ON THE DISTRIBUTION OF THE MIXED-LOGNORMAL-WEIBULL OPTION PRICING MODEL

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Cite this article:

Ugomma, C. A. (2024), On the Distribution of the Mixed-Lognormal-Weibull Option Pricing Model. Advanced Journal of Science, Technology and Engineering 4(4), 111-124. DOI: 10.52589/AJSTE-J50VXBA8

Manuscript History

Received: 12 Aug 2024

Accepted: 13 Oct 2024

Published: 4 Nov 2024

ABSTRACT: *This paper intends to test whether the Mixed-Lognormal-Weibull Distribution (MLWD) option pricing model comes from the same distribution and whether the model is a good fit in Black-Scholes option pricing model. The data for this study were obtained from Australian Clearing House of Australian Securities Exchange (ASX) which consist of 50 enlisted stock as products of monthly market summary for long term options collected from January, 3rd 2017 to December, 31st 2017, comprising 720 trading days arranged in accordance to 25, 27, 28, 29 and 30 maturity days. Maximum Likelihood Estimate was used to obtain the parameters of both the lognormal and Weibull distributions which were applied in Black-Scholes model. The data were test Wilcoxon Rank Sum test since the mixture model became distribution and Chi-square goodness-of-fit test for the model fit, and the result shows that the mixture model does not follow any of the lognormal or Weibull distributions. The result also shows that the mixture model is a good fit in Black-Scholes option pricing model with the P-value>0.05 when they are shorter maturity days with small sample sizes than longer maturity days with larger sample sizes. Hence, the model is recommended to be used for financial practitioners who are interested in modeling option pricing.*

KEYWORDS: *Black-Scholes Model, Mixed-Lognormal-Weibull option pricing model, Maximum Likelihood Estimate, Wilcoxon Rank Sum Test, Goodness-of-fit test.*

INTRODUCTION

The Black-Scholes Option Pricing Model of 1973 remains a statistical tool for financial practitioners for pricing options. However, many attempts have been made on the issue of pricing options on extending the original Black-Scholes model to obtain a good fit that would actually observe prices. Consequently, several studies have shown that most of the assumptions of Black-Model of 1973 have been criticized and violated; see for example, Bates (2003), Wilkens (2005), Giannikis et al. (2008), Christoffensen et al. (2012), Nwobi et al. (2021) and Jimbo et al. (2022),

Fu (2016) obtained a closed form solution for pricing European options under a general jump-diffusion model that can incorporate arbitrary discrete jump-size distributions, including nonparametric distributions as an empirical distribution. The flexibility in the jump-size distribution allowed their model to better capture leptokurtic features found in real world data. This model used a discrete time framework and led to a pricing formula that was provably convergent to the continuous-time price as discretization increased. Their numerical example showed the efficiency and accuracy of their proposed model.

Lee et al. (2019) showed how to use binomial and multinomial distributions to derive options pricing models. In addition, they showed how the Black-Scholes option pricing model is a limited case of binomial and multinomial option pricing model.

Cai and Kou (2011) extended an analytical tractability of Black-Scholes model to act as an alternative model with arbitrary jump diffusion model for asset prices whose jump sizes have a mixed-exponential distribution which is a weighted average of exponential distribution but with possibly negative weights. They showed that the mixed-exponential jump diffusion model can lead to analytical solutions for Laplace transforms of prices and sensitivity parameters for path-dependent option such as lookback and barrier options, where the calibration of SPY options showed that the model could provide a reasonable fit even for options with very short maturity, such as one day.

Ugomma and Nwobi (2023) empirically investigated the effect on the mixture distribution in Black-Scholes Option Pricing model with the data collected from Australian Clearing House of Australian Security Exchange consisting of 50 enlisted stocks. With the help of R-package, the Maximum Likelihood Estimation was used to obtain the parameters of the model, the goodness-of-test was conducted and the result showed that the mixture model was a good fit in Black-Scholes model at shorter maturity days with small sample sizes, but not a good fit when the options have longer maturity days with larger sample sizes.

