Advanced Journal of Science, Technology and Engineering
ISSN: 2997-5972
Volume 5, Issue 1, 2025 (pp. 85-93)
www.abjournals.org

EVALUATING THE PERFORMANCE OF LAPLACE AND ITS VARIANTS IN MODELLING ECONOMIC DATA

Okafor S. E.[1] and Aronu C. O.[2*]

[1]Census and Survey Department, Anambra State Bureau of Statistics, Anambra State, Nigeria.
[2]Department of Statistics, Chukwuemeka Odumegwu Ojukwu University, Uli Campus, Anambra State, Nigeria.

[*]Corresponding Author's Email: amaro4baya@yahoo.com

Cite this article:

Okafor, S. E., Aronu, C. O. (2025), Evaluating the Performance of Laplace and Its Variants in Modelling Economic Data. Advanced Journal of Science, Technology and Engineering 5(1), 85-93. DOI: 10.52589/AJSTE-0YVU9C0V

Manuscript History

Received: 15 Jan 2025

Accepted: 27 Feb 2025

Published: 19 Mar 2025

ABSTRACT: *The Laplace distribution and its extensions have been widely utilized in statistical modelling due to their ability to capture real-world data characteristics such as skewness and heavy tails. This study evaluated the performance of the classical Laplace (L) distribution against three of its variants: the Transmuted Laplace (TL), Alternative Laplace (AL), and Asymmetric Laplace (ASL) distributions. While these extensions introduce additional parameters to enhance flexibility, their empirical performance remains a subject of interest. Using three datasets Rent prices, Voltage Drop, and Nigeria's Unemployment Rate, this study assessed model fit based on the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Mean Squared Error (MSE). Findings revealed that the standard Laplace (L) distribution consistently outperforms its counterparts. In the Rent dataset, it achieves the lowest AIC (613.636), BIC (609.2266), and a reasonable MSE (2343.761), whereas the TL and AL distributions yield significantly higher AIC and BIC values, and the ASL distribution demonstrates an extremely high MSE (9.34×10^{12}), indicating poor fit. A similar trend is observed in the Voltage Drop dataset, where the L distribution records the lowest AIC (201.1564), BIC (197.7293), and MSE (132.7978), while TL and ASL show excessive model instability. In the Unemployment Rate dataset, the L distribution again provides the best fit, with an AIC of 349.7985, a BIC of 345.896, and a moderate MSE of 186.4666. On average, across all datasets, the L distribution remains the most robust model, with the lowest AIC (388.197), BIC (384.284), and MSE (887.6751). The AL distribution follows closely with an MSE of 888.9518 but exhibits significantly higher AIC (2426.027) and BIC (2424.071). The ASL distribution, while demonstrating moderate AIC (1443.016) and BIC (1448.885), suffers from poor predictive accuracy with an extremely high MSE (3.19E+12). The TL distribution performs the worst, with the highest AIC (34,686.77), BIC (20,112.08), and an MSE of 76,038.22, highlighting its instability. In conclusion, this study established that the standard Laplace (L) distribution provides the most reliable and accurate fit across diverse datasets. While alternative forms introduce additional flexibility, their increased complexity does not necessarily yield superior model performance. Future research should explore modifications to improve the parameter stability of Laplace extensions and investigate alternative estimation techniques to enhance predictive accuracy in real-world applications.*

KEYWORDS: Laplace distribution, Transmuted laplace, Alternative laplace, Asymmetric laplace, Model fit, Laplace extensions.

Article DOI: 10.52589/AJSTE-0YVU9C0V
DOI URL: https://doi.org/10.52589/AJSTE-0YVU9C0V

