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STRATEGIC SECURITY ZONING VIA CHROMATIC GRAPH THEORY: AN OPTIMIZED PATROL ALLOCATION FOR NIGERIA'S GEOPOLITICAL REGIONS

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ABSTRACT: *Geographically distributed security risks throughout Nigeria tend to spread from one state to another via shared borders and movement routes, ensuring the requirement for an optimized and conflict-free security deployment strategy. In this work, a framework for allocating and deploying available security resources in a manner that avoids inter-unit conflicts in a Nigerian setting is proposed by formulating a graph-coloring model that represents the six geopolitical zones in a two-dimensional coordinate plane as map graphs and allocates states as vertices and borders between them as edges. For each zone, a backtracking algorithm based on an adjacency matrix approach is applied with a view to determining the chromatic numbers $\chi(G)$ that represent the optimal minimum needed non-conflicting security patrol units and schedules needed to avoid interference and overlap on interstate levels for enhanced operational efficiency and effectiveness. From the results, it can be observed that while the Northeast geopolitical zone has the highest connectivity and insurgent activities, security deployment needs $\chi(G) = 4$ units, whereas for the North-Central, Northwest, Southeast, South-South, and Southwest zones, security deployment needs $\chi(G) = 3$ units. This indicates the immediate applicability of security resource allocation into partitioned time schedules for nationwide security patrol network deployment and regional security force coordination. By appropriating map graphs on a plane into a real-time applicability tool, the research highlights the applicability of computational graph theory concepts for optimizing security resource planning from conventional-based approaches toward efficient strategies involving minimum resource spending with the highest operational ranges to achieve the UN Sustainable Development Goal.*

MSC 2020 SUBJECT CLASSIFICATION: 05C50, 05C15, 05C10

KEYWORDS: Map graph, Chromatic number, Graph coloring, Backtrack algorithm, Security zoning, Patrol scheduling, Geopolitical zones.

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INTRODUCTION

Spatially distributed security challenges, such as criminality, banditry, and terrorism, are relational problems where regions influence one another through proximity, mobility corridors, shared borders, and communication pathways [1, 2]. Mathematical graph theory provides a modelled framework for modelling interactions by abstracting geographic regions as vertices and their interactions as edges to yield a map graph. A map graph is a mathematical model where geographical territories (countries, states, or local government areas) are represented as vertices and shared borders as edges to form an undirected graph [3, 4]. Map graphs preserve the essential structure of territorial adjacency while enabling rigorous analytical tools that go beyond visual cartography. In recent years, graph-based models have increasingly been recognized as instruments for security analysis, infrastructure protection, and counterterrorism planning, particularly in environments where threats propagate spatially rather than randomly [5-7]. In regions like Nigeria, where criminal networks exploit porous state boundaries, map graphs facilitate the visualization and analysis of territorial adjacencies as undirected graphs, allowing security agencies to detect clusters of high-risk areas through matrix operations.

A representation of a map graph is its adjacency matrix, which encodes the pairwise border-sharing relationships between regions in algebraic form. Through adjacency matrices, spatial security systems become amenable to spectral analysis, connectivity assessment, and algorithmic intervention, which allow analysts to quantify how disturbances in one region may influence others through the underlying network topology [8]. Thus, an adjacency matrix $A = [a_{ij}]$ of dimension $n \times n$ is a binary square matrix, such that if $a_{ij} = 1$ indicates a shared border between regions i and j, otherwise 0. The adjacency matrix is useful in determining the connectivity of the map graph, see [9, 10]. Connectivity of a map graph is the degree to which regions remain linked after the removal of certain routes or nodes [11]. High connectivity, while beneficial for trade and integration, equally presents a vulnerability by allowing threats to diffuse rapidly across jurisdictions [12].