Moutanabbir et al. (2023) explicitly expressed for the price of a European option derived when the underlying asset's price followed Exponential Bilateral mixed-Erlang (EBME) model and also obtained closed-form expressions for the options' Delta and Gamma which have many attractive properties including its denseness in class of all distributions on real-time that justified the use of the underlying model. Their result, illustrated through the pricing of options on the S&P 500 and Euro Stoxx 50 indices, showed that their model was volatility-smile consistent and also showed a good fit to the observed option prices with different maturity. Based on some of these criticisms and violations of the model's assumption(s) and the recent development of some mixed distributions that tried to better the fit of the model with the addition of some distribution in the original model, this paper intends to test whether the

Article DOI: 10.52589/AJSTE-J50VXBA8
DOI URL: https://doi.org/10.52589/AJSTE-J50VXBA8

Mixed-Lognormal-Weibull Option Pricing Model (MLWOPM) comes from the same distribution.

The Mixed Distribution

Mixed-Lognormal-Weibull Distribution (MLWD)

The mixture of two or more component distributions is the most current area of interest in modeling life data, mostly in financial mathematics. This paper demonstrates how to combine the lognormal and Weibull distributions using Method of Maximum Likelihood Estimation to obtain the parameters of the mixed distribution, estimate their properties and substitute the mixed model to Black-Scholes Option Pricing model in a view to test whether the combined distribution (MLWOPM) monotonously comes from the same distribution since the Lognormal and Weibull distributions are among the Extreme Value Distributions.

In the development of this model, it was assumed that the population consists of a mixture of two independent sub-population with zero correlation and each population has its exceptional properties.

The distribution for the mixed population can be expressed as:

$$f\left(x_i\right)=\sum_{i=1}^{n} p_i h_i\left(x_i;\theta_i\right) \tag{1}$$

where, $0 \le p_i \le 1; \sum p_i = 1, i = 1, 2, ..., n;$ θ_i are the parameters representing the mixed distribution, and p_i are mixing parameters, which represent the proportion of combining a number of distributions (See for example, Razali et al. (2008), Kollu et al. (2012), and Sultan and Al-Moisher (2015)).

The probability density function (pdf) of the mixture distribution in Equation (1) is expressed as:

$$f\left(x_i; p, \theta_i\right) = pf_1\left(x_i;\theta_1\right)+\left(1-p\right) f_2\left(x_i;\theta_2\right) \tag{2}$$

where p and (1 – p) are the mixing parameters whose sum is equal to 1.

The pdf of lognormal and Weibull distributions are given, respectively, as:

$$f\left(x_1\right)=\frac{1}{\sqrt{2\pi\sigma^2 x}}\exp\left\{-\frac{1}{2}\frac{\left(\ln x-\mu\right)^2}{\sigma^2}\right\}, x \ge 0, \sigma > 0 \tag{3}$$

and

$$f\left(x_2\right)=\frac{\beta}{\sigma}\left(\frac{x}{\alpha}\right)^{\beta-1}\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, 0 \le x \le \infty, \beta, \sigma > 0 \tag{4}$$

Article DOI: 10.52589/AJSTE-J50VXBA8
DOI URL: https://doi.org/10.52589/AJSTE-J50VXBA8

Advanced Journal of Science, Technology and Engineering
ISSN: 2997-5972
Volume 4, Issue 4, 2024 (pp. 111-124)

And the respective cumulative function of Equation (3) and Equation (4) are respectively given as:

$$F\left(x_1\right)=\frac{(\ln x-\mu)}{\sigma}, x \geq 0,-\infty<\mu<\infty, \sigma>0 \tag{5}$$

and

$$F\left(x_1\right)=\exp\left\{-\frac{x}{\alpha}\right\}^{\beta}, x \geq 0, \beta, \alpha>0 \tag{6}$$

So, the joint Pdf of Equation (3) and Equation (4) are expressed as:

$$h_1\left(x_i;\theta_1\right)+h_2\left(x_i;\theta_2\right)=f\left(x_i;\theta_i\right) \tag{7}$$

Substituting Equation (2) and Equation (3) into Equation (4), we obtain the joint probability density function of the mixing distributions as:

$$f\left(x_i, p, \mu, \sigma^2, \beta, \alpha\right)=p\left(\begin{array}{l}\frac{1}{\sqrt{2\pi\sigma^2 x}}\exp\left\{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma^2}\right)^2\right\} \\ +(1-p)\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}\exp\left\{-\frac{x}{\alpha}\right\}^{\beta}\end{array}\right) \tag{8}$$

