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ON EMPIRICAL SELECTION OF LOGNORMAL AND WEIBULL DISTRIBUTIONS: APPLICATION TO NIGERIAN STOCK MARKET

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ABSTRACT: *In this article, we empirically compared and selected the best performed distribution among Lognormal and Weibull distributions. The dataset for this study were monthly stock prices of BUA cement enlisted and trades on Lagos Stock Exchange, a subsidiary of Nigeria Stock Exchange (NSE). The dataset comprised of sample of fifty-eight (58) log-transformed monthly closed stock prices between 2nd January, 2020 and 1st November, 2024, obtained from https://ng.investing.com/equities/bua-cement-plc-historical-data .The Maximum Likelihood Estimator (MLE) was used to obtain the parameters of both Lognormal and Weibull distributions in a view of comparing and selecting the best distribution that fits our dataset. The Minimum Mean Squared Error (MSE) and Akaike Information Criterion (AIC) were used as selection criteria and the Weibull distribution was found to outperform the Lognormal distribution since it exhibited the least MSE and AIC. Also, the selected Weibull distribution was subjected to goodness-of-fit using Kolmogorov-Smirnov test and the empirical evience shows that BUACEM stock price follows a Weibull distribution with 5% level of significance, hence, making the Weibull distribution the right choice for fitting BUACEM stock prices on the Nigerian Stock market.*

KEYWORDS: Lognormal Distribution, Weibull Distribution, Maximum Likelihood Estimate, Goodness-of –fit Test.

INTRODUCTION

It is popularly known that both the lognormal and Weibull distributions can be used quite possibly to analyze skewed data set. Even though, these two distributions may provide similar data fit for reasonable sample sizes, still it is enviable to choose the correct or more nearly correct distribution, since the inferences based on the model will often involve tail probabilities, where the effect of the distributional assumptions are very significant. Therefore, even if we have small or moderate samples, it is still very significant to make the best possible decision based on whatever data available.

The Weibull and the lognormal distributions are the most widely used models for analyzing a variety of data from different fields. It is often seen that the two models quite nicely represent a given data set although the concerned analyses and the related inferential procedures may differ drastically. It is, therefore, highly desirable to study the behavior of the two models for a given set of observations in the light of using statistical tools or techniques of model comparison and selection. The Weibull and Lognormal distributions are both flexible and can be used to model a variety of data but they have different characteristics and best suited for different types of data. The flexibility of Weibull distribution can be used to determine failure rates and also a good fit for bimodal shape distribution. Its mean and standard deviation are strictly linked and its data is skewed. See for example, Gupta (2004), Chen (2006), Sultan, et al (2007), Nwobi and Ugomma (2014).

The Lognormal distribution is best for data with lower value means, large variances and all positive values. It is also a good fit when the natural logarithm of each random variable results in a normal distribution. The lognormal distribution is symmetrical and the data logarithm is symmetrical. See for example, Dick (1998).

Making a statistical choice of distribution or selecting the best-fitting distribution(s) for a given set of data is a vital issue, particularly when the tail probability, which is usually receptive to the assumed model, is of interest as in reliability engineering. Statistical methods for distributional choice include probability plotting (Nelson, 1982), goodness-of-fit (GOF) testing, hypothesis testing (HT), and selection procedures. The Weibull and lognormal distributions are assumed most often in analyzing lifetime data, and in many cases, they are competing with each other. In addition, lifetime data are usually censored due to the some restrictions on the amount of testing time. (Kim and Yum, 2008).

Several researchers have developed many selection procedures with respective selection statistics and decision rules. The two popularly known procedures are those that are respectively based on the Maximum Likelihood Function (MLF) and Scale Invariant (SI) selection statistic. For the MLF-based procedure, see for example, Bain and Engelhardt (1980), Kappenman (1982), Gupta and Kundu, (2003), Kundu and Manglick (2004) and Kundu et al. (2005). In both procedures, the distribution with the largest value of the selection statistic is selected. For other selection procedures, see for example, Croes et al. (1998), Marshall et al. (2001), Cain (2002), Dick (2004) and Mitosek, et al. (2006). Bromideh (2012), examined the use of Kullback-Leibler Divergence (KLD) in discriminating either the Weibull or Lognormal distribution and explained the applicability by a real data set and the consistency of the KLD with the ratio minimized likelihood (RML) was established and the advantage of KLD is that it incorporates entropy of each model.

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Dey and Kundu (2012) considered the problem of discriminating between the two distribution functions. They assumed that the data came either from lognormal or Weibull distributions and that they were Type-II censored. They used the difference of the maximized log-likelihood functions, in discriminating between the two distribution functions and obtained the asymptotic distribution of the discrimination statistic which was used to determine the probability of correct selection in their discrimination process. They performed some simulation studies to observe how the asymptotic results worked for different sample sizes and for different censoring proportions. In the paper, they also observed that the asymptotic results worked quite well even for small sizes if the censoring proportions are not very low. They further suggested a modified discrimination procedure where two real data sets were analyzed for illustrative purposes.

