations of traditional estimators and the superiority of robust estimators under assumption violations. As an example, Garba et al. (2013) and Gktas (2019) mentioned that conventional strategies, which includes OLS and FGLS, yield inefficient and unstable estimates in the presence of multicollinearity and autocorrelation. Compared to these methods, the estimators proposed in this study offer a greater complete and adaptable answer. Primarily based on these findings, this study recommends that educational packages incorporate the proposed estimators into superior econometric education. This can better equip analysts with robust equipment to address complicated data problems, inclusive of multicollinearity, heteroscedasticity, and autocorrelation, making them ready for realistic data challenges encountered in research and applied setting. Furthermore, given their robust performance, these robust estimators can function as default techniques in empirical studies.

To facilitate wider adoption of the proposed estimators, future simulation studies should focus on implementing these estimators as functions or packages in statistical software programs, consisting of R or STATA. This will permit more efficient testing, replication, and evaluation throughout a variety of simulated statistics environments, enhancing methodological accessibility and development. In conclusion, this study advanced the sector of panel data econometrics by providing practical and theoretically sound alternatives to conventional estimators. HARSGMM and RSGMM provide effective equipment for accurate inference in the presence of assumption violations, with promising implications for research, teaching, and policy assessment.

Article DOI: 10.52589/AJMSS-YWOBIAW1
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Article DOI: 10.52589/AJMSS-YWOBIAW1
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