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RESPONSE SURFACE OPTIMIZATION OF SUGARCANE YIELD UNDER VARYING AGRONOMIC INPUTS IN CHIREDZI DISTRICT, ZIMBABWE

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ABSTRACT: *Sugarcane production plays a critical role in Zimbabwe's agro-industrial economy, particularly in the irrigated low-veld region of Chiredzi District. Optimizing yield under conditions of high temperature and limited rainfall requires precise management of key soil fertility parameters. This study applied Response Surface Methodology (RSM) to model and optimize sugarcane yield as a function of soil pH and organic matter content using data from thirteen stratified plots across Triangle, Hippo Valley, Mkwasine, and Mwenezana estates during the 2023/24 production season. A Central Composite Design (CCD) was employed to estimate a second-order polynomial model, and optimization was conducted using the method of steepest ascent. The fitted quadratic model was highly significant ($F = 58.09$, $p < 0.001$), explaining 97.65% of the variation in yield ($R^2 = 0.9765$), with strong predictive capability ($R^2\_pred = 83.42\%$). Both soil pH and organic matter exhibited significant positive linear effects ($p < 0.001$), while significant negative quadratic terms confirmed diminishing returns and the presence of a stationary maximum within the experimental region. Area planted was not statistically significant in the preliminary linear model, indicating that productivity differences were primarily attributable to soil quality rather than scale of cultivation. Optimization results indicated a maximum predicted sugarcane yield of approximately 91.27 tonnes per hectare at a soil pH of 7.04 and organic matter content of 1.93%. Response surface and contour analyses confirmed that near-neutral pH combined with elevated organic matter levels provides optimal agronomic conditions for sugarcane production under irrigated low-veld systems. The findings demonstrate the practical applicability of RSM as a precision optimization tool in structured commercial sugarcane production systems and underscore the importance of integrated soil fertility management for sustainable yield improvement in semi-arid environments.*

KEYWORDS: Response Surface Methodology, Steepest Ascent, Optimization, Central Composite Design (CCD).

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INTRODUCTION

Sugarcane is a key cash crop in Zimbabwe, playing a significant role in both the national economy and the agricultural sector. In Zimbabwe, sugarcane is cultivated under canal irrigation in the low-veld region, an area at an altitude of just under 610 meters above sea level, specifically in Triangle and Hippo Valley, located in the Chiredzi District of Masvingo Province. With global demand for sugar on the rise, improving sugarcane yields has become increasingly crucial for enhancing food security, supporting farmer livelihoods, and promoting sustainable agriculture.

Sugarcane productivity in Zimbabwe is affected by several agronomic factors, including soil fertility, water availability, pest infestations (such as Eldana, Sugarcane Yellow Aphid, and Black Maize Beetle), and diseases (including Smut, Ratoon Stunt Disease, Leaf Scald, Brown Rust, Orange Rust, and Sugarcane Yellow Leaf). Additionally, crop management practices, the extent of land under cultivation, and the specific sugarcane varieties used are all important determinants of yield. Response Surface Methodology is best suited for modelling continuous, controllable input variables within a constrained experimental design. Given the small sample size of thirteen plots and the need to avoid over-parameterization, the study adopted a parsimonious model focusing on key soil-related variables that exhibited measurable variation across plots. The exclusion of additional agronomic, biotic, and management factors was therefore guided by the statistical considerations (degrees of freedom and over fitting risk), the limited variability in certain factors and the non-availability of data on those factors.

Masood et al. (2011) note that declining yields are often linked to issues such as low-performing varieties, water shortages, degraded soil fertility, poorly timed planting, and ineffective pest and weed control. Fertilizer application and plant density are also critical to maximizing yield, with nitrogen fertilization, in particular, being essential for sugarcane growth. Despite this knowledge, many farmers still rely on traditional farming techniques that may not achieve optimal productivity, highlighting the urgent need for the adoption of improved and innovative agronomic practices.

Response Surface Methodology (RSM) is a robust statistical tool used to explore the relationships between multiple variables and their interactions. It comprises a suite of mathematical and statistical techniques for designing experiments aimed at identifying optimal values of independent variables that enhance one or more desired outcomes (Myers et al., 2016; Montgomery, 2019). RSM is particularly valuable in modelling complex processes where interaction and quadratic effects are expected, enabling researchers to efficiently determine optimal operating conditions while minimizing experimental runs. According to Myers et al. (2009), RSM is instrumental in product formulation and process optimization. This study leverages RSM to systematically evaluate how soil pH and organic matter impact sugarcane yield. The goal is to generate practical, evidence-based recommendations to guide farmers in adopting best practices tailored to their specific conditions.

