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# UNSTEADY MAGNETOHYDRODYNAMIC FLOW OF NON-NEWTONIAN FLUID IN AN INCLINED PLANE WITH JOULE HEATING

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**ABSTRACT:** *This present paper is on numerical study of unsteady magnetohydrodynamic flow of non-Newtonian fluid in an Inclined plane with Joule heating. The set of coupled non-linear partial differential equations is a solved collocation technique, the effects of some physical parameters examined. Third grade parameter is introduced to account for the non-Newtonian fluid. Results from the investigation reveal that increase in the third grade parameter increases the flow velocity and decreases the temperature and increase in Eckert number leads to an increase in the temperature of the cylindrical walls. Results further show that increase in Prandtl number enhances viscous dissipation. This implies that the boundary layer thickness decreases with increase in Prandtl number, thereby reducing the temperature profiles.*

**KEYWORDS:** Magnetohydrodynamic, Heat transfer, Cylindrical pipe, Joule heating.


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## INTRODUCTION

Within the past several years, generalization of the Navier-Stokes model which is highly non-linear constitutive laws have been proposed and examined by many researchers. Because of the deficiency of the classical Navier-Stokes theory in describing rheological complex fluids such as paints, blood, oils and greases and the applications in engineering and technology as well as the pulp industries, has led to the development of many theories of non-Newtonian fluids.

Many researchers have done some work to explain some of the complex nature of the non-Newtonian fluids of the differential types, amongst whom are: Rivlin and Erickson [18], on stress deformation relations for isotropic materials, and Fosdic and Rajagopal [6], on thermodynamics, stability of fluids of third grade. Okedayo *et al.* [15] carried out a computational study of reactive flow of an electrically conducting fluid with temperature dependent viscosity and axial magnetic field using the semi-implicit finite difference scheme.

Ramachandran *et al.* [17] analyzed numerically laminar mixed convection in the two-dimensional stagnation flows around heated surfaces for both cases of an arbitrary wall temperature and arbitrary surface heat flux variations. Hayat *et al.* [8] applied homotopy perturbation and numerically obtained the solution of the third grade fluid past a porous channel with suction and injection at the walls. Massoudi and Christie [10] examined the effects of variable viscosity and viscous dissipation on the flow of a third grade fluid in a pipe. Shirkhani *et al.* [19] examined the unsteady time-dependent incompressible Newtonian fluid flow between two parallel plates by homotopy analysis method (HAM), homotopy perturbation method (HPM) and collocation method (CM). They transformed the Navier-Stokes equation into ordinary differential equation using similarity transformation and investigated the effects of Reynolds number and suction or injection characteristic parameter on the velocity field.

Oosthuizen [16] investigated numerically the two-dimensional mixed convection flow over a horizontal plate in a saturated porous medium mounted near an impervious adiabatic horizontal surface and subject to a horizontal forced flow. Muralidhar [12] performed a numerical calculation for buoyancy-assisted mixed convection in a vertical annulus by using the Darcy model. The results show that the Nusselt number increases with the Rayleigh number. Kumari *et al.* [9] incorporated the Forchheimer term to account for the inertia effect on mixed convection from horizontal surfaces. Aldoss *et al.* [2] reported non-similarity solutions using the Darcy's law for the problem of mixed convection along the horizontal surface in porous media by dividing the entire mixed convection regime into two regions: one covers the forced-convection-dominated regime and the other covers the free-convection-dominated regime. Obi *et al.* [14] analyzed the flow of incompressible MHD third grade fluid in an inclined rotating cylindrical pipe with isothermal wall and joule heating. Obi [13] applied perturbation technique to analyze magnetohydrodynamic flow of third grade fluid in an inclined cylindrical pipe.

Hadim and Chen [7] investigated the effect of Darcy number on the buoyancy-assisted mixed convection in the entrance region of a vertical channel with asymmetric heating at fixed values of the Reynolds number, Forchheimer number, Prandtl number and modified Grashof number. Chen *et al.* [4] considered the non-Darcian mixed Convection flow along non-isothermal vertical Surfaces and Presented heat transfer results for the entire regime of mixed convection.