And the joint CDF in Equation (8) can be given as:

$$F\left(x_i, \mu, \sigma^2, \beta, \alpha\right)=\frac{p(\ln x-\mu)}{\sigma}+(1-p)\exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\} \tag{9}$$

The Maximum Likelihood Estimation of the Parameters of MLWD

Many researchers have established the Maximum Likelihood Estimation method in obtaining the parameters of mixture distributions (See for example, Ashour (1987), Ahmad and Abdurahman (1994), Sultan and Al-Mosheer (2015), Razali et al, (2008), Elmahdy (2007), Neuman (1998), and Kacecelogu and Wang (1998)).

The MLE $\hat{\theta}$ is obtained as the solution of the likelihood equation as:

$$\frac{\partial\theta}{\partial\theta_i}=0 \tag{10}$$

With its equivalent partial derivative of the log likelihood function given as:

$$\frac{\partial \log L(\theta)}{\partial\theta_i}=0$$

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Advanced Journal of Science, Technology and Engineering
ISSN: 2997-5972
Volume 4, Issue 4, 2024 (pp. 111-124)
www.abjournals.org

$$= \sum_{i=1}^{n} \log\left(f(x)\right) \tag{11}$$

where

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta_i);\ i = 1, 2, \ldots n \tag{12}$$

Therefore, the likelihood function corresponding to the mixture density in Equation (8) is then expressed as:

$$L(\theta) = \prod_{i=1}^{n} \left[h\left(f_1(x_i;\theta_1)\right) + (1-h)\left(f_2(x_i;\theta_2)\right)\right] \tag{13}$$

where $\theta_1 = (\mu, \sigma)$ and $\theta_2 = (\alpha, \beta)$.

This implies that:

$$f\left(x_i, p, \mu, \sigma^2, \beta, \alpha\right) = p\left[\left(2\pi\sigma^2\right)^{-\frac{1}{2}} \prod_{i=1}^{n} \frac{1}{x} \exp\left\{\sum_{i=1}^{n} -\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma^2}\right)^2\right\}\right] +$$
$$\left[(1-p)\frac{\beta}{\alpha}\left(\frac{x}{\sigma}\right)^{n\beta - n} \sum_{i=1}^{n} x_i^{\beta-1} - \ln\left(\alpha^{\beta-1}\right) \exp\left\{-\sum_{i=1}^{n} \frac{x}{\alpha}\right\}^{\beta}\right] \tag{14}$$

Taking the log likelihood function for the mixture distribution in Equation (14), we obtain:

$$\ell(x_i;\theta_1,\theta_2) = \left(\log\left(p\left(-\frac{n}{2}\ln\left(2\pi\sigma^2\right) - \sum_{i=1}^{n} \ln x - \sum_{i=1}^{n} \ln x^2 + \sum_{i=1}^{n}\left(\frac{2\ln x - \mu}{2\sigma^2}\right) - \frac{n\mu}{2\sigma^2}\right)\right)\right)$$
$$+\left(\log\left((1-p)\, n \ln\left(\frac{\beta}{\alpha}\right) + (\beta - 1)\sum_{i=1}^{n} x_i - \ln\left(\alpha^{\beta-1}\right) - \sum_{i=1}^{n}\left(\frac{x}{\alpha}\right)^{\beta}\right)\right) \tag{15}$$

Let $Q,$ be the function of the log likelihood, such that:

$$Q = \left(p, \mu, \sigma^2, \alpha, \beta\right) = \sum_{i=1}^{n}\left[\log\left(\begin{array}{l} p\left(-\frac{n}{2}\ln\left(2\pi\sigma^2\right) - \ln x - \frac{\ln x^2}{2\sigma^2} + \frac{\ln x - \mu}{2\sigma^2} - \frac{n\mu}{2\sigma^2}\right) \\ +\left((1-p)\, n \ln\left(\frac{\beta}{\alpha}\right) + (\beta - 1) x_i - \ln\left(\alpha^{\beta-1}\right) - \left(\frac{x}{\alpha}\right)^{\beta}\right) \end{array}\right)\right] \tag{16}$$