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MATERIALS AND METHODS

Study Area

This study was conducted at Triangle, Hippo Valley, Mwenezana and Mkwasine Estates in Chiredzi District, Masvingo Province, located in the low-veld region of Zimbabwe. The area is characterized by low and highly variable summer rainfall, typically receiving less than 450 mm annually, along with high temperatures averaging around 35°C. Sugarcane is thus grown under irrigation, which supports optimal growth within a 12-month cycle. Approximately 80 percent of Zimbabwe's sugarcane is produced by two major estates, Triangle and Hippo Valley, which are owned by the South Africa-based Tongaat Hulett Company. The remaining 20 percent is contributed by private growers, including both large-scale and newly resettled farmers. Estate sugarcane production is typically characterized by centralised and standardized agronomic protocols, controlled irrigation systems, uniform input application, established varietal recommendations, and structured disease management programs. In such systems, variability in management practices is reduced, which enhances the suitability of RSM. Since RSM performs best under controlled and measurable input conditions, estate-managed fields provide an environment where response surfaces can be estimated with greater precision because noise from uncontrolled variables is minimized. Therefore, the dominance of large estates supports the methodological relevance of RSM as a tool for fine-tuning input combinations within already structured production systems.

Sampling, Data Collection and Analysis

Understanding the factors that influence crop yield is essential for optimizing agricultural production. To this end, a ground-truthing exercise was conducted to collect sugarcane production data, using a stratified random sampling method to ensure representative coverage across different levels of production. The sample of thirteen farmers was used primarily due to methodological, logistical, and contextual considerations. The plots were selected within a relatively homogeneous irrigated production system in Chiredzi, reducing environmental variability and allowing meaningful estimation of key effects with fewer observations. Additionally, limited variability in certain management factors and restricted access to farmer-managed land constrained sample expansion. Given the objective of testing the feasibility of RSM rather than producing population-level estimates, a parsimonious design with thirteen carefully measured plots was adopted as a proof-of-concept framework to inform future, larger-scale studies (Myers et al., 2016; Montgomery, 2019). In experimental and agronomic research, small but well-controlled designs are often appropriate when the primary aim is methodological validation rather than statistical generalization (Kozak & Piepho, 2018). This approach allowed the sample to accurately reflect the variability in sugarcane output among farmers while maintaining experimental efficiency (Box et al., 2016). Stratification was based on historical yield levels, enabling targeted data collection across a range of production capacities, a strategy commonly recommended to improve representativeness and precision in agricultural field experiments (FAO, 2021).. Farmers were grouped according to their past sugarcane yields, which guided the selection of plots for the study. Out of 27 identified sugarcane plots, thirteen were randomly selected (Trianlge (4), Hippo Valley (3), Mkwasine (3), Mwenezana (3)). Each of these plots covered between 20 and 24 hectares, offering a comprehensive snapshot of the production landscape.

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Data collection focused on the 2023/24 sugarcane season and included yield (Y), soil pH (X_1), soil organic matter(X_2) and organic matter content (X_3). Actual yield data was obtained from local milling facilities where farmers deliver their harvests, ensuring accuracy and consistency. Soil pH levels were measured in each plot, given their significant influence on sugarcane growth. Additionally, the total cultivated area per plot and the amount of organic matter present in the soil were recorded, as both are critical factors affecting soil fertility and overall yield. Including area planted as a variable provided a comprehensive view of the factors influencing sugarcane yield, leading to more effective optimization strategies tailored to the specific conditions of Chiredzi District. Additionally, the interaction of agronomic inputs and yield may differ based on area planted. Area planted was also crucial for resource allocation, for better statistical modelling of yield responses and was believed to have an impact on overall yield potential.

After data collection, various analytical techniques were applied to the data. Multiple regression models were developed to assess the influence of soil pH, organic matter content, and planted area on sugarcane yield. Response surface plots were used to visualize the interactions between soil pH and organic matter, helping to identify optimal conditions for maximizing yield. Contour plots were also generated to graphically represent how different combinations of pH and organic matter levels affect production. These visual tools provided intuitive insights into the relationships among the variables, supporting more informed decision-making for yield optimization.

