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ENSEMBLING CAPUTO, CAPUTO–FABRIZIO AND ATANGANA–BALEANU FRACTIONAL-ORDER MODELS TO STUDY HIV DYNAMICS IN NIGERIA

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ABSTRACT: *This study investigates HIV dynamics in Nigeria using fractional-order epidemic models formulated with Caputo, Caputo–Fabrizio, and Atangana–Baleanu fractional derivatives, together with an ensemble framework integrating their predictions. A five-compartment SEIDT model representing susceptible, exposed, infected, diagnosed, and treated populations is developed to incorporate memory effects inherent in HIV transmission and treatment. Numerical simulations covering 1990–2030 are conducted using parameter values from the literature and Nigerian demographic data. Results show that the Caputo–Fabrizio model produces rapid early responses, the Atangana–Baleanu model exhibits improved long-term stability, and the ensemble model provides conservative and robust forecasts. Sensitivity analysis confirms the stabilizing advantage of the ensemble approach. These findings highlight the relevance of fractional-order models for realistic HIV forecasting and public health planning in Nigeria.*

KEYWORDS: HIV dynamics, Fractional calculus, Caputo derivative, Caputo–Fabrizio derivative, Atangana–Baleanu derivative, Ensemble modeling, Nigeria.

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INTRODUCTION

Human Immunodeficiency Virus (HIV) continues to pose a formidable global public health challenge, with Nigeria experiencing a disproportionately high burden of the epidemic (World Health Organization (WHO), 2020). Effective intervention strategies to mitigate transmission and improve health outcomes necessitate a deep understanding of the virus's complex transmission and progression dynamics. The impact of HIV/AIDS in Nigeria is multifaceted, extending beyond health to encompass significant socioeconomic dimensions. Studies have documented its role in exacerbating healthcare disparities(Akinyemi et al., 2021) , deepening poverty and disrupting families (Ijeoma, 2022), imposing psychosocial burdens such as stigma (Okeowo et al., 2022), creating economic barriers to treatment (Olatunji et al., 2023), affecting workforce productivity (Nwankwo et al., 2023), and contributing to adverse mental health outcomes among affected individuals (Eze et al., 2024).

Traditionally, the dynamics of infectious diseases like HIV have been studied using integer-order epidemiological models, such as the Susceptible-Infectious-Recovered (SIR) framework (Brauer et al., 2019). Beyond traditional compartmental models, advanced frameworks like reaction-diffusion systems are essential for capturing the spatial and biochemical complexities of HIV progression (Samuel & Gill, 2018b, 2018a). Research has demonstrated this through diffusion-chemotaxis and cross-diffusion models that analyze how stress hormones, such as cortisol and norepinephrine, modulate immune responses and viral dynamics. These studies underscore the importance of incorporating endocrine-immune interactions and spatial processes to fully understand HIV pathogenesis (Samuel et al., 2019). However, these conventional models are often limited in their ability to capture the memory-driven, non-local, and hereditary properties inherent in biological systems, leading to potential oversimplifications of complex disease behaviors (Ghaffari et al., 2022; Nascimento et al., 2023). Fractional-order calculus has emerged as a powerful alternative, offering a more nuanced mathematical framework that incorporates memory and history-dependent effects, thereby providing a more accurate representation of real-world disease dynamics (Alsaedi et al., 2021; Bakker et al., 2020). Advanced fractional operators, including the Caputo, Caputo-Fabrizio, and Atangana-Baleanu derivatives, have been successfully applied to model various infectious diseases, demonstrating superior capability in capturing complex transmission patterns (Atangana & Baleanu, 2020; Ghaffari et al., 2022; Santos et al., 2024).

Recent research highlights the growing application and comparative analysis of these fractional operators in HIV modeling. Studies have utilized fractional-order models to analyze HIV/AIDS dynamics (Unaegbu et al., 2021), explore co-infection scenarios (Naik et al., 2024), forecast epidemiological trends (Kumar et al., 2024), and assess stability (Ahmad et al., 2023). Comparative analyses further suggest that while different operators have distinct strengths—for instance, Caputo-Fabrizio may better fit transmission rates, whereas Atangana-Baleanu excels in scenarios involving memory and reinfection (Akinwande & Ojo, 2023; Bashir & Qureshi, 2022) -an ensemble approach that integrates multiple fractional models can significantly enhance predictive accuracy and provide a more comprehensive understanding (Ahmad et al., 2023; Zhang et al., 2022).