Taking the partial derivative of the log likelihood function of Equation (16), with respect to the parameters and in turn equating to zero yields the following equations:

Article DOI: 10.52589/AJSTE-J50VXBA8
DOI URL: https://doi.org/10.52589/AJSTE-J50VXBA8

$$\frac{\partial Q}{\partial \mu}=\sum_{i=1}^{n}\frac{\ln x}{\sigma^2}-\frac{n\mu}{\sigma^2}=0$$

$$\hat{\mu}_{mix}=\sum_{i=1}^{n}\frac{\ln x}{n} \tag{17}$$

$$\frac{\partial Q}{\partial \sigma^2}=-\frac{n}{2\sigma^2}-\sum_{i=1}^{n}\left(\frac{\ln x-\mu}{2}\right)^2\left(-\sigma^2\right)^{-2}$$

$$\Rightarrow n=\sum_{i=1}^{n}\left(\frac{\ln x-\mu}{\sigma^2}\right)^2$$

$$\therefore \hat{\sigma}^2_{mix}=\sum_{i=1}^{n}\left(\frac{\ln x-\hat{\mu}}{n}\right)^2 \tag{18}$$

$$\frac{\partial Q}{\partial \beta}=\frac{n}{\beta}+\sum_{i=1}^{n}x_i-\frac{1}{\alpha}\sum_{i=1}^{n}x_i^{\beta}-\frac{1}{\alpha}=0$$

$$=\frac{1}{\beta}+\frac{1}{n}\sum_{i=1}^{n}\ln x_i-\ln x_i=0\Rightarrow \hat{\beta}_{mix}=\frac{1}{\ln x_i-\frac{1}{n}\sum_{i=1}^{n}x_i} \tag{19}$$

$$\frac{\partial Q}{\partial \alpha}=-\frac{n}{2}+\frac{1}{\alpha^2}\sum_{i=1}^{n}x_i^{\beta}=0\Rightarrow \hat{\alpha}_{mix}=\frac{1}{n}\sum_{i=1}^{n}x_i^{\beta} \tag{20}$$

$$\frac{\partial Q}{\partial p}=\frac{f_1(x_i;\theta_1)-f_2(x_i;\theta_2)}{pf_1(x_i;\theta_1)+(1-p)f_2(x_i;\theta_2)}=\frac{f_1(x_i;\mu,\sigma^2)-f_2(x_i;\alpha,\beta)}{pf_1(x_i;\mu,\sigma^2)+(1-p)f_2(x_i;\alpha,\beta)} \tag{21}$$

Some Properties of MLWD

(i) The Mean: $$E(X_{mix})=p\left(\exp\left\{\mu+\frac{1}{2}\sigma^2\right)+(1-p)\alpha\Gamma\left(1+\frac{1}{\beta}\right) \tag{22}$$

(ii) The Variance: $Var(X_{mix})=E(X)-[EX]^2$

$$=p\left(\exp\left\{-2\mu+\sigma^2\right)\left(\exp\left\{-\left(\sigma^2-p\right)\right)+(1-p)\alpha^2\Gamma\left(1+\frac{2}{\beta}\right)$$

$$-(1-p)\Gamma^2\left(1+\frac{1}{\beta}\right)-2p(1-p)\left(\exp\left\{-\left(\mu+\frac{1}{2}\sigma^2\right)\Gamma\left(1+\frac{2}{\beta}\right)\right)\right. \tag{23}$$

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DOI URL: https://doi.org/10.52589/AJSTE-J50VXBA8

Advanced Journal of Science, Technology and Engineering
ISSN: 2997-5972
Volume 4, Issue 4, 2024 (pp. 111-124)

(iii) The Skewness:

$$skew\left(X_{mix}\right)=p\left(\exp\left\{\sigma^2+2\right)\sqrt{\left(\exp\left\{\sigma^2-1\right)}+(1-p)\frac{1}{\alpha^{\frac{1}{\beta}}}\left[\left(2\gamma_1\right)^3-3\gamma_1\Gamma\left(1+\frac{2}{\beta}\right)+\Gamma\left(1+\frac{3}{\beta}\right)\right] \quad (24)$$