In map-based security modelling, graph coloring is one of the most relevant problems, particularly map coloring, where adjacent regions are assigned different color labels. Thus, graph coloring is the assignment of colours to vertices or regions such that no two adjacent vertices share the same color [13]. The minimum number of colors required to color the map is the chromatic number $\chi(G)$. Chromatic number is important for implementing security zoning, patrol allocation, surveillance scheduling, and jurisdictional deconfliction, ensuring that neighboring regions do not suffer coordination conflicts that criminals or terrorists could exploit [14, 15]. In banditry-prone settings, chromatic number enables strategic zoning to isolate criminal hotspots, integrating adjacency data for proactive threat containment.

Algorithmically, determining optimal or near-optimal colorings of large geographic graphs is nontrivial and is known to be NP-complete in general. Backtracking map coloring algorithm, however, remains the most effective exact method for planar graphs typical of national and subnational maps [16]. The application of the backtracking algorithm moves the problem of territorial security from intuition-based zoning to an optimization challenge. The algorithm systematically explores feasible color assignments while pruning inconsistent partial solutions, making it specifically suitable for security applications where correctness and constraint satisfaction are more critical than approximate speed [17]. In dynamic security scenarios, such as responding to evolving terrorist networks in real-time, backtracking adapts to planar graph

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updates, ensuring connectivity-driven adjustments for minimal chromatic solutions.

For security purposes, graph coloring can model the non-overlapping deployment of surveillance drones, specialized task forces, or the scheduling of coordinated operations without overlap in adjacent high-risk zones, thereby mitigating banditry through optimized coverage or preventing jurisdictional conflict. Then apply the chromatic number to determine the smallest set of distinct resources or units needed to cover all regions without operational interference.

The rise in insecurity, such as kidnapping, robbery, banditry, and terrorism in Nigeria's geopolitical zones has been mapped out with security outfits in each region to tackle the security issues. Western Nigeria Security Network or Amotekun (South-West), Eastern Security Network or Ebube Agu and Operation Iron Fence (South-East), Safe Haven (North-Central), Operation Lafiya Dole (North-East), Pulo Shield (South-South), and Operation Sharan Daji (North-West) can be combined with the Nigeria Police Force, Vigilante Group of Nigeria, Joint Task Force, and Nigeria Security and Civil Defence Corps. Therefore, the goal of the research is to ensure the security outfits will be distributed based on the obtained chromatic numbers from the map graphs.

The map graph of Nigeria has been considered in [12]; however, the author has not examined the map graph of the six geopolitical zones in Nigeria, nor has it considered tackling the security challenge of the regions using graph coloring and its chromatic number. Thus, in Section 2, preliminaries on the Nigerian map graph, including algorithms, are considered for the foundation of the research. Then Section 3 examines only the map graph of the six regions in Nigeria and applies the chromatic number to determine the patrol timetable in combating security in each region.

PRELIMINARY ON GRAPHS

This section will discuss the planar and the map graph, including Euler's formula. The section will also pinpoint the research gap as well as discuss the already established map graph of Nigeria.

Planar and map graph

An undirected graph is a planar graph if it can be drawn on a plane surface in such a way that no two edges intersect except at a common vertex [18]. All map graphs derived from planar maps are planar graphs, but not all planar graphs are map graphs. A map graph is a planar graph since it is conceptualizing actual divisions of land (countries and states) that exist in a plane.

For any connected planar map graph, Euler's formula holds as follows

$$V - E + F = 2$$

where V = vertices, E = edges, F = faces which include the infinite face [19]. For any map graph with $V \geq 3$, then $E \leq 3V - 6$ and $\chi(G) \leq 4$. The minimum degree of a graph, δ(G), is the minimum number of edges connected to any one vertex in a graph, while the maximum degree of a graph, $\Delta(G)$, is the largest degree among all its vertices.

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Specifically, map graphs have constraints, such as regions must be simply connected, borders must be well-defined and multiple adjacency is not allowed since regions must share at most one border [20]. Appel and Haken [21] stated that *"Any map on a plane (or sphere) can be colored using at most four colors such that no two adjacent regions share the same color."* This is equivalent to saying that the chromatic number $\chi(G)$ of any planar graph G is at most 4, see the proofs in [22, 23].