This ensemble approach is in line with contemporary trends in hybrid modeling, as demonstrated in related applications to tumor-immune responses (Gill et al., 2023), COVID-19 (Shikaa, 2024), anthrax (Shikaa et al., 2024), tuberculosis (Manu et al., 2025; Samson et al.,

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2025), and Monkeypox (Igiri & Shikaa, 2025), which integrate fractional calculus with computational techniques like neural networks for improved predictive accuracy.

Despite these advancements, there remains a need to systematically apply and evaluate an ensemble of leading fractional-order models specifically to understand HIV dynamics in Nigeria, a setting characterized by its significant epidemic burden. This study therefore aims to address this gap by developing and simulating an ensemble model combining the Caputo, Caputo-Fabrizio, and Atangana-Baleanu fractional-order frameworks. The objective is to provide a refined approach to understanding HIV transmission dynamics in Nigeria, leading to optimal control and management of the disease.

BASIC CONCEPTS

The following definitions are useful in this study:

Definition 1: Let $\mathrm{h}: [a,b] \to \mathbb{R}, n-1 < \theta \leq n$, and $n \in \mathbb{N}$. The Caputo fractional derivative of order θ is defined by

$$ {}_{a}^{\mathrm{C}}\mathcal{D}_t^{\theta}\mathrm{h}(t) = \frac{1}{\Gamma(n-\theta)}\int_a^t \mathrm{h}^{(n)}(\gamma)(t-\gamma)^{n-\theta-1}d\gamma. \tag{1}$$

The corresponding fractional integral is given by

$$ {}_{a}^{\mathrm{C}}I_t^{\theta}\mathrm{h}(t) = \frac{1}{\Gamma(\theta)}\int_a^t \mathrm{h}(\gamma)(t-\gamma)^{\theta-1}d\gamma. \tag{2}$$

Definition 2: Let $\mathrm{h} \in H^1(a,b)$ and $0 < \theta < 1$. The Caputo-Fabrizio (CF) fractional derivative is defined as

$$ {}_{a}^{\mathrm{CF}}\mathcal{D}_t^{\theta}\mathrm{h}(t) = \frac{1}{1-\theta}\int_a^t \frac{d\,\mathrm{h}(\gamma)}{d\gamma}e^{-\beta(t-\gamma)}d\gamma, \tag{3}$$

where $\beta = \frac{\theta}{1-\theta}$.

The corresponding CF fractional integral is given by

$$ {}_{a}^{\mathrm{CF}}I_t^{\theta}\mathrm{h}(t) = (1-\theta)\mathrm{h}(t) + \theta\int_a^t \mathrm{h}(\gamma)d\gamma. \tag{4}$$

Definition 3: Let $\mathrm{h} \in H^1(a,b)$ and $0 < \theta < 1$. The Atangana-Baleanu (AB) fractional derivative of order θ is defined by

$$ {}_{a}^{\mathrm{AB}}\mathcal{D}_t^{\theta}\mathrm{h}(t) = \frac{\mathrm{B}(\theta)}{1-\theta}\int_a^t \frac{d\,\mathrm{h}(\gamma)}{d\gamma}E_\theta\left[-\beta(t-\gamma)^\theta\right]d\gamma \tag{5}$$

where $\mathrm{B}(\theta)$ is the normalization function with $\mathrm{B}(0) = \mathrm{B}(1) = 1$, $\beta = \frac{\theta}{1-\theta}$ and $E_\theta(\cdot)$ denotes the one-parameter Mittag-Leffler function,

$$ E_\theta(z) = \sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\theta n+1)}, \theta > 0, z \in \mathbb{C} \tag{6}$$

For $\theta = 1$, this function reduces to the classical exponential function,

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$E_1(z) = e^z$

The corresponding AB fractional integral is given by

$${}_{a}^{\mathrm{AB}}I_t^{\theta}\mathrm{h}(t) = \frac{1-\theta}{\mathrm{B}(\theta)}\mathrm{h}(t) + \frac{\theta}{\mathrm{B}(\theta)\Gamma(\theta)}\int_a^t \mathrm{h}(\gamma)(t-\gamma)^{\theta-1}d\gamma \quad (7)$$

METHODOLOGY

Integer-Order System

A deterministic SEIDT compartmental model by Priya & Ganesan, (2023) is considered to describe HIV transmission dynamics in Nigeria. The total population $N(t)$ is subdivided into susceptible $S(t)$, exposed $E(t)$, infected $I(t)$, diagnosed $D(t)$, and treated $T(t)$ individuals. The model incorporates recruitment into the susceptible class, HIV transmission, disease progression, diagnosis, treatment initiation, treatment discontinuation, and both natural and disease-induced mortality.