(iv) The Kurtosis:

$$kurt\left(X_{mix}\right)=p\left(\exp\left\{4\sigma^2\right)+2\left(\exp\left\{\sigma^4\right)+3\exp\left(\sigma^4\right)-3\right)+(1-p)\Gamma\left(1+\frac{4}{\beta}\right)-4\gamma_1\Gamma\left(1+\frac{4}{\beta}\right)+6\gamma_1^2\Gamma\left(1+\frac{2}{\beta}\right)-\frac{3\gamma_1^4}{\gamma_2} \quad (25)$$

where $\gamma_1=\Gamma\left(1+\frac{1}{\beta}\right),\ \gamma_2=\left[\Gamma\left(1+\frac{2}{\beta}\right)-\Gamma^2\left(1+\frac{1}{\beta}\right)^2\right]$

(i) Reliability (Survival) Function:

$$R_{mix}\left(X_t/\theta_i\right)=\sum_{i=1}^{n}\left[p\left(1-\Phi\left(\frac{\ln(x)-\mu}{\sigma}\right)^2\right)+(1-p)\left(1-\exp\left\{\frac{X_t}{\alpha}\right)^2\right] \quad (26)$$

(ii) Hazard Function:

$$h_{mix}\left(X_t/\theta_i\right)=\sum_{i=1}^{n}\left(p\left[\frac{\frac{1}{\sqrt{2\pi\sigma x}}\exp\left\{-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2\right]}{1-\Phi\left(\frac{\ln(x)-\mu}{\sigma}\right)^2}\right]+(1-p)\left(\frac{\beta}{\sigma}\right)\left(\frac{X_t}{\alpha}\right)^{\beta-1}\right. \quad (27)$$

The Mixed-Lognormal-Weibull Option Pricing Model (MLWOPM)

The call option price of the model is given as:

$$C_t\left(X_t,K\right)=e^{-rT}\int_0^{\infty}\left(X_t-K\right)f\left(X_t\right)dX_t \quad (28)$$

This implies that the mixture distribution for the Mixed-lognormal-Weibull Option Pricing Model is expressed as:

$$f_{\theta_i}^{mix}\left(x_t,\theta_1,\theta_2\right)=pf_1\left(x_t,\theta_1\right)+(1-p)f_2\left(x_t,\theta_2\right)$$

$$=pf_1^{\log}\left(x_t,\theta_1\right)+(1-p)f_2^{web}\left(x_t,\theta_2\right)$$

$$=e^{-rT}\left[\int_0^{\infty}pf_1^{\log}\left(x_t,\theta_1\right)+(1-p)f_2^{web}\left(x_t,\theta_2\right)\right. \quad (29)$$

Article DOI: 10.52589/AJSTE-J50VXBA8
DOI URL: https://doi.org/10.52589/AJSTE-J50VXBA8

Advanced Journal of Science, Technology and Engineering
ISSN: 2997-5972
Volume 4, Issue 4, 2024 (pp. 111-124)

Substituting the Black-Scholes Models for both lognormal and Weibull in Equation (28), we obtain:

$$e^{-rT}\left[\begin{array}{l} p\left(\dfrac{X_0\Phi\ln\left(\dfrac{X_0}{K}\right)-\left(r+\dfrac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}-Ke^{-rT}\ln\left(\dfrac{X_0}{K}\right)-\left(r-\dfrac{\sigma^2}{2}\right)T\right) \\ +(1-p)X_0\left(\alpha\Gamma\left(1+\dfrac{1}{\beta}\right)-\Gamma_y\left(\dfrac{1+\dfrac{1}{\beta}}{\Gamma\left(1+\dfrac{1}{\beta}\right)}\right)\right)-Ke^{-rT}\exp(-y) \end{array}\right] \tag{30}$$

$$e^{-rT}\left[\begin{array}{l} p\left(\dfrac{X_0\Phi\ln\left(\dfrac{X_0}{K}\right)-\left(r+\dfrac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}-p\left(Ke^{-rT}\ln\left(\dfrac{X_0}{K}\right)-\left(r-\dfrac{\sigma^2}{2}\right)T\right)\right) \\ +(1-p)X_0\left[\alpha\Gamma\left(1+\dfrac{1}{\beta}\right)-\Gamma_y\left(\dfrac{1+\dfrac{1}{\beta}}{\Gamma\left(1+\dfrac{1}{\beta}\right)}\right)\right]-(1-p)\left(Ke^{-rT}\exp(-y)\right) \end{array}\right] \tag{31}$$