The Nigerian map graph

The construction of the map graph for Nigeria and the backtracking color algorithm are well represented in Algorithms 1 and 2, respectively.

Algorithm 1. Map graph algorithm

```
function Map_Graph(G):
def construct_map_graph(map_regions):
    G = Graph()
    for region in map_regions:
        G.add_vertex(region.id, attributes=region.attributes)
    for i, region1 in enumerate(map_regions):
        for j, region2 in enumerate(map_regions[i+1:], i+1):
            if share_border(region1, region2):
                G.add_edge(region1.id, region2.id,
                          weight=border_length(region1, region2))
  return G
```

Algorithm 2. Graph coloring backtrack algorithm

```
class GraphColoringBacktracking:
    def init(self, adjacency_matrix: np.ndarray, state_names: List[str] =
None):
        self.adj_matrix = adjacency_matrix
        self.n = len(adjacency_matrix)
        self.state_names = state_names or [f"Region_{i}" for i in
range(self.n)]
        self.colors = [-1] * self.n
        self.solutions = []
        self.best_solution = None
        self.best_num_colors = float('inf')
        self.nodes_visited = 0
        self.backtracks = 0
    def is_safe(self, vertex: int, color: int) -> bool:
        for neighbor in range(self.n):
            if self.adj_matrix[vertex][neighbor] == 1 and
self.colors[neighbor] == color:
                return False
        return True
    def basic_backtrack(self, vertex: int, max_colors: int) -> bool:
        self.nodes_visited += 1
        if vertex == self.n:
            return True
        for color in range(max_colors):
            if self.is_safe(vertex, color):
                self.colors[vertex] = color
                if self.basic_backtrack(vertex + 1, max_colors):
                    return True
```