The integer-order model governing the dynamics of the population is given by:

$$\left.\begin{aligned}
\frac{\mathrm{ds}}{\mathrm{dt}} &= \alpha\mathrm{N} - (\beta\mathrm{I} + \delta)\mathrm{S} \\
\frac{\mathrm{dE}}{\mathrm{dt}} &= \beta\mathrm{SI} - \gamma\mathrm{E} - \delta\mathrm{E} \\
\frac{\mathrm{dI}}{\mathrm{dt}} &= \gamma\mathrm{E} - (\omega + \delta + \mu)\mathrm{I} \\
\frac{\mathrm{dD}}{\mathrm{dt}} &= \omega\mathrm{I} - (\delta + \mu + \eta)\mathrm{D} + \lambda\mathrm{T} \\
\frac{\mathrm{dT}}{\mathrm{dt}} &= \eta\mathrm{D} - (\varepsilon + \lambda + \mu + \delta)\mathrm{T}
\end{aligned}\right\} \quad (8)$$

with initial conditions; $\mathrm{S} \geq 0,\ \mathrm{E} \geq 0,\ \mathrm{I} \geq 0,\ \mathrm{D} \geq 0,\ \mathrm{T} \geq 0.$

The integer-order system is extended to fractional order by replacing the classical derivative with Caputo, Caputo–Fabrizio, and Atangana–Baleanu fractional derivatives of order $0 < \theta \leq 1$.

Fractional-Order System

Let $0 < \theta < 1$ and let ${}_{a}^{*}D_t^{\theta}$ denote any of the fractional derivatives ${}_{a}^{\mathrm{C}}D_t^{\theta}$, ${}_{a}^{\mathrm{AB}}D_t^{\theta}$, or ${}_{a}^{\mathrm{CF}}D_t^{\theta}$ defined in Equations (1), (3) and (5). The fractional-order model is given by

$$\begin{aligned}
{}_{a}^{*}D_t^{\theta}S(t) &= \alpha N - (\beta I(t) + \delta)S(t), \\
{}_{a}^{*}D_t^{\theta}E(t) &= \beta S(t)I(t) - (\gamma + \delta)E(t), \\
{}_{a}^{*}D_t^{\theta}I(t) &= \gamma E(t) - (\omega + \delta + \mu)I(t), \\
{}_{a}^{*}D_t^{\theta}D(t) &= \omega I(t) - (\delta + \mu + \eta)D(t) + \lambda T(t), \\
{}_{a}^{*}D_t^{\theta}T(t) &= \eta D(t) - (\varepsilon + \lambda + \mu + \delta)T(t).
\end{aligned} \quad (9)$$

with initial conditions; $\mathrm{S} \geq 0,\ \mathrm{E} \geq 0,\ \mathrm{I} \geq 0,\ \mathrm{D} \geq 0,\ \mathrm{T} \geq 0.$

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Ensembled Model

An ensemble model is subsequently formulated as a weighted combination of the three fractional-order models, namely the Caputo, Atangana–Baleanu, and Caputo–Fabrizio formulations, which are presented in compact form in Equation (9) and denoted by ${}_{a}^{\mathrm{C}}D_{t}^{\theta}$, ${}_{a}^{\mathrm{AB}}D_{t}^{\theta}$, and ${}_{a}^{\mathrm{CF}}D_{t}^{\theta}$ respectively.

$$\hat{\mathrm{y}}_{\mathrm{t}} = \sum_{\mathrm{i}=1}^{3} \mathrm{w}_{\mathrm{i}}\hat{\mathrm{y}}_{\mathrm{t,i}} \tag{10}$$

where $\hat{\mathrm{y}}_{\mathrm{t,i}}$ is prediction from the i-th model and w_{i} is weight assigned to the i-th model.