INTRODUCTION

The Laplace distribution, originally formulated to model double-exponential decay phenomena, has undergone substantial theoretical and methodological advancements (Laplace, 1774). These developments have led to various modifications and extensions, significantly enhancing its adaptability to complex real-world data. Variants of the Laplace distribution offer improved flexibility, better fit, and enhanced modelling capabilities across diverse domains, particularly in economic and financial applications. Despite these advancements, a comprehensive evaluation of the relative performance of these variants remains limited, highlighting a critical research gap that this study aims to address. One of the notable extensions, the beta Laplace distribution introduced by Cordeiro and Lemonte (2011), incorporates a beta transformation to extend the distribution's structural properties and applicability. Mahmoudvand et al. (2015) modified the Laplace distribution by introducing a symmetric variant, emphasizing its robustness through real-world applications. Other significant contributions include the transmuted Laplace distribution by Hady and Shalaby (2016), which enhances flexibility in capturing complex data structures, and the alternative Laplace distribution by Kumar and Jose (2019), known for its bimodal and unimodal characteristics. Furthermore, the q-Esscher-transformed Laplace distribution (Rimsha & George, 2019) and the spherical Laplace distribution (You & Shung, 2023) have broadened the application of the Laplace family in areas such as entropy optimization and directional statistics. Additionally, advancements in parameter estimation methodologies (Wright, 2024) and novel skewness parameterizations (Khandeparkar & Dixit, 2023) further highlight the increasing relevance of Laplace distribution variants in handling complex datasets.

While these developments have contributed valuable insights, there remains a lack of a unified framework for assessing the comparative performance of these Laplace variants. With their increasing adoption in fields such as finance, econometrics, reliability analysis, and environmental studies, it is crucial to systematically evaluate their effectiveness under varying data conditions. In particular, a direct comparison of key variants including the standard Laplace (L), transmuted Laplace (TL), alternative Laplace (AL), and asymmetric Laplace (ASL) distributions remains absent in existing literature. Such an analysis is essential for understanding their relative strengths and weaknesses, thereby guiding model selection for specific applications. For example, while the transmuted Laplace distribution is recognized for its flexibility in capturing complex distributions (Hady & Shalaby, 2016), the alternative Laplace distribution offers tunable kurtosis properties (Kumar & Jose, 2019). The asymmetric Laplace distribution, on the other hand, is particularly suited for skewed data and has found extensive applications in econometrics and finance (Wright, 2024).

In addition, recent studies have explored various extensions and applications of the Laplace distribution, demonstrating its adaptability in handling asymmetric noise, parameter estimation challenges, and real-world data complexities. Xu et al. (2021) enhanced the robustness of linear parameter varying (LPV) models by integrating the asymmetric Laplace distribution into expectation-maximization algorithms. Their approach improved parameter estimation in the presence of asymmetric noise and outliers, outperforming traditional Gaussian noise models. The study also introduced the transmuted Laplace distribution (TLD), leveraging the quadratic rank transmutation map to refine probabilistic models. Applied to ball-bearing lifetime data, TLD exhibited superior reliability over conventional Laplace models. Natido and Kozubowski (2023) expanded statistical modelling by combining uniform and Laplace distributions, tackling computational challenges in expectation-maximization techniques. Their stochastic

Article DOI: 10.52589/AJSTE-0YVU9C0V
DOI URL: https://doi.org/10.52589/AJSTE-0YVU9C0V

model demonstrated improved parameter estimation accuracy through synthetic data analysis. Similarly, Bapat et al. (2023) introduced a two-parameter distribution merging Laplace and a modified binomial component. Their model, which preserved key statistical properties such as skewness and kurtosis, showed superior predictive performance in finance and healthcare datasets. Kozubowski et al. (2023) extended the generalized asymmetric Laplace (GAL) distribution to matrix variate cases, unveiling Type I and Type II GAL distributions. This extension provided new tools for analyzing complex panel data, with implications for finance and econometrics. Thakur et al. (2023) explored the Neutrosophic Laplace distribution, designed for heavy-tailed data modeling. Applied to NIFTY50 stock market returns, their model effectively captured uncertainty in financial time series. Ibrahim and Khan (2024) further developed the Neutrosophic Laplace Distribution (LDN) as a flexible tool for real-world problems involving imprecise information. Using maximum likelihood estimation and simulation studies, they demonstrated LDN's efficacy in reliability analysis and environmental modelling, particularly for pollutant concentration data. Okafor et al. (2025) conducted an empirical study comparing the performance of four key Laplace variants such as L, TL, AL, and ASL using model selection criteria such as the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Mean Squared Error (MSE). Their findings, based on simulations from an approximately normal distribution, suggested that L and AL perform well for symmetric data, while TL and ASL are better suited for handling skewed distributions. These insights underscore the importance of further evaluating the practical implications of these models on real-world datasets. These advancements highlight the growing versatility of Laplace-based distributions, offering robust solutions for noise handling, financial modelling, and uncertainty quantification.