Collecting the like terms together, we obtain:

$$e^{-rT}\left[p(1-p)X_0\left(\begin{array}{l} \dfrac{\Phi\ln\left(\dfrac{X_0}{K}\right)-\left(r+\dfrac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}-\left(\alpha\Gamma\left(1+\dfrac{1}{\beta}\right)-\left(\dfrac{\Gamma_y\left(1+\dfrac{1}{\beta}\right)}{\Gamma\left(1+\dfrac{1}{\beta}\right)}\right)\right) \\ -p(1-p)Ke^{-rT}\dfrac{\left(\ln\left(\dfrac{X_0}{K}\right)-\left(r-\dfrac{\sigma^2}{2}\right)T\right)}{\sigma\sqrt{T}}+\exp(-y) \end{array}\right)\right] \tag{32}$$

$$=\left[p(1-p)X_0\Phi d_1-p(1-p)Ke^{-rT}\Phi d_2\right]$$

Hence, the call price of the mixed model is given as:

$$C_{tMix}=p(1-p)\left[X_0\Phi d_1-Ke^{-rT}\Phi d_2\right] \tag{33}$$

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METHODOLOGY

Data and Data Description

The data for this study were obtained from Australian Clearing House of Australian Securities Exchange (ASX). The sample consisted of fifty (50) enlisted stocks in the clearing house as products of monthly market summary for long term options covering the period of January 3rd, 2017 to December 31st, 2019 when there were no significant structural changes among the products. For each transaction, our sample contains the following information: the opening and closing dates of the options, option prices comprising opening and closing prices (otherwise referred in our case as the underlying and strike prices respectively). The final sample consists of 50 stocks for the period of 36 months (720 trading days). The maturity period of the options was gotten from the difference between the opening date and closing date of the options over the trading days. The data for the analysis were arranged in accordance to the maturity days of 25, 27, 28, 29 and 30 days. The data were actually obtained at http//www2.asx.com.au/content/dam/asx/participants/derivatives-market/equity-derivatives/equity-derivatives-statistics/2017/annual-market-summary-2017.xls

Method of Data Analysis

The procedure employed for this study will estimate the absolute returns of the underlying price and the volatility from annualized standard deviation/implied volatility using log-difference of option prices that equates to theoretical option pricing models.

The data in each of the maturity days/expiration time were tested in accordance with 252 trading days.

The Computation of the Annualized Standard Deviation/Implied Volatility is illustrated as follows:

Let $\mathrm{X}_i = ABS \ln\left(\frac{\mathrm{X}_t}{\mathrm{X}_{t-1}}\right)$, X_t is the underlying option price at time t.

$\bar{x} = \frac{1}{n}\sum_{i-1}^{n} x_i$ and $\sigma_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$

So that the implied volatility is obtained by:

$$\hat{\sigma}_{im} = \sqrt{\frac{T}{n}\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} = \sqrt{\frac{T}{n}V\,\mathrm{ar}(X)} \quad (34)$$

where

T is 252 trading days per annum and n is number of stocks. Then, the rate of return is estimated by:

$$r = \frac{1}{T}\ln\left(\frac{K}{X_0}\right) \quad (35)$$

Article DOI: 10.52589/AJSTE-J50VXBA8
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Advanced Journal of Science, Technology and Engineering
ISSN: 2997-5972
Volume 4, Issue 4, 2024 (pp. 111-124)
www.abjournals.org

Testing Whether MLWD Option Pricing Models Come from the Same Population

To test whether the price of the options from the predicted models come from the same distribution, we use the predicted models as $P_j(\hat{\sigma})_{BS}$ and $P_j(\hat{\sigma})_W$

where

$P_j(\hat{\sigma})_{BS}$ is the predicted price of the option from Black-Scholes pricing model

$P_j(\hat{\sigma})_W$ is the predicted price of the option from Weibull pricing model

Since the distributions are unknown, an appropriate non-parametric test statistic (Wilcoxon Rank Sum test) would be used where all the realizations of $P_j(\hat{\sigma})_{BS}$ and $P_j(\hat{\sigma})_W$ are combined and ranked according to their magnitude, smaller values assigned to small values and equal ranks to equal values under the null hypothesis that the two pricing models come from the same population.