Let the state variables associated with each fractional formulation be denoted by S_i, E_i, I_i, D_i, and T_i, where $i = 1$ corresponds to the Caputo model, $i = 2$ to the Caputo-Fabrizio model, and $i = 3$ to the Atangana-Baleanu model. The ensemble state of each epidemiological compartment is then defined as a weighted average of the corresponding states across the three fractional models.

$$\left.\begin{aligned}
\mathrm{S}_{\mathrm{ensemble}}(\mathrm{t}) &= \mathrm{w}_1\mathrm{S}_1(\mathrm{t}) + \mathrm{w}_2\mathrm{S}_2(\mathrm{t}) + \mathrm{w}_3\mathrm{S}_3(\mathrm{t})\\
\mathrm{E}_{\mathrm{ensemble}}(\mathrm{t}) &= \mathrm{w}_1\mathrm{E}_1(\mathrm{t}) + \mathrm{w}_2\mathrm{E}_2(\mathrm{t}) + \mathrm{w}_3\mathrm{E}_3(\mathrm{t})\\
\mathrm{I}_{\mathrm{ensemble}}(\mathrm{t}) &= \mathrm{w}_1\mathrm{I}_1(\mathrm{t}) + \mathrm{w}_2\mathrm{I}_2(\mathrm{t}) + \mathrm{w}_3\mathrm{I}_3(\mathrm{t})\\
\mathrm{D}_{\mathrm{ensemble}}(\mathrm{t}) &= \mathrm{w}_1\mathrm{D}_1(\mathrm{t}) + \mathrm{w}_2\mathrm{D}_2(\mathrm{t}) + \mathrm{w}_3\mathrm{D}_3(\mathrm{t})\\
\mathrm{T}_{\mathrm{ensemble}}(\mathrm{t}) &= \mathrm{w}_1\mathrm{T}_1(\mathrm{t}) + \mathrm{w}_2\mathrm{T}_2(\mathrm{t}) + \mathrm{w}_3\mathrm{T}_3(\mathrm{t})
\end{aligned}\right\} \quad (11$$