Thus, this study aims to bridge the existing research gap by systematically comparing these variants using real-world datasets, including voltage drop, rent variations, and Nigeria's unemployment rate. By assessing their performance using key model selection criteria, this research seeks to provide practical insights into the selection of appropriate Laplace variants for statistical and econometric applications. The objectives of the study are to assess the suitability of Laplace and its variant distributions (Transmuted Laplace, Alternative Laplace, and Asymmetric Laplace) for modelling different types of real-world datasets; to compare the performance of these distributions using model selection criteria such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Mean Squared Error (MSE); and to determine the best-performing distribution for each dataset, including voltage drop, rent variations, and Nigeria's unemployment rate.

METHODS

Method of Data Collection

The present study relied on secondary data sources to ensure access to relevant and reliable information. The datasets used include voltage drop data, rent data, and unemployment rate data. The voltage drop dataset, which captures battery voltage drop in a guided missile motor during flight, was sourced from Montgomery et al. (2015). Rent variation data for agricultural land planted to Lucerne across 67 counties in Minnesota, as discussed in 1977, was obtained from Weisberg (2005). Additionally, unemployment rate data in Nigeria from 1970 to 2021 was extracted from the Central Bank of Nigeria (CBN) Statistical Bulletin (2022).

Article DOI: 10.52589/AJSTE-0YVU9C0V
DOI URL: https://doi.org/10.52589/AJSTE-0YVU9C0V

Method of Data Analysis

Table 1 presents the probability density functions (PDFs) of Laplace and its related distributions, highlighting their structural variations. These distributions accommodate asymmetry, transmutation, and alternative parameterizations for broader statistical applications.

Table 1: Overview of Laplace Variants and Their Probability Density Functions

S/No.	Distribution	PDF
1.	Laplace (L) Distribution	$\frac{1}{2b}e^{\left(-\frac{\lvert x-\mu\rvert}{b}\right)}, b > 0, \mu \in R, x \in R$
2	Transmuted Laplace (TL) Distribution	$\frac{1}{2\beta}e^{\left(-\frac{1}{\beta}\lvert x\rvert\right)\left\{1+\lambda\, sgn(x)\left[e^{\left(-\frac{1}{\beta}\lvert x\rvert\right)}-1\right]\right\}}, \beta > 0, \lambda \in [-1,1], x \in R$
3	Alternative Laplace (AL) Distribution	$\frac{1}{2(\alpha+1)}(1+\alpha\lvert x\rvert)e^{-\lvert x\rvert}, \alpha > -1, \mathrm{x} \in \mathrm{R}$
4	Asymmetric Laplace (ASL) Distribution	$\left(\frac{\lambda}{k+\frac{1}{k}}\right)e^{-(x-m)\lambda\, sgn(x-m)k^{sgn(x-m)}}, \lambda > 0, \mathrm{k} > 0, \mathrm{m} \in \mathrm{R}, x \in R$

Source: Okafor et al. (2025)

The classical Laplace (L) distribution is defined by a symmetric exponential decay around the location parameter μ with scale parameter b. The Transmuted Laplace (TL) distribution introduces an additional parameter λ that adjusts tail behaviour, allowing more flexibility in modelling skewness. The Alternative Laplace (AL) distribution modifies the standard Laplace form by incorporating a shape parameter α, impacting peak sharpness and tail thickness. Lastly, the Asymmetric Laplace (ASL) distribution generalizes the Laplace distribution with parameters kg and λ, enabling different decay rates on either side of the mode mmm, making it useful for skewed data modelling. These extensions enhance adaptability for real-world applications requiring flexible distributional assumptions (Okafor et al., 2025).