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```
                self.colors[vertex] = -1
                self.backtracks += 1
        return False
    def find_chromatic_number_bruteforce(self, max_colors: int = 10) ->
int:
        for k in range(1, max_colors + 1):
            self.colors = [-1] * self.n
            self.nodes_visited = 0
            self.backtracks = 0
            if self.basic_backtrack(0, k):
                self.best_num_colors = k
                self.best_solution = self.colors.copy()
                return k
        return -1
```

To consider the map graph of Nigeria, using the states and the Federal Territory Capital Abuja (FCT), the map of Nigeria was well-defined in Figure 1.

Figure 1: Map of Nigeria.

For the vertex list of Figure 1, let Abia = v_1, Adamawa = v_2, Akwa Ibom = v_3, Anambra = v_4, Bauchi = v_5, Bayelsa = v_6, Benue = v_7, Borno = v_8, Cross River = v_9, Delta = v_{10}, Ebonyi = v_{11}, Edo = v_{12}, Ekiti = v_{13}, Enugu = v_{14}, Gombe = v_{15}, Imo = v_{16}, Jigawa = v_{17}, Kaduna = v_{18}, Kano = v_{19}, Katsina = v_{20}, Kebbi = v_{21}, Kogi = v_{22}, Kwara = v_{23}, Lagos = v_{24}, Nasarawa = v_{25}, Niger = v_{26}, Ogun = v_{27}, Ondo = v_{28}, Osun = v_{29}, Oyo = v_{30}, Plateau = v_{31}, Rivers = v_{32}, Sokoto = v_{33}, Taraba = v_{34}, Yobe = v_{35}, Zamfara = v_{36}, F.C.T = v_{37}.

Babarinsa [12] represented all 36 states and the FCT by creating a connection between the borders (edges) and the states (vertices) in a map graph using Algorithms 1 and 2 on the vertex list to yield Figure 2.

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Figure 2: Map graph of Nigeria [12].

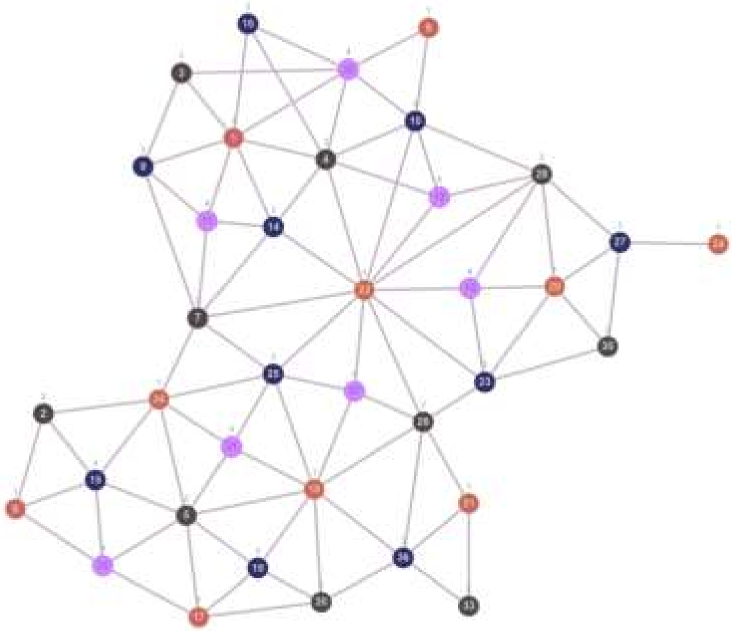

Figure 2 has a chromatic number of 4, and vertex 22 (Kogi State) has the maximum degree of 11. That is, vertex 22 has the largest number of neighbors in the graph. This denotes that Kogi State might be susceptible to terrorist attacks.

It can be seen that the map of Nigeria obeys Euler's formula in the following manner:

$V = 37, E = 85, F = 50$

Thus,

$$V - E + F = 2$$

$$37 - 85 + 50 = 2$$

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METHODOLOGY

This section considers the 6 geopolitical zones in Nigeria, which are: North-Central, North-East, North-West, South-East, South-South and South-West, see Figure 3.

Figure 3: Six geopolitical zones in Nigeria.

Map graph of the North-Central region

The North-Central region consists of 7 states, given in the vertex list as

NC_1 = Benue state

NC_2 = F.C.T

NC_3 = Kogi state

NC_4 = Kwara state

NC_5 = Nasarawa state

NC_6 = Niger state

NC_7 = Plateau state

From the vertex list, an adjacency matrix and the map graph can be constructed; see Table 1 and Figure 4.

Table 1: Adjacency matrix of states in North-Central Nigeria

	NC_1	NC_2	NC_3	NC_4	NC_5	NC_6	NC_7
NC_1	0	0	1	0	1	0	0
NC_2	0	0	1	0	1	1	0
NC_3	1	1	0	1	1	1	0

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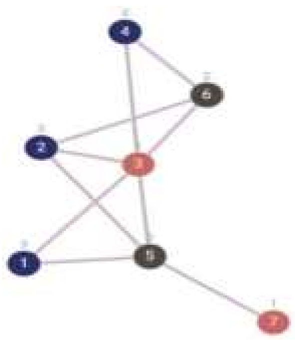
NC_4	0	0	1	0	0	1	0
NC_5	1	1	1	0	0	0	1
NC_6	0	1	1	1	0	0	0
NC_7	0	0	0	0	1	0	0

Figure 4: Map graph of North-Central Nigeria



The chromatic number of the North-Central region is three, signifying that a minimum of three distinct patrol units is required to prevent overlapping patrol operations in adjacent states; see the patrol timetable in Table 2.

Table 2: North-central zone patrol timetable

Groups	**States**
Group 1	NC_3, NC_7
Group 2	NC_5, NC_6
Group 3	NC_1, NC_2, NC_4