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Chen [3] presented the effects of non-Darcian flow phenomena on mixed convection in a porous medium adjacent to a non-isothermal horizontal plate. Chen *et al.* [5] investigated the fully developed mixed convection in a vertical porous channel with a uniform heat flux imposed at the plates by using the Brinkman-Forchheimer-extended Darcy model. Mishra *et al.* [11] studied the Mixed Convection flow in a porous medium bounded by two vertical walls using analytical methods.

## Mathematical Formulation

Considering an unsteady incompressible magnetohydrodynamic flow of third grade fluid in a cylindrical pipe. The velocity field is of the form

$$v=\left(0,0,u\left(0\right)\right) \tag{1}$$

$$\nabla .v=0 \tag{2}$$

$$S=s\left(r\right) \tag{3}$$

$$\rho \frac{Dv}{Dt}=-\nabla p+\rho B+divT \tag{4}$$

The equation in a third grade fluid is expressed as in Rajagopal and Na [11] as

$$T=-p_1+\mu A_1+\alpha_1 A_2+\alpha_2 A_1^2+\beta_1 A_3+\beta_2\left(A_1A_2+A_2A_1\right)+\beta_3\left(trA_1^2\right)A_1 \tag{5}$$

Where

$\rho$ is the fluid density, B is the body force, V is the velocity, p is pressure, T is the Cauchy stress tensor, $\mu$ is the dynamic viscosity, $\frac{Dv}{Dt}$ is the material –time derivative, $\nabla$ is the gradient operator and S is the extra tensor for the third grade. $B=B_0+b\left(B_0\right.$ and b are applied and induced magnetic field$\left.\right)$

$$\text{J}=\sigma\left(E+V\times B\right) \tag{6}$$

$$A_0=I, A_1=L+L^T \tag{7}$$

$$A_n=\frac{dA_{n-1}}{dt}+A_{n-1}L+L^TA_{n-1},\ n=2,3,4,... \tag{8}$$

$$L=\nabla v \tag{9}$$

The incompressibility criterion is satisfied by eqn (1). In applying the eqns (7-9) and substituting the values of $v$ and $T$ in eqns (1) and (2), neglecting body force, the momentum equation is given as

$$\rho\frac{\partial u}{\partial t}=\mu\frac{\partial^2 u}{\partial r^2}+\alpha\frac{\partial^2 u}{\partial r^2\partial t}+6\left(\beta_2+\beta_3\right)\left(\frac{\partial u}{\partial r}\right)^2\frac{\partial^2 u}{\partial r^2}-\sigma \mathrm{B}_0^2u+\rho g\sin\phi \tag{10}$$


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The energy balance for the thermodynamically compatible non-Newtonian fluid with viscous dissipation, work done due to deformation and Joule heating is given as:

$$\rho Cp\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial r^2}+\mu\left(\frac{\partial u}{\partial r}\right)^4+\alpha\frac{\partial^2 u}{\partial r^2\partial t}+2\left(\beta_2+\beta_3\right)\left(\frac{\partial u}{\partial r}\right)^4+\sigma \mathrm{B}_0^2\mathrm{u}^2 \tag{11}$$

With the boundary and initial conditions as

$$u(r,t)=0,\ r=0,t>0;u(r,t)=0,\ r=1,t>0;u(r,0)=0,\ 0<\mathrm{r}<1 \tag{12}$$

$$T(r,t)=T_w,\ r=0,t>0;T(r,t)=T_\delta,\ r=\delta,t>0;T(r,0)=0,\ 0<\mathrm{r}<1 \tag{13}$$

where u is the fluid velocity, T is the temperature, $\mathrm{T}_\delta$ is the ambient temperature of the fluid, $\mathrm{C}_p$ is the specific heat capacity, k is the thermal conductivity of the fluid, $\rho$ is the fluid density, r is the radius of the cylinder and t is the time.