The analysis involves evaluating the performance of Laplace distribution variants (Laplace, Transmuted Laplace, Alternative Laplace, and Asymmetric Laplace) using the following metrics: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Mean Squared Error (MSE). Each metric is calculated based on the fitted models to the simulated datasets, and decision rules are applied to determine the best-performing variant.

These measures are used for understanding the balance between model fit, complexity, and predictive accuracy.

i. Akaike Information Criterion (AIC)

The AIC can be calculated using the formula presented as

$$AIC = 2k - 2log(L) \quad (1)$$

Where, k is the number of parameters, and L is the maximum value of the likelihood function.

Article DOI: 10.52589/AJSTE-0YVU9C0V
DOI URL: https://doi.org/10.52589/AJSTE-0YVU9C0V

ii. Bayesian Information Criterion (BIC)

The BIC can be calculated using the formula presented as equation (2):

$$BIC = klog(n) - 2log(L) \quad (2)$$

Where, n is the number of data points.

iii. Mean Squared Error (MSE)

The MSE can be calculated using the formula presented as

$$MSE = \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n} \quad (3)$$

Where, y_i are the observed values and $\hat{y}_i$are the predicted values.

For each metric, compare the values across all distribution variants:

AIC and BIC:

a. Lower values indicate a better balance between model fit and complexity.
b. Both metrics penalize overfitting, with BIC being more stringent for larger datasets.

MSE:

Lower MSE values indicate better predictive accuracy and a closer fit to the data.

Decision Rule

i. AIC: Select the model with the lowest AIC as it indicates the best trade-off between goodness-of-fit and model complexity.

ii. BIC: Select the model with the lowest BIC, particularly for datasets where penalizing overfitting is critical (e.g., large sample sizes).

iii. MSE: Select the model with the lowest MSE as it reflects the highest predictive accuracy.

Aggregate Decision

i. Rank the models based on each metric and compute an overall performance score by weighting the metrics equally or as determined by the study's goals (Okafor, 2025).

ii. If a single model consistently performs well across AIC, BIC, and MSE, it is deemed the most robust.

iii. In case of conflicts, prioritize metrics based on the study's focus (e.g., prioritizing MSE for predictive tasks or BIC for large datasets).

Article DOI: 10.52589/AJSTE-0YVU9C0V
DOI URL: https://doi.org/10.52589/AJSTE-0YVU9C0V

RESULTS

Table 2 presents the parameter estimates and model evaluation metrics for different Laplace-based distributions applied to three datasets: Rent, Voltage Drop, and Unemployment Rate in Nigeria. The table includes Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Mean Squared Error (MSE), providing insights into the comparative performance of each distribution.

Table 2: Model Fit Comparison for Laplace Variants across Different Datasets

Dataset	Distributions	Parameter estimates	AIC	BIC	MSE
Rent	L	$\mu = 44.5601$, b=18.4370	613.636	609.2266	2343.761
	TL	$\beta = -0.2445, \lambda = -8.10\ e + 09$	50951.74	50956.15	Inf
	AL	$\alpha = 1677723$	5449.531	5447.326	2344.776
	ASL	$\mu = -1.84\ e + 05, \lambda = 3549687, k = 2.29\ e - 13$	2006.953	2013.567	9336374000000
Voltage drop	L	$\mu = 11.1499$, b=2.2434	201.1564	197.7293	132.7978
	TL	$\beta = -0.0438, \lambda = -733270.5$	43797.32	64.9400	Inf
	AL	$\alpha = 6710887$	787.4366	785.723	134.8959
	ASL	$\mu = -85878.72, \lambda = 119735.6, k = 4.79\ e - 12$	960.7896	965.9303	12988789743
Unemployment Rate in Nigeria	L	$\mu = 8.5097$, b=5.5209	349.7985	345.896	186.4666
	TL	$\beta = -0.2736, \lambda = -55.1433$	9311.244	9315.146	76038.22
	AL	$\alpha = 6710887$	1041.114	1039.163	187.1835
	ASL	$\mu = -342308.3, \lambda = 474304.6, k = 2.33\ e - 13$	1361.304	1367.158	208560294899