Statistical Methods

Response Surface Methodology was used to optimize sugarcane yield (tons per hectare) given the soil pH and organic matter for the thirteen selected plots. Response Surface Methodology, as a set of statistical techniques, is applicable in the design, development, and formulation of new products, as well as the improvement of existing product designs. One of the most widespread applications of these techniques is to model and analyse problems in which a response of interest (there may be more than one) is influenced by several quantitative factors, the objective being to optimize this response by determining the optimal values of the factors involved (Montgomery, 2004). The relationship will be given by:

$$y = f(x_1, x_2) + \varepsilon \quad (1)$$

which is assumed to be continuous and ε represents the noise or error observed in the response, whose distribution is assumed to be normal with zero mean. The variables in the equation (1) are natural predictor variables since they are expressed in the natural units of measurement. y is a function of x_1 and x_2 However, it is common to transform them into coded variables without dimensions but with zero mean and the same standard deviation. Thus, the actual value expected to be taken by the response variable implies a relationship that can be represented by a hyper-surface called response surface

$$\eta = f(x_1, x_2, \dots, x_k) \quad (2)$$

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The Central Composite Design (CCD)

A Central Composite Design is a response surface design which apart from the 3 level factors has axial or star points that allow for the estimation of the curvature effect. The axial or star point increases the number of levels to 5 levels. The design allows the experimental designer to know what effect the factors have on the response variable if the experimental designer goes beyond or below the chosen levels of factors. It also provides high quality predictions of linear and quadratic interaction effects of parameters affecting the process. The minimum numbers of numerical or continuous factors a CCD can accommodate is two. The number of experimental runs obtained at each number of factors is given by the formula

$$N = 2^n + 2n + n_0 \qquad (3)$$

where N is the number of runs,

n is the number of factors and

n_0 is the number of centre points the designer desires.

The Central Composite Design (CCD) was employed in the study to optimize sugarcane yield by analysing the effects of agronomic factors, soil pH, area planted, and soil organic matter content. A design matrix was established incorporating different levels for the factors, allowing systematic exploration of their impact on yield. Data collection involved multiple trials across the thirteen plots, with sugarcane yield recorded as the primary response variable. A second-order polynomial model was fitted to the yield data, aided by regression analysis and ANOVA to assess the significance of each factor and their interactions. Response surface plots were generated to visualize the relationships between the factors and yield, helping identify optimal conditions for cultivation.

RESULTS AND DISCUSSION

Model building

Area planted, organic matter content, and soil pH were identified as key variables potentially influencing sugarcane yield. These factors were selected based on their agronomic relevance and incorporated into a linear regression model to evaluate their individual and combined effects on yield. Land size was considered to assess whether the scale of cultivation had a direct relationship with productivity, while organic matter and soil pH were included due to their known impact on soil fertility and crop growth. By fitting these variables into the regression model, the study aimed to determine the extent to which each factor contributes to variations in sugarcane yield and to identify which parameters are statistically significant in predicting higher productivity.

The results, presented in Table 1, indicate the Analysis of Variance (ANOVA) for soil pH, organic matter and area planted as well as their statistical significance shown by their p-values.

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Table 1: Regression Analysis of Variance: Yield versus pH, Organic Matter, Area planted

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	529.871	176.624	15.56	0.001
pH	1	127.593	127.593	11.24	0.008
Organic Matter	1	93.486	93.486	8.23	0.018
Area planted	1	2.670	2.670	0.24	0.639
Error	9	102.179	11.353		
Total	12	632.049			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
3.36945	83.83%	78.45%	66.61%

Table 2: Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-5.2	31.6	-0.16	0.874	
pH	7.98	2.38	3.35	0.008	1.00
Organic Matter	12.34	4.30	2.87	0.018	3.27
Area planted	0.00152	0.00314	0.48	0.639	3.27

The analysis of variance indicated that the overall regression model was statistically significant ($F = 15.56$, $p = 0.001$), demonstrating that the included predictors collectively explain a substantial proportion of the variability in sugarcane yield. The model accounted for 83.83% of the total variation ($R^2 = 0.8383$), with an adjusted R^2 of 78.45%, indicating strong explanatory power after accounting for model complexity. The predicted R^2 (66.61%) suggests reasonable external predictive capability, although some reduction relative to R^2 indicates moderate shrinkage when applied to new data. The residual standard error ($S = 3.37$) further indicates acceptable model precision within the experimental domain.