Strategically, Niger, Kogi, and Kwara states serve as major transit corridors connecting the north and south of Nigeria. Strengthening security presence around these high-traffic boundaries will significantly reduce the movement of insurgents, banditry, and smuggling across regions.

Map graph of the North-East region

The states in the North-East region are labelled as follows:

NE_1 = Adamawa state

NE_2 = Bauchi state

NE_3 = Borno state

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NE_4 = Gombe state

NE_5 = Taraba state

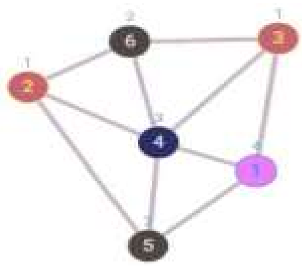
NE_6 = Yobe state.

Then the adjacency matrix of the region is obtained in Table 3 and its map graph constructed in Figure 3.

Table 3: Adjacency matrix of North-East Nigeria

	NE_1	NE_2	NE_3	NE_4	NE_5	NE_6
NE_1	0	0	1	1	1	0
NE_2	0	0	0	1	1	1
NE_3	1	0	0	1	0	1
NE_4	1	1	1	0	1	1
NE_5	1	1	0	1	0	0
NE_6	0	1	1	1	0	0

Figure 5: Map graph of North-East Nigeria



The chromatic number of the map graph in Figure 5 is four. This implies that 4 is the minimum number of patrol slots required to schedule non-overlapping patrols in adjacent states; see Table 4.

Table 4: North-East zone patrol timetable

Groups	**States**
Group 1	NE_2, NE_3
Group 2	NE_5, NE_6
Group 3	NE_4
Group 4	NE_1

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It means that 4 patrol slots or units are enough to assign non-conflicting patrol schedules across the 6 Northeast states. With a chromatic number of 4, the Northeast zone requires four patrol units to guarantee conflict-free patrol scheduling among neighboring states. The Borno, Yobe, and Gombe clique forms a critical joint-security corridor due to insurgency concentration. Prioritizing intelligence-driven patrol operations across these borders enhances national border protection and rapid response capability.

Map graph of the North-West region

The Northwest region consists of 7 states, which are labelled as follows:

NW_1 = Sokoto

NW_2 = Zamfara

NW_3 = Kano

NW_4 = Katsina

NW_5 = Kebbi

NW_6 = Kaduna

NW_7 = Jigawa.

Thus, converting the region list into an adjacency matrix is given in Table 5 and its map graph in Figure 6.

Table 5: Adjacency matrix of North-West Nigeria

	NW_1	NW_2	NW_3	NW_4	NW_5	NW_6	NW_7
NW_1	0	1	0	0	1	0	0
NW_2	1	0	0	1	1	1	0
NW_3	0	0	0	1	0	1	1
NW_4	0	1	1	0	0	1	1
NW_5	1	1	0	0	0	0	0
NW_6	0	1	1	1	0	0	0
NW_7	0	0	1	1	0	0	0

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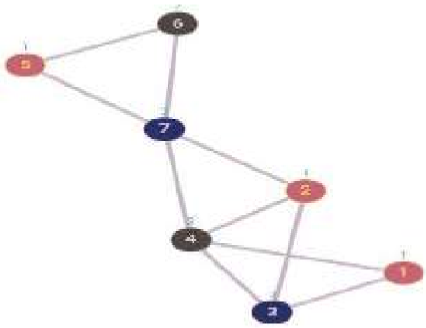
Figure 6: Map graph of Northwest Nigeria



The chromatic number of three, from Figure 6, indicates that three patrol units are sufficient for the Northwest region, which is analyzed in Table 6.

Table 6: North-West zone patrol timetable

Groups	States
Group 1	NW_1, NW_2, NW_5
Group 2	NW_4, NW_6
Group 3	NW_3, NW_7

From Table 6, banditry and cattle rustling remain major security concerns, especially across Zamfara, Katsina and Sokoto boundaries. Enhanced air surveillance, forest monitoring, and coordinated border patrols are necessary for the region's stability.

Map graph of the South-East region

South-East has vertex list:

SE_1 = Abia state

SE_2 = Anambra state

SE_3 = Ebonyi state

SE_4 = Enugu state

SE_5 = Imo state

Representing the South-East region from the vertex list into an adjacency matrix is given in Table 7 and its map graph in Figure 7.

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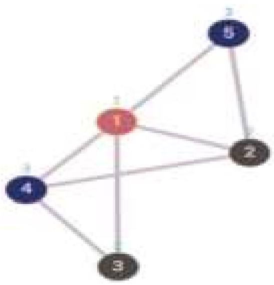
Table 7: Adjacency matrix of South-East Nigeria

	SE_1	SE_2	SE_3	SE_4	SE_5
SE_1	0	1	1	0	1
SE_2	1	0	0	1	1
SE_3	1	0	0	1	0
SE_4	1	1	1	0	0
SE_5	1	1	0	0	0

Figure 7: Graph of South-East region of Nigeria.



From Figure 6, the chromatic number of the South-East region is three indicates that three patrol units can be grouped into three for better analysis; see Table 8.