The result obtained in Table 2 presents the AIC, BIC, and MSE values across the three datasets used in the study. The findings showed that the standard Laplace (L) distribution consistently demonstrates the best model fit. For the Rent dataset, the Laplace distribution has the lowest AIC (613.636) and BIC (609.2266), with a reasonable MSE (2343.761), whereas the Transmuted Laplace (TL) and Alternative Laplace (AL) distributions show significantly higher AIC and BIC values, and the ASL distribution exhibits an extremely high MSE (9.34×10^{12}), indicating poor fit. Similarly, for the Voltage Drop dataset, the Laplace distribution yields the lowest AIC (201.1564) and BIC (197.7293) with the lowest MSE (132.7978), whereas the TL and ASL distributions yield unrealistic or excessively high values. In the Unemployment Rate dataset, the Laplace distribution again achieves the lowest AIC (349.7985) and BIC (345.896) with a moderate MSE (186.4666), suggesting it is the most suitable model. The consistently poor performance of the TL and ASL distributions, particularly their excessively high MSE values, suggests instability in parameter estimation. Overall, the results indicate that the Laplace distribution provides the best model fit across all datasets based on lower AIC, BIC, and MSE values.

Article DOI: 10.52589/AJSTE-0YVU9C0V
DOI URL: https://doi.org/10.52589/AJSTE-0YVU9C0V

Table 3 presents the average AIC, BIC, and MSE values for four Laplace variant distributions. Lower values indicate better model fit, with the standard Laplace (L) distribution outperforming others in overall efficiency.

Table 3: Comparison of Average Model Selection Criteria for Laplace Variants across Datasets

Distributions	Average AIC	Average BIC	Average MSE
L	388.197	384.284	887.6751
TL	34686.77	20112.08	76038.22
AL	2426.027	2424.071	888.9518
ASL	1443.016	1448.885	3.19E+12

Based on the result of the average model selection in Table 3, the standard Laplace (L) distribution demonstrates the best overall fit, with the lowest average AIC (388.197), BIC (384.284), and MSE (887.6751). The Alternative Laplace (AL) distribution follows closely, exhibiting a slightly higher MSE (888.9518) but significantly higher AIC (2426.027) and BIC (2424.071), indicating a comparatively weaker fit. The Asymmetric Laplace (ASL) distribution shows moderate performance in terms of AIC (1443.016) and BIC (1448.885), but its extremely high MSE (3.19E+12) suggests poor predictive accuracy. The Transmuted Laplace (TL) distribution performs the worst, with the highest AIC (34686.77) and BIC (20112.08), alongside a large MSE (76038.22), indicating instability and poor fit. Thus, the standard Laplace distribution emerges as the most reliable model for the given datasets.

CONCLUSION

This study evaluated the performance of the standard Laplace (L) distribution and its variants Transmuted Laplace (TL), Alternative Laplace (AL), and Asymmetric Laplace (ASL) in modelling three distinct economic datasets: rent variations, voltage drop, and Nigeria's unemployment rate. The assessment was conducted using the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Mean Squared Error (MSE) to determine the best-fitting model across different data structures.

The results consistently highlight the superiority of the standard Laplace (L) distribution over its extensions. For all three datasets, the L distribution demonstrated the lowest AIC and BIC values, indicating superior model parsimony and fit, alongside the lowest MSE values, reflecting its predictive accuracy. This result is in line with the findings of Okafor et al. (2025) who employed a simulated dataset to evaluate the performances of the same variants of Laplace distribution. In contrast, the TL and ASL distributions exhibited unstable parameter estimation, with excessively high MSE values, particularly in the rent and voltage drop datasets. The AL distribution, while performing relatively better than TL and ASL, still showed higher AIC and BIC values, suggesting weaker model efficiency.

Article DOI: 10.52589/AJSTE-0YVU9C0V
DOI URL: https://doi.org/10.52589/AJSTE-0YVU9C0V

Overall, the standard Laplace distribution emerged as the most reliable model, maintaining robust performance across varying economic data types. The findings suggest that despite the enhanced flexibility of its variants, their added complexity does not necessarily translate into better model performance. This study underscores the importance of model selection based on empirical evidence rather than theoretical flexibility alone.