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Examination of individual predictors revealed that soil pH had a statistically significant positive effect on yield ($F = 11.24$, $p = 0.008$). The estimated coefficient ($\beta = 7.98$) indicates that, holding other factors constant, a one-unit increase in pH is associated with an average increase of approximately 7.98 yield units. Similarly, organic matter exerted a significant positive effect ($F = 8.23$, $p = 0.018$), with a coefficient of 12.34, suggesting a stronger marginal contribution to yield relative to pH within the observed range. In contrast, area planted was not statistically significant ($F = 0.24$, $p = 0.639$), indicating that field size does not materially influence yield under the conditions of this study. This confirms that productivity differences are primarily driven by soil quality attributes rather than scale of production. Variance inflation factors (VIFs) were below commonly accepted thresholds, indicating no serious multicollinearity concerns among predictors. The non-significant intercept ($p = 0.874$) has no substantive agronomic interpretation and does not affect inference regarding the explanatory variables.

Overall, the results demonstrate that sugarcane yield is significantly influenced by soil pH and organic matter content, while area planted does not contribute meaningfully to yield variation. These findings underscore the importance of soil fertility management in enhancing productivity, with organic matter improvements offering comparatively larger marginal gains than pH adjustments within the studied range.

Table 3: Response Surface Regression: Yield versus coded X_1 , X_2

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	5	617.175	123.435	58.09	0.000
Linear	2	526.866	263.433	123.98	0.000
X_1	1	129.316	129.316	60.86	0.000
X_2	1	397.550	397.550	187.09	0.000
Square	2	90.219	45.110	21.23	0.001
X_1^2	1	50.987	50.987	24.00	0.002
X_2^2	1	50.987	50.987	24.00	0.002
$X_1 \times X_2$	1	0.090	0.090	0.04	0.843
Error	7	14.874	2.125		
Lack-of-Fit	3	14.702	4.901	113.97	0.000
Pure Error	4	0.172	0.043		
Total	12	632.049			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
1.45769	97.65%	95.97%	83.42%

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Table 4: Coded Coefficients

Term	Effect	Coef SE	Coef	T-Value	P-Value	VIF
Constant	86.940	0.652	133.36		0.000	
X_1	11.371	5.685	0.729	7.80	0.000	1.00
X_2	19.937	9.969	0.729	13.68	0.000	1.00
X_1^2	-10.83	-5.41	1.11	-4.90	0.002	1.02
X_2^2	-10.83	-5.41	1.11	-4.90	0.002	1.02
$X_1 \times X_2$	-0.60	-0.30	1.46	-0.21	0.843	1.00

Fits and Diagnostics for Unusual Observations

Obs	Yield	Fit	Resid	Resid
1	72.400	70.303	2.097	2.35 R

The regression equation in coded units was

$$Yield = 86.94 + 4.021X_1 + 7.050X_2 - 2.708X_1^2 - 2.708X_2^2 - 0.150X_1X_2 \ldots\ldots\ldots\ldots(4)$$

where X_1 and X_1 denote the coded levels of the two significant experimental factors pH and organic matter.

The fitted second-order response surface model, equation (4), was highly significant ($F = 58.09$, $p < 0.001$), explaining 97.65% of the total variation in yield ($R^2 = 0.9765$). The adjusted R^2 (95.97%) confirms excellent explanatory strength after accounting for model terms, while the predicted R^2 (83.42%) indicates good predictive capability within the experimental domain. The residual standard error was low ($S = 1.46$), demonstrating high model precision.