Defining the dimensionless variables as:

$$u=\mu\bar{u},\ r=\frac{\mu\bar{r}}{\delta},T=\theta\left(T_\delta-T_w\right),t=\frac{\delta\bar{t}}{\mu^2} \tag{14}$$

Using eqn (14) in eqns (10-13), yields

$$\rho\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial r^2}+\beta\left[\lambda\frac{\partial^2 u}{\partial r^2\partial t}+6\left(\frac{\partial u}{\partial r}\right)^2\frac{\partial^2 u}{\partial r^2}\right]-Mu+k \tag{15}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\mathrm{Pr}}\frac{\partial^2 T}{\partial r^2}+E_c\left(\frac{\partial u}{\partial r}\right)^2+\beta\left[\lambda\frac{\partial u\partial^2 u}{\partial r\partial r\partial t}+2E_c\left(\frac{\partial u}{\partial r}\right)^4\right]+ME_c\mathrm{u}^2 \tag{16}$$

$$u(0,t)=0,\ t>0;\frac{\partial u(1,t)}{\partial r}=0,\ t>0;u(r,0)=0,\ 0<\mathrm{r}<1 \tag{17}$$

$$\theta(0,t)=0,\ t>0;\ \theta(1,t)=1,\ t>0;\ \theta(r,0)=0,\ 0<\mathrm{r}<1 \tag{18}$$


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## SOLUTION OF THE PROBLEM

In order to solve equations (15-18), we apply the weighted residual collocation method. This method is a function approximation procedure in which the set of partial differential equations are reduced to algebraic equations . These algebraic equations in turn will be solved by any known procedure or fixed point iterate.

The function approximation procedure is of the form

$$u(r)=\sum_{i=0}^{n} a_i\phi(r) \tag{19}$$

Equation (19) is the trial or basis function over the domain of the boundary and initial conditions (17) and (18). It is required that the residual function be orthogonal to a set of weighted functions.

In order to ascertain the convergence of the method, the one, two term coefficients are applied which satisfies the boundary conditions. The functions are:

$$u(r)=1+a_0\left(r^2-1\right)+a_1\left(r^3-1\right),\theta(r)=1+b_0\left(r^2-1\right)+b_1\left(r^3-1\right) \tag{20}$$

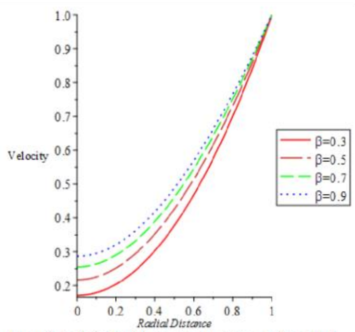

Figure 1: Velocity Profiles For Various Values of The Third Grade Parameter (β) At t=1

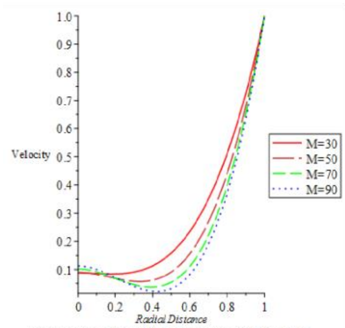

Figure 2: Velocity Profiles For Various Values of The Magnetic Field Parameter (M) At t=1


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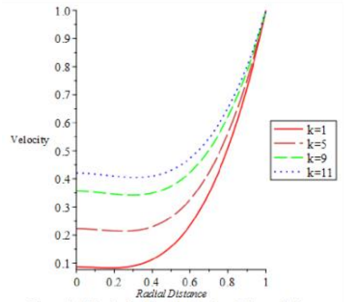

Figure 3: Velocity Profiles For Various Values of The Gravitational Parameter (k) At t=1

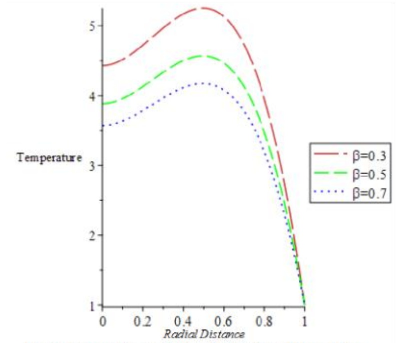

Figure 4: Temperature Profiles For Various Values of The Third Grade Parameter (β) At t=1

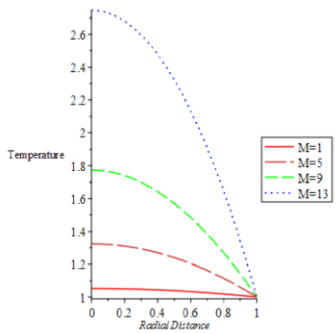

Figure 5: Temperature Profiles For Various Values of The Magnetic Field Parameter (M) At t=1