Future research could explore modifications to the Laplace variants that improve parameter stability or investigate their applicability to more diverse datasets, particularly in cases where pronounced skewness or heavy tails are expected. Additionally, alternative estimation techniques, such as Bayesian methods or robust optimization frameworks, could enhance the performance of Laplace extensions in practical applications.

By providing an evaluation of Laplace-based models, this study contributes valuable insights for researchers and practitioners in statistical modelling and econometrics, guiding informed choices in selecting appropriate distributions for economic and financial data analysis.

REFERENCES

Bapat, S. R., Bakouch, H. S., & Chesneau, C. (2023). A distribution on via perturbing the Laplace distribution with applications to finance and health data. *Stat, 12*(1).

Cordeiro, G. M., & Lemonte, A. J. (2011). The beta Laplace distribution. *Statistics and Probability Letters, 81*(8), 973-982.

Hady, D. H. A., & Shalaby, R. M. (2016). Transmuted Laplace distribution: Properties and applications. *American Journal of Applied Mathematics and Statistics, 4*(3), 94-98. http://pubs.sciepub.com/ajams/4/3/5

Ibrahim, A. M. M., & Khan, Z. (2024). Neutrosophic Laplace distribution with properties and applications in decision making. *International Journal of Neutrosophic Science*, *23*(1), 73-84.

Khandeparkar, P., & Dixit, V. (2023). Tests for skewness parameter of skew log Laplace distribution. *Model Assisted Statistics and Applications*, *18*(4), 331-346.

Kozubowski, T., Mazur, S., & Podgórski, K. (2023). Matrix variate generalized asymmetric Laplace distributions. *Theory of Probability and Mathematical Statistics, 109*(0), 55-80.

Kumar, C. S., & Jose, R. (2019). An alternative Laplace distribution. *Journal of Statistical Research, 53*(2), 111-127.

Laplace, P. S. (1774). Mémoire sur la probabilité des causes par les évènements. *Mémoires de l'Académie Royale des Sciences, 6*, 621-656.

Mahmoudvand, R., Faradmal, J., Abbasi, N., & Lurz, K. (2015). A new modification of the classical Laplace distribution. *Journal of the Iranian Statistical Society*, *14*(2), 93-118.

Montgomery, D. C., Peck, E. A., & Vining, G. G. (2015). *Introduction to linear regression analysis*. John Wiley & Sons.

Natido, A., & Kozubowski, T. J. (2023). A uniform-Laplace mixture distribution. *Journal of Computational and Applied Mathematics, 429*.

Okafor, S. E., Aronu, C. O. and Ugwu, N. D. (2025). A comparative study of selected Laplace distribution variants for statistical modelling. *International Journal of Advanced Scientific Research*, 10(1): 5-9

Article DOI: 10.52589/AJSTE-0YVU9C0V
DOI URL: https://doi.org/10.52589/AJSTE-0YVU9C0V

Rimsha, H., & George, D. (2019). q-Esscher transformed Laplace distribution. *Communications in Statistics - Theory and Methods*. https://doi.org/10.1080/03610926.2018.1435812

Shaw, W. T., & Buckley, I. R. (2009). The alchemy of probability distributions: Beyond Gram-Charlier expansions, and a skew-kurtosis-normal distribution from a rank transmutation map. *arXiv preprint arXiv:0901.0434*.

Thakur, R., Malik, S. C., & Raj, M. (2023). Neutrosophic Laplace distribution with application in financial data analysis. *Neutrosophic Sets and Systems, 57*, 224-233.

Weisberg, S. (2005). *Applied linear regression*. John Wiley & Sons.

Wright, S. E. (2024). A note on computing maximum likelihood estimates for the three-parameter asymmetric Laplace distribution. *Applied Mathematics and Computation, 464*.

Xu, C., Yang, X., & Yu, M. (2021). Robust LPV models identification approach based on shifted asymmetric Laplace distribution. *Measurement and Control (United Kingdom), 54*(9-10), 1336-1346.

You, K., & Shung, D. (2023). On the spherical Laplace distribution. In 2023 26th International Conference on Information Fusion, FUSION 2023. IEEE. https://doi.org/10.23919/FUSION52260.2023.10224108