Significance of Model Components

Partitioning of the model into linear, quadratic, and interaction components revealed that the linear effects were highly significant ($F = 123.98$, $p < 0.001$), accounting for the majority of explained variation. Both factors contributed significantly:

$$X_1 (F = 60.86, p < 0.001) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

$$X_2 (F = 187.09, p < 0.001) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The magnitude of the coded coefficients indicates that X_2 exerted a stronger linear influence on yield than X_1, suggesting greater sensitivity of yield to changes in X_2 within the studied range. The quadratic (square) terms were also statistically significant ($F = 21.23$, $p = 0.001$), with both X_1^2 and X_2^2 significant ($p = 0.002$). The negative coefficients for these terms (−2.708 in the regression equation) indicate concave curvature, confirming the presence of a true maximum within the design space rather than a boundary optimum. In contrast, the two-way interaction (X_1X_2) was not significant ($F = 0.04$, $p = 0.843$), indicating that the combined effect

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of X_1 and X_2 does not deviate significantly from additivity. This suggests that each factor primarily influences yield independently within the studied region.

Regression Model

The positive linear coefficients and negative quadratic coefficients in the fitted response surface equation (4) in coded units confirm a rising response followed by diminishing returns, characteristic of an agronomically realistic optimum. Variance inflation factors (VIF ≈ 1.0) indicate absence of multicollinearity, ensuring stable coefficient estimates. Despite the strong overall fit, the lack-of-fit test was statistically significant ($F = 113.97$, $p < 0.001$). The very small pure error (MS = 0.043) relative to lack-of-fit mean square (4.901) suggests that the experimental measurements were highly precise, making even small model deviations statistically detectable. This may indicate minor unexplained curvature or structural variability not captured by the quadratic model, although the high R^2 and low residual error suggest that the model remains practically adequate for optimization purposes.

Diagnostic Assessment

One observation showed a moderately large standardized residual (2.35), flagged as unusual. However, it does not exceed typical critical thresholds (±3), and therefore does not indicate severe outlier influence. The second-order response surface model adequately described yield behaviour across the experimental region, with highly significant linear and quadratic effects and negligible interaction. The negative quadratic terms confirm the existence of an interior optimum, validating the application of response surface methodology for optimization. Although the lack-of-fit test was significant due to very low pure error, the model demonstrates strong predictive performance and practical suitability for determining optimal factor levels.

Optimization using the Method of Steepest Ascent

The method of steepest ascent is an efficient optimization technique for complex processes used to find the optimal settings for process variables based on iteratively adjusting input variables in the direction of the steepest ascent to maximize a response variable. Common designs used in the Steepest Ascent method include Factorial Designs and Fractional Factorial Designs, and the method is especially useful in industrial settings with complex processes and multiple variables for optimizing discrete simulations of complex industrial plants. When starting with a linear model, RSM uses the path of steepest ascent (for maximization) or steepest descent (for minimization) to move toward optimal regions (Myers et al., 2022). This then calls for careful production of experimental designs.

The core steps involve identifying the response variable, screening input variables, fitting a first-order model, calculating the steepest ascent direction, and conducting experiments along that path. If x_0 is the initial approximation of the maximum point then x_1 is obtained by a positive multiple of ∇f_{x_0} to x_0. This is repeated for $x_2,\ x_3, x_4$ .... we summarize the method by

$$x_{n+1} = x_n + \alpha \nabla f_{x_n} \tag{7}$$

However, there are challenges anticipated with this method. Determining when to stop the search is crucial. Researchers have studied stopping rules to optimize the process and avoid wasting resources. The selection of an appropriate step size is crucial for the algorithm's

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stability and convergence. As the experimental region approaches optimal conditions, curvature and interactions among factors may become more prevalent.

Using the second order model developed in (4) an optimal solution was estimated in table 5 below. This was done to find the combinations of pH and organic matter that maximize sugarcane yield and the predictions of the yield too. Because both quadratic coefficients (−2.708-2.708−2.708) are negative, the response surface is concave, indicating the existence of a stationary maximum. The gradient vector was derived from the first-order partial derivatives:

$$\frac{\partial Yield}{\partial X_1} = 4.021 - 5.416X_1 - 0.150X_2 \quad (8)$$

$$\frac{\partial Yield}{\partial X_2} = 7.050 - 5.416X_2 - 0.1501 \quad (9)$$

At (0,0) $\nabla Yield(0,0) = (4.021, 7.050)$

Eleven iterations of the method of steepest ascent were done with a step size of $\propto = 0.1$ starting at point $(X_1^0, X_2^0) = (0,0)$.