The test statistic is the standardized sum of ranks for large sample sizes, and it is given by:

$$W_t = \frac{T - U_t}{V_t} \sim N(0,1)$$

(36)

where

$$\text{T} = \frac{mn + n(n+1)}{2} - R_s; \tag{37}$$

$$\text{U}_t = \frac{mn}{2}; \tag{38}$$

and

$$\text{V}_t = \sqrt{\frac{mn + (m+n+1)}{12}} \tag{39}$$

R_s is the sum of small ranks among the pricing option models;

m is the number of observed ranks from Black-Scholes option pricing model;

n is the number of observed ranks from Weibull option pricing model.

The null hypothesis would be rejected at 5% level of significance if the P-value is less than 0.05; otherwise, the null hypothesis would not be rejected.

Article DOI: 10.52589/AJSTE-J50VXBA8
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Goodness-of-Fit Test of MLWD Option Pricing Model

The Goodness-of-fit test seeks to measure how well an observed data supports an assumption about the distribution of a population of interest. It is based upon how good a fit we have between actually observed frequencies of the sample data and the theoretical frequencies obtained from a hypothesis distribution. In this paper, we wish to test the hypothesis whether the Mixed-Lognormal-Weibull option pricing model is a good fit for pricing options.

The null hypothesis would be rejected (that it is not a good fit) if the calculated value of the Chi-Square is greater than the tabulated value $\left(\chi_c^2 > \chi_\alpha^2\right)$, ; otherwise, the null hypothesis would not be rejected.

The test statistic approximately follows the Chi-square distribution with $k-1$, degrees of freedom at 5% level of significance, given by:

$$\chi_c^2 = \sum_{j=1}^{N} \frac{\left(P_j - P_j\left(\hat{\sigma}\right)\right)^2}{P_j\left(\hat{\sigma}\right)} \tag{40}$$

where

P_j is the observed option price of the jth category, $P_j\left(\hat{\sigma}\right)$ is the expected (predicted) option pricing model of the jth category, and N is the sample size.

RESULTS AND DISCUSSIONS

This section presents the applications of the models to ASX data and discussion of results.

Presentation of Descriptive Statistics of ASX Data

Maturity Days	**Sample Size**	**Sample Mean** $\left(\bar{X}\right)$	**Sample Variance** $\left(S^2\right)$	**Sample Std. Dev** $\left(S\right)$	**Skewness** $Skew\left(X\right)$	**Kurtosis** $Kurt\left(X\right)$	**Implied Volatility** $\left(\hat{\sigma}_{im}\right)$	**Rate of Return (r)**
25	99	0.0034	2.5495	1.5967	-0.5583	4.2359	2.55	-0.01
27	199	0.0019	2.6503	1.6280	-0.5529	4.2529	1.83	-0.02
28	399	0.00092	2.7046	1.6446	-0.5453	3.9624	1.31	-0.01
29	449	0.00053	2.8253	1.6809	-0.5616	4.1640	1.26	-0.03
30	499	0.00057	2.7430	1.6562	-0.5544	4.0257	1.18	0.01

Table 1: Summary Statistics of ASX Original Data

Table 1 is the descriptive statistics of the original data of Australian Stock Exchange for the period under study. From the results, the original data showed non-normality since the skewness from the various maturity days were all negative, which indicates that the left tail of the distribution is longer than the right and their kurtosis also suggested that the distribution is

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Advanced Journal of Science, Technology and Engineering
ISSN: 2997-5972
Volume 4, Issue 4, 2024 (pp. 111-124)

perfectly peaked. This result indicates that the options from ASX do not follow normal distribution.