Table 8: South-East zone patrol timetable

Groups	**States**
Group 1	SE_1
Group 2	SE_2, SE_3
Group 3	SE_4, SE_5

From Table 8, high-density trade and transportation routes in Abia and Imo require prioritized patrol scheduling. Increased presence along boundary zones, particularly the Ebonyi and Enugu corridors, helps prevent the movement of illegal firearms and other criminal infiltration routes.

Map graph of the South-South region

The vertex list of South-South is highlighted as follows

SS_1 = Akwa Ibom state

SS_2 = Bayelsa State

SS_3 = Cross River State

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SS_4 = Delta State

SS_5 = Edo State

SS_6 = Rivers state

Table 9 represents the adjacency matrix of the South-South region, which is obtained from the vertex list, and then constructs the map graph; see Figure 8.

Table 9: Adjacency matrix of South-South Nigeria

	SS_1	SS_2	SS_3	SS_4	SS_5	SS_6
SS_1	0	0	1	0	0	1
SS_2	0	0	0	1	0	1
SS_3	1	0	0	0	0	0
SS_4	0	1	0	0	1	1
SS_5	0	0	0	1	0	0
SS_6	1	1	0	1	0	0

Figure 8: Graph of South-South region of Nigeria

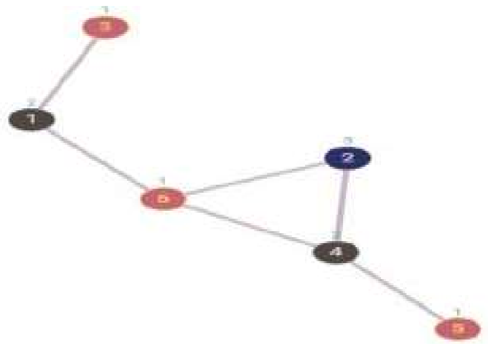

Figure 8 indicates the chromatic number of the South-South region is three, thus the patrol grouping is given in Table 10.

Table 10: South-south zone patrol timetable

Groups	States
Group 1	SS_3, SS_6, SS_5
Group 2	SS_1, SS_4
Group 3	SS_2

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From Table 10, the Niger Delta state (creeks) presents major challenges, especially around Rivers, Bayelsa, and Delta states. Marine-supported patrols and surveillance technology are essential in reducing illegal bunkering, piracy, and militancy.

Map graph of the South-West region

There are 6 states in the Southwest region as follows:

SW_1 = Lagos state

SW_2 = Ondo state

SW_3 = Osun state

SW_4 = Ogun state

SW_5 = Oyo state

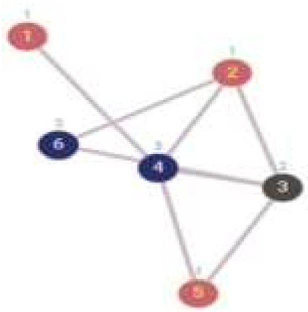
SW_6 = Ekiti state.

Therefore, the adjacency matrix, including its map graph, obtained from the list, is given in Table 11 and Figure 9.

Table 11: Adjacency matrix of South-West Nigeria

	SW_1	SW_2	SW_3	SW_4	SW_5	SW_6
SW_1	0	0	0	1	0	0
SW_2	0	0	1	1	0	1
SW_3	0	1	0	1	1	1
SW_4	1	1	1	0	1	0
SW_5	0	0	1	1	0	0
SW_6	0	1	1	0	0	0

Figure 9: Map graph of South-West region



From Figure 9, the chromatic number of South-West is three. This indicates that the South-West zone requires three patrol units; see Table 12.

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Table 12: South-West zone patrol timetable

Groups	States
Group 1	SW_1, SW_2, SW_5
Group 2	SW_3
Group 3	SW_4, SW_6

Lagos, Ogun, and Oyo feature major mobility centres vulnerable to criminal exploitation. Coordination through shared intelligence enables interception of illicit activities along the Lagos-Ibadan corridor and porous Benin Republic borders. Thus, security operations are partitioned into four non-adjacent zones, each managed by a dedicated command, thereby reducing coordination overhead and minimizing gaps exploited by agile non-state actors.

ANALYSIS AND DISCUSSION