Table 5: Steepest Ascent Optimal Point Estimation

Step	pH(x_1)	Organic Matter (x_1)	Yield
0	0.000	0.000	86.94
1	0.0495	0.0869	87.66
2	0.0990	0.1738	88.34
3	0.1485	0.2607	88.97
4	0.1980	0.3476	89.54
5	0.2475	0.4345	90.05
6	0.2970	0.5214	90.48
7	0.3465	0.6083	90.83
8	0.3960	0.6952	91.08
9	0.4455	0.7821	91.23
10	0.4950	0.8690	91.27
11	0.5445	0.9559	91.18

The decoding formula is:

$$X_{natural} = X_0 + (x \times \Delta) \quad (10)$$

Soil pH Conversion

$$pH_{optimal} = 6.8 + (0.485 \times 0.5) = 6.8 + 0.2425 = 7.0425 = 7.04$$

Organic Matter Conversion

$$Organic\ Matter_{opt} = 1.5 + (0.869 \times 0.5) = 1.5 + 0.4345 = 1.9345 = 1.93\%$$

Optimization of the fitted quadratic response surface indicated that maximum predicted sugarcane yield (91.27) is achieved at a soil pH of approximately 7.04 and an organic matter content of 1.93%. These results suggest that peak productivity occurs under slightly neutral

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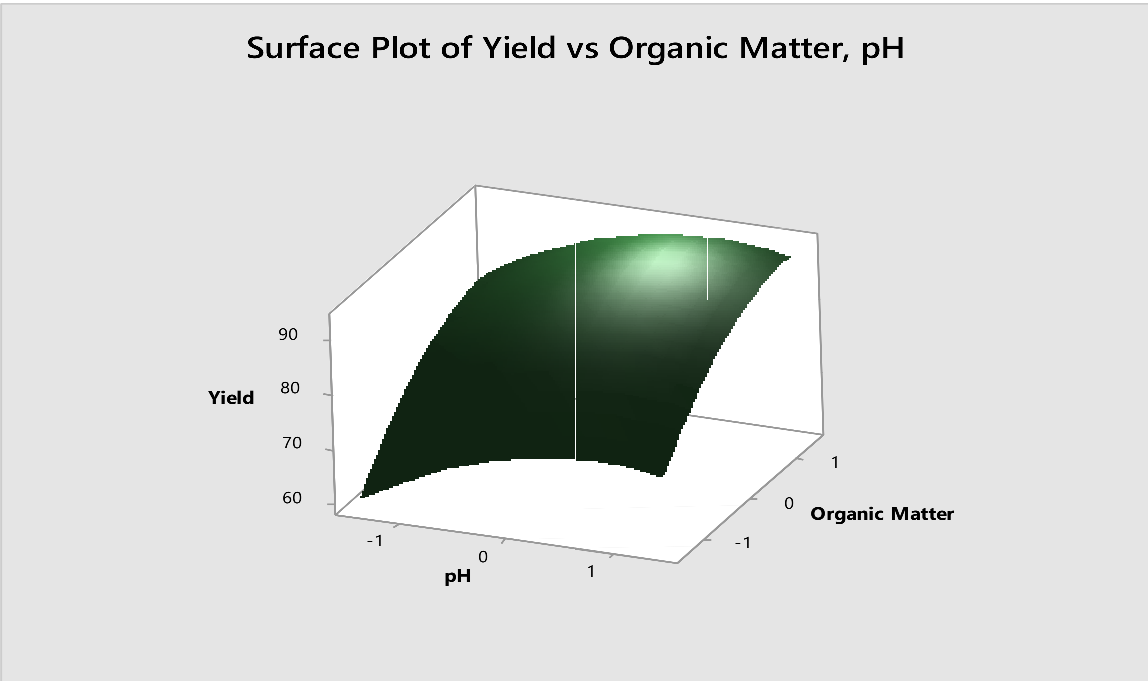
soil conditions combined with elevated organic matter levels approaching 2%. The proximity of the optimal pH to neutrality aligns with established agronomic understanding that nutrient availability, particularly phosphorus, calcium, and magnesium, is maximized within a narrow neutral range, while extreme acidity or alkalinity constrains uptake efficiency.