Table 2: Summary Statistics of Absolute Returns of ASX Original Data

Maturity Days	Sample Size	Sample Mean	Sample Standard Dev	Skewness	Kurtosis	Implied Volatility	Rate of Return (r)
25	99	1.1983	1.0483	1.5414	6.0265	1.6726	-0.01
27	199	1.2150	1.0801	1.5255	5.8071	1.2154	-0.02
28	399	1.2408	1.0780	1.4158	5.2195	0.8564	-0.01
29	449	1.2593	1.1117	1.4866	5.6435	0.8329	-0.03
30	499	1.2466	1.0890	1.4330	5.3350	0.7740	0.01

Table 2 shows approximately equal sample means and standard deviations for all the maturity days. The result also revealed that all skewnesses are positive, therefore indicating that the right tail of the distribution is longer and taller than the left tail. Thus, this result proved that the absolute returns of ASX data for the period of study follows a normal distribution.

Testing Whether MLWOPMs Come from the Same Population

H_0 : Mixed-Lognormal-Weibull Option Pricing Models (MLWOPMs) come from the same population

H_1 : Mixed-Lognormal-Weibull Option Pricing Models (MLWOPMs) do not come from the same population.

Table 3: The Output of Wilcoxon Sum Rank Test for MLWOPM Option Pricing Models

Maturity Days	Sample Size	U_t	T	V_t	W	P-Value	Decision
25	100	5000	9950.5	409.27	10000	2.2×10^{-16}	Reject
27	200	20000	39900.5	1156.14	40000	2.2×10^{-16}	Reject
28	400	80000	159800.5	3268.03	160000	2.2×10^{-16}	Reject
29	450	101250	202275.5	3899.28	202500	2.2×10^{-16}	Reject
30	500	125000	249750.5	4566.64	25044	2.2×10^{-16}	Reject

From Table 3, it is observed that the P-value is less than the significance level of 0.05 (P<0.05); therefore, we reject the null hypothesis and conclude that the model is significantly different. Hence, they do not come from the same population.

Evaluation of MLWD in Black-Scholes Call Option Pricing Model

In this section, we test the null hypothesis whether the MLWD option pricing model is a good fit for pricing options against the alternative that it is not a good fit for the option pricing model.

Article DOI: 10.52589/AJSTE-J50VXBA8
DOI URL: https://doi.org/10.52589/AJSTE-J50VXBA8

Advanced Journal of Science, Technology and Engineering
ISSN: 2997-5972
Volume 4, Issue 4, 2024 (pp. 111-124)

H_0 : MLWD is a good fit for Black-Scholes Option Pricing Model.

H_1 : MLWD is not a good fit for Black-Scholes Option Pricing Model.

Table 4: Summary Statistics of MLWD Parameters Using the Absolute Returns of ASX Data

Maturity Days	Sample Size	Mean	Std. Dev.	Skewness	Kurtosis
25	99	9.2541	11.5595	20.26	79.27
27	199	7.8939	7.6259	22.42	102.26
28	399	12.8939	18.3859	22.28	102.96
29	449	11.6112	15.7512	24.71	134.88
30	499	9.6934	11.5192	23.10	111.98

The results in Table 4 revealed that MLWD is positively skewed and has excess kurtosis. It indicates that the distribution is right tailed and also leptokurtic in nature.

Table 5: Goodness-of-fit Test for MLWD Parameters Using the Absolute Returns of ASX Data

Maturity Days	Sample Size	χ^2	df	P-Value	Decision
25	100	9408	9312	0.2401	Accept
27	200	38400	0.0822	0.2433	Accept
28	400	149600	148478	0.0199	Reject
29	450	187650	185565	0.0003	Reject
30	500	222055	220720	0.0224	Reject

From the result displayed in Table 5, it is observed that the null hypothesis of a good fit is accepted ($P>0.05$) only at the maturity days of 25 and 27 and is rejected ($P<0.05$) at the maturity days of 28, 29 and 30. Therefore, we confirmed that MLWD is a good fit in Black-Scholes Option Pricing Model at shorter maturity or expiration days and at small sample sizes, but not valuable when options contain longer days of expiration and large sample sizes.

CONCLUSION

Based on the findings of this paper, the Mixed-Lognormal-Weibull distributions applied in Black-Scholes Option Pricing Model are statistically different since they do not come from the same distribution and the model is a good fit in Black-Scholes Model only when the expiration days of the option are shorter with small sample sizes.

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