Figure 1: Schematic diagram of the HIV SEIDT model

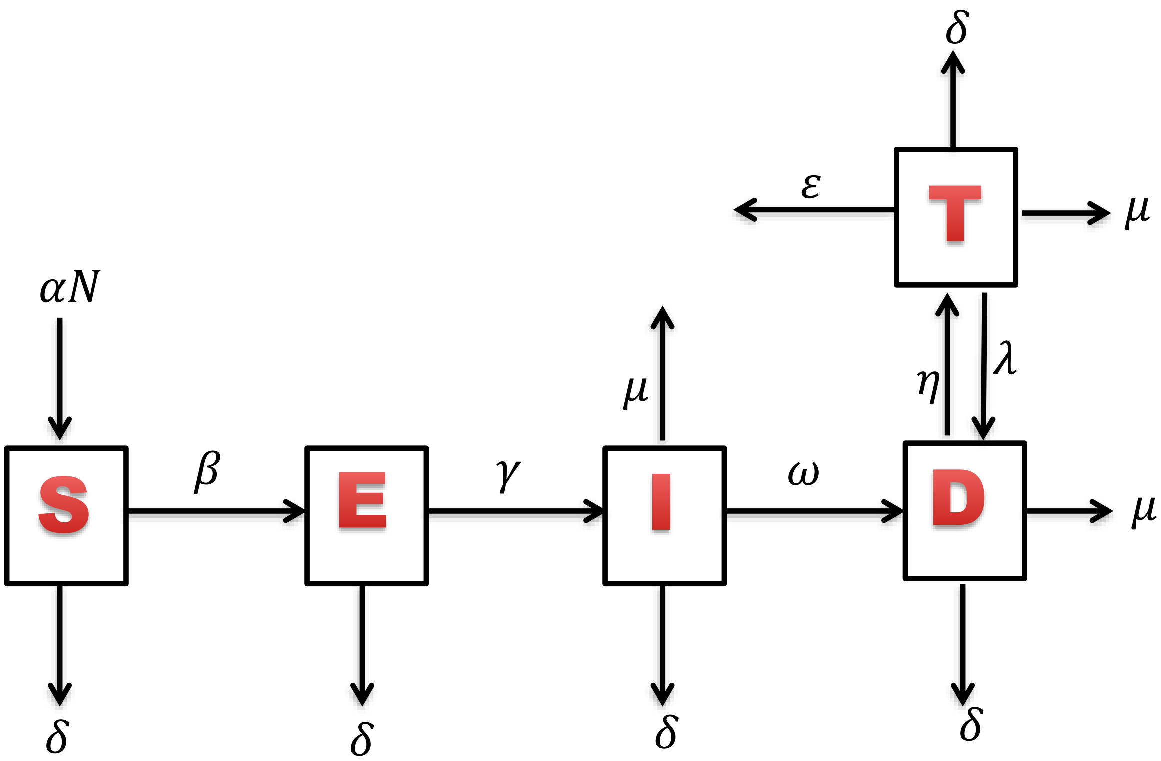

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Table 1: Description of model parameters

Parameter	Description
α	Natural birth rate
δ	Natural death rate
μ	Death rate due to HIV/AIDS
ε	Rate at which patients discontinue treatment
β	Rate of exposure
γ	Infection rate
ω	Rate at which patients undergo test
η	Rate at which diagnosed person undergo treatment
λ	Rate at which people under treatment undergo diagnosis
θ	Fractional parameters

RESULTS

Numerical Schemes

The following schemes are used to perform the numerical experiments.

a. *Caputo Derivative*

$$y(t_n) = y(0) + \frac{h^{\theta}\Gamma(2-\theta)}{n^{1-\theta}} f\big(t_n, y(t_n)\big)$$

$$-\frac{h^{\theta}\Gamma(2-\theta)}{n^{1-\theta}}\left[\sum_{j=1}^{n-1} \omega_{n-j+1}^{(\theta)}\Big(y(t_j) - y(t_{j-1})\Big)\right] \tag{12}$$

where the memory weights are

$$\omega_j^{(\theta)} = j^{1-\theta} - (j-1)^{1-\theta}$$

b. Caputo-Fabrizio Derivative

$$y_{n+1} = y_0 + \frac{1-\theta}{M(\theta)} f(t_n, y_n) + \frac{\theta h}{M(\theta)} \sum_{j=0}^{n} \frac{f(t_j, y_j) + f(t_{j+1}, y_{j+1})}{2} \exp\left(-\frac{\theta(t_n - t_j)}{1-\theta}\right). \tag{13}$$

c. Atangana-Baleanu Derivative

$$y_{n+1} = y_0 + \frac{1-\theta}{M(\theta)} f(t_n, y_n) + \frac{\theta h^{\theta}}{M(\theta)\Gamma(\theta+2)} \sum_{j=0}^{n} w_{j,n} f(t_j, y_j), \tag{14}$$

with the Mittag-Leffler kernel weights given by

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$$w_{j,n} = \begin{cases} n^{\theta} - (n-1)^{\theta}, & j = 0, \\ (n-j+1)^{\theta} - 2(n-j)^{\theta} + (n-j-1)^{\theta}, & 1 \le j \le n-1, \\ 1, & j = n. \end{cases}$$

Numerical Results

Numerical simulations are performed for the period 1990–2030 using initial conditions based on Nigerian population data. The initial conditions are set at $(94{,}999{,}422, 0, 214{,}834, 0, 0)$, reflecting Nigeria's population and early HIV prevalence (Ajao et al., 2023). For the simulation, the disease parameters including the natural birth rate ($\alpha = 0.0$), natural death rate ($\delta = 0.022$), HIV/AIDS-induced death rate ($\mu = 0.33$), treatment discontinuation rate ($\varepsilon = 0.201$), exposure rate ($\beta = 0.0785$), infection rate ($\gamma = 0.3$), testing rate ($\omega = 0.1$), treatment initiation rate ($\eta = 0.89$), and diagnosis rate during treatment ($\lambda = 0.29$) were held constant, while the fractional order parameters were sampled uniformly between 0.8 and 1.0 using Monte Carlo sampling across 50 iterations.