The comparatively higher coded displacement for organic matter indicates that yield response is more sensitive to improvements in soil organic carbon than to marginal pH adjustments within the experimental domain. Increasing organic matter enhances soil structure, water-holding capacity, microbial activity, and nutrient retention, all of which are critical for sustained sugarcane growth in semi-arid production systems. The concave nature of the response surface further confirms diminishing returns beyond this range, implying that excessive amendments may not proportionally increase yield. Collectively, the results demonstrate that integrated soil fertility management strategies emphasizing organic matter enhancement, alongside maintenance of near-neutral pH, provide the greatest leverage for optimizing sugarcane productivity within the studied agro-ecological conditions.

Response Surface Plots

Data for the two operating parameters, soil pH and Organic matter content were taken and a three-dimensional (3D) plot produced as shown in Figure 1 below:

Figure 1: Surface Plot of Yield versus Organic Matter, pH



The response surface illustrates a clear nonlinear relationship between yield, soil pH, and organic matter content. Yield increases markedly as both pH and organic matter move from low (−1 coded units) toward moderate-to-high levels, indicating strong positive main effects within the studied range. The curvature of the surface suggests the presence of quadratic

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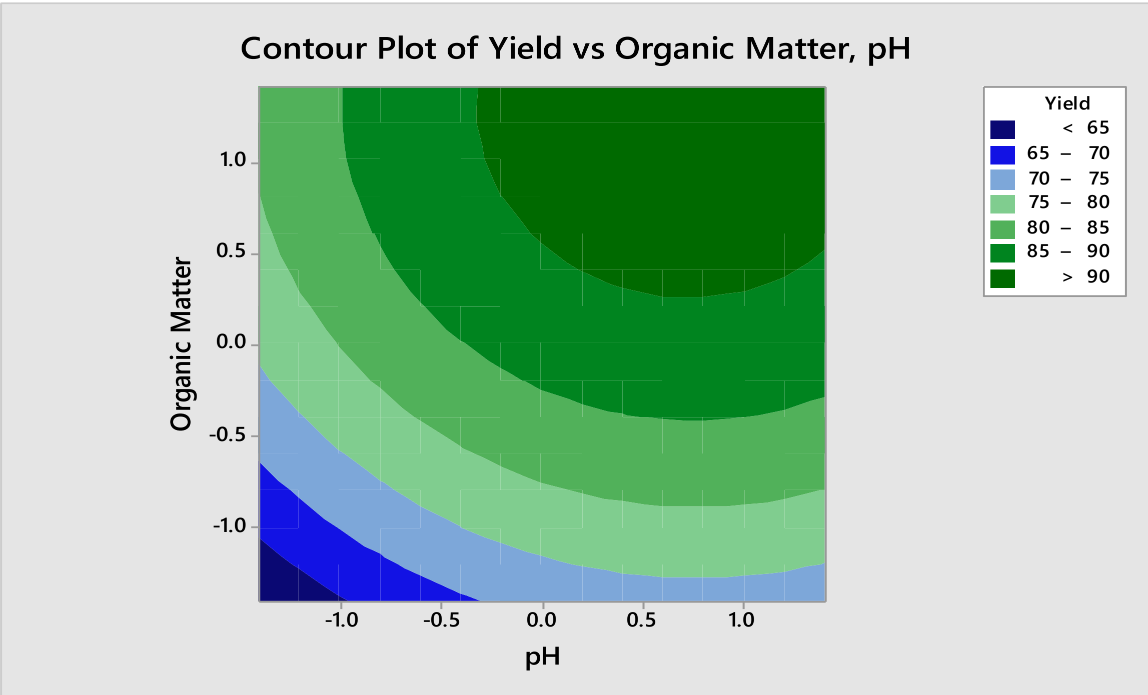
effects, as the rate of yield increase diminishes at higher levels of both factors, forming a plateau region rather than a sharp peak.

The maximum yield is observed in the upper region of the design space, where both pH and organic matter are at relatively high coded levels (approximately +0.7 to +1.0). At this combination, predicted yield approaches approximately 92 units, representing the optimal response within the experimental domain. The surface does not show a pronounced saddle shape; instead, it indicates a synergistic improvement in yield when both factors are increased simultaneously. In practical terms, the optimal agronomic condition corresponds to maintaining soil pH towards the upper end of the tested range together with elevated organic matter content.

Response surface contours

Figure 2 is a contour plot illustrating the relationship between Yield (Y) and the two factors: pH and Om (organic matter).