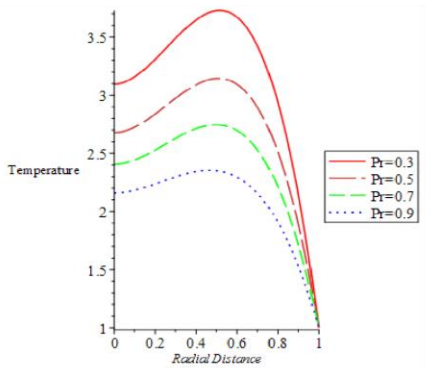

Figure 6: Temperature Profiles For Various Values of The Prandtl Number (Pr) At t=0.5

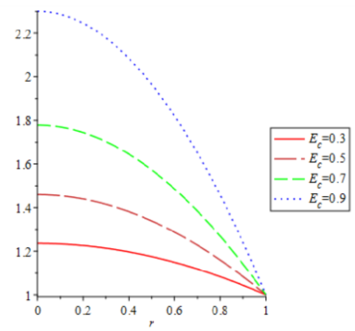

Figure 7: Temperature Profiles For Various Values of The Eckert Number ($E_c$) At t=1


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## RESULTS AND DISCUSSION

In this section, the solutions for velocity and temperature profiles are obtained by specifying the values of the thermo-physical parameters to ascertain the effects of various dimensionless numbers on the velocity field and energy balance. Equations (15) to (18) form the set of coupled non-linear partial differential equations with time development of the velocity u and temperature profile $\theta$ for a fixed time t.

Figure 1 shows the velocity profiles for different values of the non-Newtonian parameter represented by the third grade parameter ($\beta$). Results indicate that as the parameter $\beta$ increases at a steady rate, the velocity of the fluid flow also increases. Figure 2 is the velocity profiles for various values of the magnetic field parameter (M). Result shows that velocity of the fluid decreases with increase in the magnetic field parameter. Figure 3 shows the effects of gravitational parameters on the fluid flow. It is seen that increase in gravitational parameter k increases the fluid flow. This is because as the gravitational parameter increases, the pressure decreases within the cylindrical pipe giving rise to faster movement of the fluid. Figure 4 is the temperature profiles for various values of the third grade parameter ($\beta$). Results show that increase in the third grade parameter decreases the temperature of the system. Figure 5 shows the temperature profiles for variation of the magnetic field parameter (M). It is observed that an increase in magnetic field increases the temperature within the boundary layer and at the same time reduces the flow velocity. Figure 6 shows the effects of Prandtl number on the temperature profiles. Results indicate that increase in the Prandtl number ($\Pr$) decreases the temperature because increase in the parameter leads to a stronger viscous dissipation compared to thermal diffusion. This implies that the boundary layer thickness decreases with increasing Prandtl number. Figure 7 is the temperature profile for various values of the Eckert number. Eckert number is the relationship between the differences in kinetic energy and the heat enthalpy. Result shows that increase in Eckert number leads to an increase in the temperature of the plate due to resistivity of the thermal effects.

## CONCLUSION

The study of unsteady magnetohydrodynamic flow of non-Newtonian fluid in an inclined plane with Joule heating is presented and the effects of some of the physical parameters examined. Third grade parameter is introduced to account for the non-Newtonian fluid. Results from the investigation reveal that:

- Increase in the third grade parameter increases the flow velocity and decreases the temperature.
- Increase in magnetic field parameters reduces the fluid flow and increases the temperature of the cylindrical wall.
- Increase in Prandtl number enhances viscous dissipation. This implies that the boundary layer thickness decreases with increase in Prandtl number, thereby reducing the temperature profiles.


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- It is observed that an increase in Eckert number leads to an increase in the temperature of the cylindrical walls.

- The velocity profile is greatly enhanced by increase in the gravitational parameter k.

## DECLARATIONS

1. Funding: Not applicable

2. Informed Consent Statement: Not applicable

3. Data Availability: Not applicable

4. Conflict of Interest Statement: No conflict of interest

## REFERENCES

[1] Aiyesimi Y.M. Okedayo G.T., and Lawal O.W. Unsteady magnetohydrodynamic (MHD) thin film flow of third grade fluid with heat transfer and no slip boundary condition down an inclined plane

[2] Aldoss TK, Chen TS, Armely BF. Non similarity solution for mixed convection from a horizontal surface in a porous medium-variable wall temperature. Int. J. Heat Mass Transfer 36, 471-477,(1993).