The temporal dynamics of the treated, susceptible, exposed, infected, and diagnosed populations are illustrated in Figures 2–6. Across the three fractional-order formulations (Caputo, Caputo–Fabrizio, Atangana–Baleanu), the ensemble model captures a smoothed trajectory reflecting the combined memory effects of all kernels. Figure 2 shows the treated population steadily increasing, indicating effective transition from infection to care. The susceptible population (Figure 3) declines consistently across all fractional models, while the exposed compartment (Figure 4) exhibits an early peak characteristic of the latent stage. The infected population (Figure 5) reaches a maximum before declining, reflecting the impact of recovery and treatment. Finally, Figure 6 demonstrates a continuous rise in the diagnosed population, highlighting efficient case detection.

Figure 2: Treated population dynamics

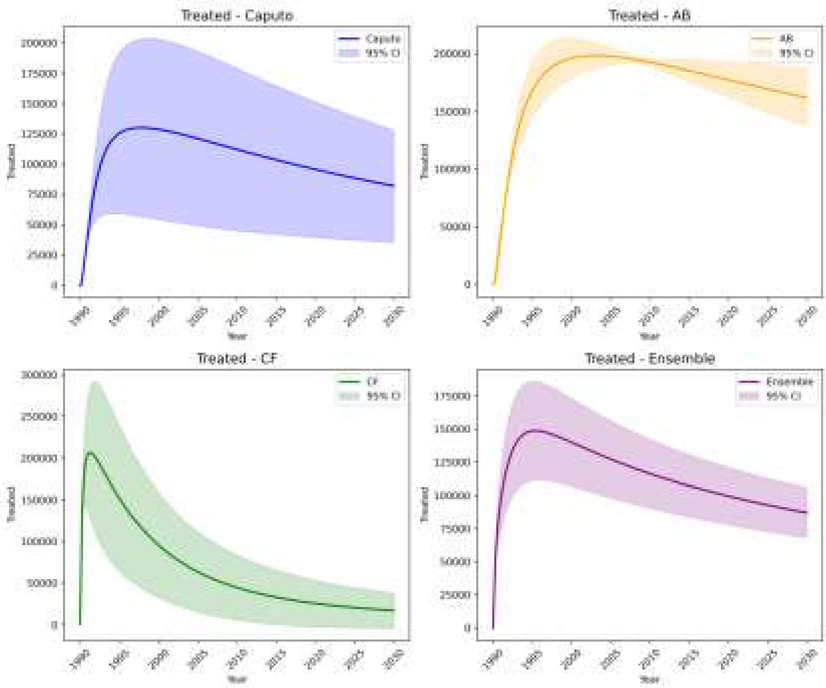

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Figure 3: Susceptible population dynamics