Figure 2: Contour Plot of Yield versus Organic Matter, pH



The contour plot illustrates the interactive effects of soil pH (X_1) and organic matter content (X_2) on sugarcane yield. The response surface demonstrates a clear collaborative relationship between the two factors, with yield increasing progressively as both pH and organic matter move from low to moderate–high levels. The elliptical contour patterns indicate the presence of significant interaction and quadratic effects, suggesting that yield response is not purely linear but characterized by curvature typical of second-order response surface models.

The lowest yields (< 70 units) are observed in the lower-left region of the plot, corresponding to simultaneously low pH and low organic matter levels. Yield improves steadily as either factor increases. However, the greatest gains occur when both variables increase together, confirming a positive interaction effect. The high yield region (> 90 units) is concentrated in

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the upper-right quadrant of the design space, where soil pH ranges approximately between +0.6 and +1.2 (coded units) and organic matter ranges between +0.4 and +1.2 (coded units).

The optimal region appears cantered approximately at a Soil pH (coded) of approximately +0.8 to +1.0, (6.9 to 7.0 in natural units), organic matter (coded) approximately +0.6 to +1.0 (1.8% to 2.0% in natural units) and a predicted maximum yield greater than 90 (approximately 91–92 units)

This indicates that sugarcane yield is maximized under moderately high pH and elevated organic matter conditions within the experimental region. The absence of declining contours at the upper boundary suggests that the stationary point lies near the upper edge of the design space, implying that further incremental increases in pH and organic matter (within agronomic feasibility limits) may sustain or slightly improve yield.

Agronomically, the results imply that yield optimization requires simultaneous soil pH correction and organic matter enhancement. Managing only one factor is unlikely to achieve the maximum attainable yield, as the contour geometry demonstrates strong interdependence between the two variables. Overall, the contour surface confirms that optimal sugarcane productivity occurs under a balanced combination of moderately alkaline pH conditions and high organic matter content, with predicted yields exceeding 90 units at the optimum region.

CONCLUSIONS

This study applied Response Surface Methodology (RSM) to model and optimize sugarcane yield as a function of soil pH and organic matter content under irrigated low-veld conditions in Chiredzi District, Zimbabwe. The fitted second-order model was highly significant (R^2 = 97.65%), demonstrating strong explanatory and predictive capability within the experimental domain. Both soil pH and organic matter exerted significant positive linear effects on yield, while their negative quadratic terms confirmed the presence of diminishing returns and the existence of a true interior optimum. Area planted was not statistically significant, indicating that productivity differences were primarily driven by soil quality attributes rather than scale of cultivation.

Optimization using the method of steepest ascent identified maximum predicted yield (approximately 91.27 tonnes per hectare) at a soil pH of approximately 7.04 and an organic matter content of 1.93%. These results indicate that near-neutral soil conditions combined with elevated organic matter levels provide the most favourable agronomic environment for maximizing sugarcane productivity under estate-managed irrigation systems. The concave response surface further suggests that excessive adjustments beyond this range are unlikely to generate proportional yield gains.

Despite the small but carefully controlled sample of thirteen plots, the study demonstrates the practical applicability of RSM as a precision optimization tool in structured commercial production systems. The findings provide empirical evidence that targeted soil fertility management, rather than expansion of cultivated area, offers the most effective strategy for yield enhancement in Zimbabwe’s low-veld sugarcane sector.

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RECOMMENDATIONS

The study recommends that

a) Farmers and estate managers should maintain soil pH within the range of approximately 6.9–7.1 through appropriate liming programs and regular soil testing to optimize nutrient availability and uptake efficiency.

b) Organic matter levels should be increased and sustained at approximately 1.8–2.0% through integrated soil fertility management practices, including residue retention, compost application, green manuring, and incorporation of organic amendments.

c) Soil pH correction and organic matter improvement should be implemented simultaneously, as optimizing only one factor is unlikely to achieve maximum yield potential.

d) Given the non-significance of area planted, productivity gains should focus on improving soil conditions rather than expanding cultivated acreage.

e) Estates and private growers should implement systematic soil testing programs to guide data-driven input adjustments and maintain optimal conditions across production cycles.

f) Future research should expand the sample size and incorporate additional agronomic variables like irrigation scheduling, nitrogen rates, varietal differences, pest pressure to refine and validate the response surface model under broader field conditions.

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