The regional map graphs obtained in this paper that adhere to the Four-Color Theorem often have chromatic numbers below 4. The backtracking algorithm proves optimal resource-efficient partition for conflict-free operations, as the problem size (6-7 vertices per zone) is manageable for exact computation, ensuring guaranteed optimality over heuristic methods. Chromatic numbers indicate the presence of vulnerabilities in the structure; for instance, the Northeast with $\chi(G)=4$ indicates a higher degree of complexity related to insurgency duration and resource requirements beyond other zones ($\chi(G)=3$). This is consistent with the four-color theorem statement while pointing to the specificity of each zone's requirements. Similarly, low values in the southern zones indicate simplicity in the structure, not a lowering of insurgency/piracy risks. The zonal breakdown allows for resource-directed interventions with scalability beyond the national map graph. A comparison with the regional map graph $\Delta(G) = 5$ and $\Delta(G) = 11$ at national map graph for Kogi state indicates the state has degree vertices compared to other states. Graph coloring goes beyond intuitive zoning by allowing precise allocations satisfying constraints, and these are very beneficial in dynamic threats, which allow backtracking based on graph modifications (like temporary border closures). Drawbacks include assuming static border conditions and zone-intracity structure constraints, but zone-between threats could be represented by assuming a supergraph over the zone graph. Further developments could improve graph coloring by adding edge weights for threat intensity and by using compromise graph-coloring algorithms for scalability.

In terms of application, the resulting patrol schedules provide a template for deconflicting combined operations among the diverse security agencies in Nigeria (such as Amotekun, Ebube Agu, the Nigerian Police, and the Civil Defence Corps). By assigning regional states to various patrol teams, it ensures no interference from its own forces, eliminates ambiguity in response times, and provides for an observable rotational coverage schedule that would be difficult for nimble adversaries to efficiently take advantage of. The adaptability features of this model include revision of adjacency considerations (in cases of conflict, for instance), or formation of new joint operating areas for combined operations

This work is relevant to the UN Sustainable Development Goal of enhancing appropriate

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national institutions for the prevention of violence and the fight against terrorist and criminal activities. Through the application of graph theory and chromatic number optimization to the Nigeria geopolitical zones, this work has developed a theoretical approach to enhancing the allocation of security patrols, coordination among jurisdictions, and conflict-free activities within the surrounding zones in an optimal manner, thereby improving the ability of national security institutions to preempt and effectively contain violence, banditry, and terrorist activities, which remain a challenge to peace within the geographical location.

CONCLUSION

This work fills a void that exists at the nexus between discrete math and operational security science. By employing a chromatic graph theory paradigm to determine the chromatic numbers of Nigeria's geopolitical districts, represented as two-dimensional graphs, we have established an optimized framework for assigning patrols and security zoning that is definitive and useful to operational security command levels in each geographic region of concern. Security command in the Northeast region must organize patrol activities in four sectors, whereas security command in all other regions may complete comprehensive security patrols in just three sectors for all regions, with no further operational conflicts or overlaps between directly abutting regions, as a function of graph theory principles that ensure minimum resource utilization.

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