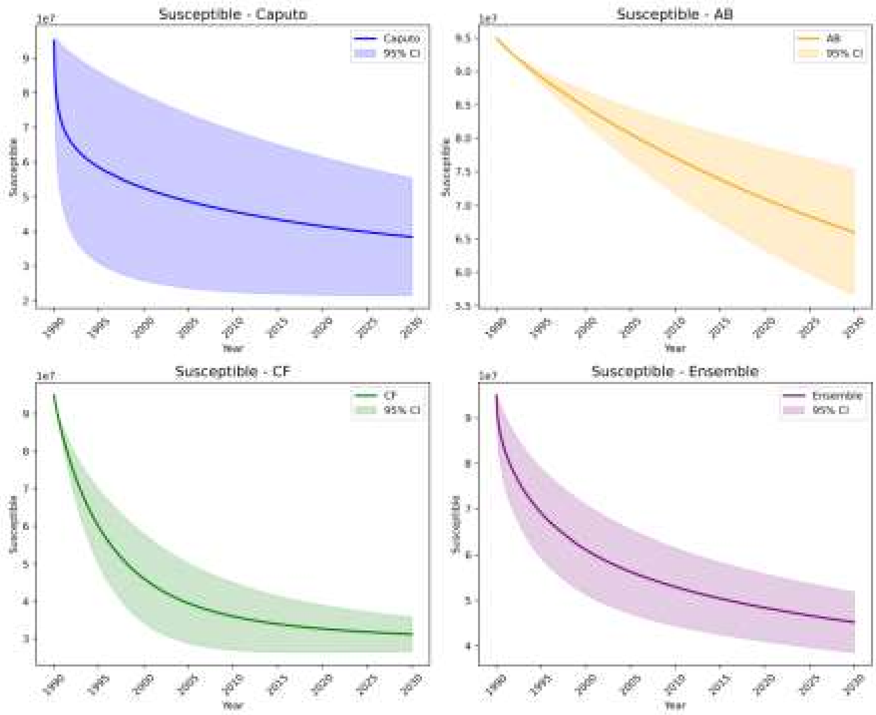

Figure 4: Exposed population dynamics

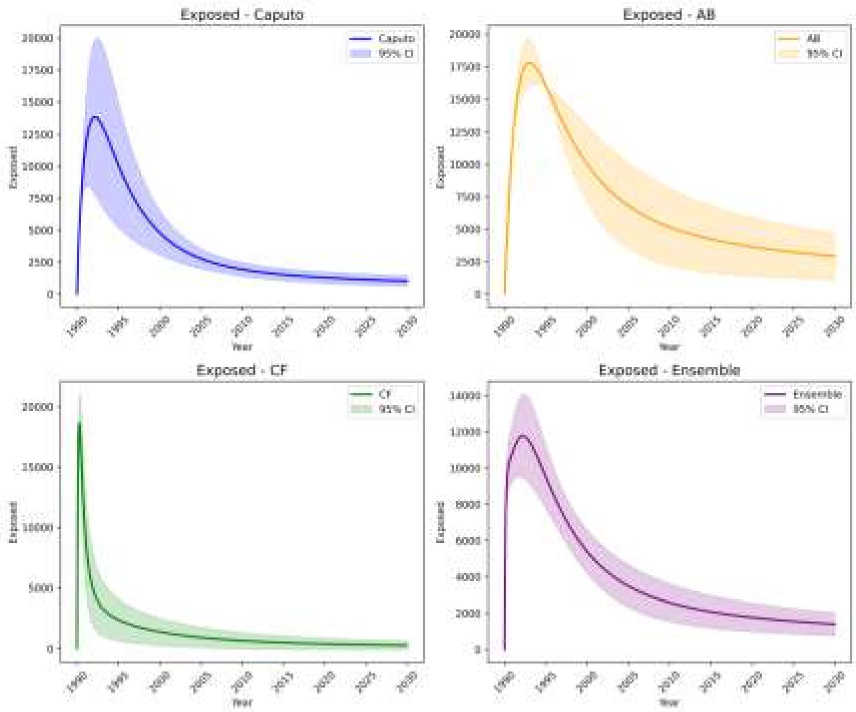

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Figure 5: Infected population dynamics