[3] Chen C.H. Analysis of non Darcian mixed convection from impermeable horizontal surface in porous media: the entire region. Int. J. Heat Mass Transfer 40, 2993-2997, (1997).

[4] Chen C.H., Chen T.S., Chen C.K. Non Darcy mixed convection along non isothermal vertical surface in porous media. Int.J. Heat Transfer 39, 1157-1164, (1996).

[5] Chen Y.C., Chung J.N., Wu C.S., Lue Y.F. Non Darcy mixed Convection in a vertical channel filled with a porous medium. Int. J. Heat Mass Transfer 43, 2421-2429, (2000).

[6] Fosdick R.L. and Rajagopal, K.R.: Thermodynamics and stability of fluids of third grade. Proc. R. Soc. Lond. 339, 351-377, (1980).

[7] Hadim A, Chen A. Non Darcy mixed convection in a vertical porous channel. J. Thermo physics and Heat transfer 8, 805-808, (1994).

[8] Hayat,T., Ellahi, R., Ariel, P.D., Asghar, S.: Homotopy solution for the channel flow of a third grade fluid. Non-linear Dyn. 45, 55-64(2006).

[9] Kumari M, Pop I, Nath G. Non similar boundary layers for non Darcy mixed about a horizontal surface in a saturated porous medium. Int. J. of Engineering Sciences 28, 253-263, (1990).

[10] Massoudi, M. and Christie, I.: Effects of variable viscosity and viscous dissipation on the flow of a third –grade fluid in a pipe. Int. J. of Nonlinear Mech.,30(5): 687-699,(1995).

[11] Mishra A.K., Paul T., Singh A.K. Mixed convection flow in a porous medium bounded by two vertical walls, (2002).

[12] Muralidhar M mixed convection flow in saturated porous sannulus. Int. J. Heat Mass Transfer 32, 881-888, (1989).


Article DOI: 10.52589/AJSTE-LFNZBFHN
DOI URL: https://doi.org/10.52589/AJSTE-LFNZBFHN

ISSN: 2997-5972
Volume 4, Issue 2, 2024 (pp. 127-135)


[13] Obi B.I.: Perturbation Analysis Of Magnetohydrodynamic Flow of Third Grade Fluid In An Inclined Cylindrical Pipe: Journal Of Mathematical Science And Computational Mathematics (JMSCM) Vol 3, No. 3(2022)

[14] Obi B.I.,Okedayo, G.T., Jiya, M. And Aiyesimi, Y.M.: Analysis of Flow of An Incompressible MHD third Grade Fluid In An Inclined Rotating Cylindrical Pipe With Isothermal Wall And Joule Heating. International Journal For Research In Mathematics And Statistics. Vol 7 Issue 6,(2021)

[15] Okedayo G. T., Abah S. O and Abah R. T.: Viscous dissipation effect on the reactive flow of a temperature dependent viscosity and thermal conductivity through a porous channel. Abacus the journal of the mathematical association of Nigeria 41(2),74-81, (2014).

[16] Oosthuizen P.H. Mixed convective heat transfer from a heated horizontal plate in a porous medium near an impermeable surface. ASME J. Heat Transfer 110, 390-394, (1988).

[17] Ramachandran N., Chen T.S., Armaly B.F. Mixed convection in stagnation flows adjacent to vertical surfaces. ASME J.Heat Transfer 110, 373-377, (1988).

[18] Rivlin, R.S., Ericksen, J.I.: Stress deformation for isotropic materials. J. Ration. Mech. Anal. 4,323-329,(1955).

[19] Shirkhani, M.R., Hoshyar, H.A., Rahimipetroudi, I., Akhavan, H., Ganji, D.D. Unsteady time-dependent incompressible Newtonian fluid flow between two parallel plates by homotopy analysis method (HAM),homotopy perturbation method (HPM) and collocation method (CM). Propulsion and Power Research . 7(3): 247-256,(2018).


Article DOI: 10.52589/AJSTE-LFNZBFHN
DOI URL: https://doi.org/10.52589/AJSTE-LFNZBFHN