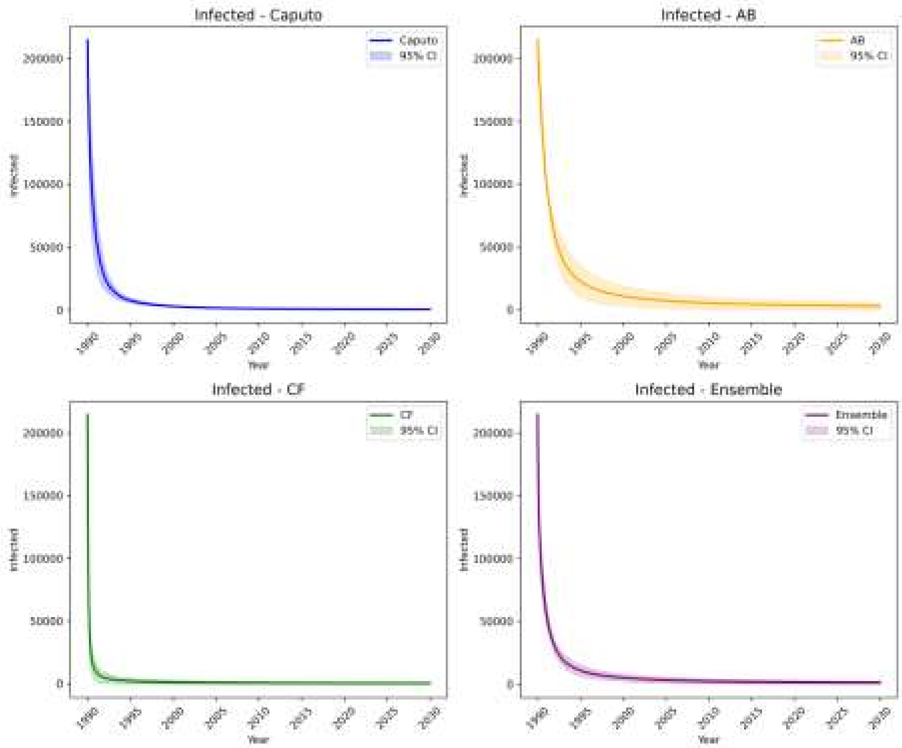
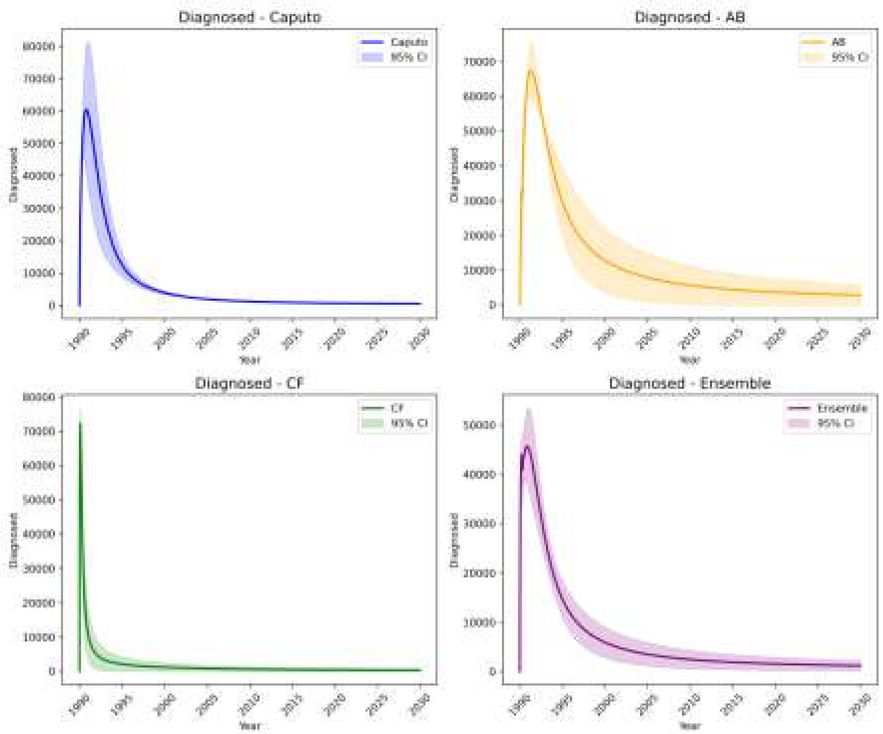


Figure 6: Diagnosed population dynamics



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DISCUSSION

The fractional-order dynamics reveal the influence of memory and hereditary effects on disease progression. Caputo, CF, and AB derivatives produce subtly different temporal patterns: CF shows a smoother, slower initial rise in infections, AB exhibits a slightly delayed peak, and Caputo captures sharper transitions.

The ensemble model integrates these behaviors, producing averaged trajectories that mitigate model-specific extremes and provide robust predictions. This approach demonstrates that fractional memory effects significantly shape the timing and magnitude of epidemic peaks, while ensemble weighting ensures more reliable representation of treated, infected, and diagnosed populations.

The predictions from the individual fractional-order models (Caputo, Caputo–Fabrizio, and Atangana–Baleanu) as well as the ensemble model are accompanied by uncertainty estimates, expressed as 95% confidence intervals. Figures 2–6 show that, while the Caputo and AB models exhibit relatively sharper peaks and slightly wider confidence bands during the initial epidemic phase, the CF model produces smoother dynamics with narrower intervals. The ensemble model integrates these behaviors, resulting in smoothed trajectories with moderate variance across all compartments. The confidence bands remain within reasonable bounds for treated, susceptible, exposed, infected, and diagnosed populations, which shows that the models consistently capture the disease and nonlocal dynamics while accounting for the uncertainty inherent in fractional-order parameterization.

CONCLUSION

This study presents a comprehensive fractional-order and ensemble modeling framework for HIV dynamics in Nigeria. By comparing Caputo, Caputo–Fabrizio, and Atangana–Baleanu derivatives, the study shows that each operator offers distinct advantages. The ensemble approach delivers balanced and reliable projections, making it suitable for long-term HIV epidemic forecasting and policy formulation.

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