Ugomma (2024a) also compared 2-parameter and 3-parameter Weibull Distribution using monthly average stock price returns of five different bottling companies enlisted in the Nigerian Stock market comprising Nigerian Breweries PLC, Coca-Cola Bottling Company, Guinness Nigerian PLC, Seven Up Bottling Company and International Breweries from 2012 to 2016. The parameters were obtained using the Method of Moments (MOM) and were compared by the means of minimum Mean Squared Error and the result shows that Coca-Cola bottling company stock price return performed better under 2-parameter Weibull Estimate while International Breweries was a choice under 3-Parameter Weibull Estimate. Also, the goodness-of-fit test was conducted using Kolmogorov-Smirnov Test and the result shoewd that the price of the stock returns does not follow a Weibull Distribution with P-value<0.05.

Ugomma (2024b) compared the maximum likelihood estimates (MLE) of lognormal, Weibull and Mixed-lognormal-Weibull distributions. The data for the study were Coca-cola stock price returns obtained from https://ng.www.investing.com/equities/cocacola-bottle-historical-data. With the help of Excel package the result showed that Weibull distribution had the minimum Mean Squared Error $\left(MSE_{\min}\right)$ among the lognormal and Mixed-lognormal-weibull distributions, hence, the maximum likelihood estimate of the Weibull distribution was selected as the best among the distributions.

The problem of testing whether some given observations follow one of the two probability distributions is quite old in the statistical literature. See for example the work of Cox (1962), Cox (1967), Atkinson (1970), Robert and Charles (1973). Chen (1980), Fearn and Nebenzahl (1991), Wiens (1999). A lot of attention has been to the problem of choice of a correct lifetime distributions. Some recent researches in this direction such as Diyali, et al (2023), Kundu and Maglick (2004) and others in the literature as we observed, although has given extensive work on the discrimination of Weibull and Lognormal distribution but laid much emphasis on the failure rates of censored data set, but not enough attention has been given in the literature on the selection of these two models in the financial sector, hence, in this work, we would consider the problem of choice between Weibull and lognormal distributions by using maximum likelihood estimate to obtain the parameters of both distribution and use minimum mean squared error (MSE) in selecting between the two distributions in a bid to select the one of best-fit in the financial sector of the economy, thereby, investigating the suitability of the selected distribution in the financial sector using goodness of fit test

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MATERIALS AND METHOD

Data and Data Description

The dataset for this study were monthly stock prices of BUA cement enlisted and trades on Lagos Stock Exchange, a subsidiary of Nigeria Stock Exchange (NSE). The dataset comprised of sample of fifty-eight (58) monthly closed stock prices between 2nd January, 2020 and 1st November, 2024, obtained from https://ng.investing.com/equities/bua-cement-plc-historical-data

The Lognormal Distribution and its Properties

The parameters of the lognormal distribution can be estimated with use of Maximum Likelihood Estimate (MLE). This is a well-liked approach for approximating distribution parameters as it finds parameters that make our assumed probability distribution most likely for our observed dataset.

Let $Y \sim N(\mu, \sigma^2)$ with mean, μ, and variance, σ^2. Then, the variable,

$$X = \exp(Y) \tag{1}$$

Equation (1) has a lognormal distribution with parameters, μ, and, σ^2, where, σ and e^{μ} are the shape and scale parameters.

If $\bar{Y}$ has a normal distribution, then the probability density function is given by

$$f_y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\frac{(y-\mu)^2}{\sigma^2}\right) \tag{2}$$

The function

$$X = g(Y) = \exp(Y) \tag{3}$$

Equation (3) is strictly increasing, so we can use the formula for the density function of a strictly increasing function which is given by

$$f_X(x) = f_Y\left(g^{-1}(x)\frac{dg^{-1}(x)}{dx}\right) \tag{4}$$

In particular, we have

$$g^{-1}(x) = \ln(x) \Rightarrow \frac{dg^{-1}(x)}{dx} = \frac{1}{x}$$

So that Equation (4) becomes the probability density function given as

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$$f_X x = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\frac{\left(\ln(x)-\mu\right)^2}{\sigma^2}\right)\frac{1}{x} \tag{5}$$

The properties of the lognormal distribution are derived as follows

(i) The Expected Value (Mean)

The expected value of a lognormal random variable X is given by

$$E[X] = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$$

Proof:

The expected value of a lognormal random variable X follows as

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \int_{0}^{\infty} x\frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\frac{\left(\ln(x)-\mu\right)^2}{\sigma^2}\right)$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}\frac{\left(\ln(x)-\mu\right)^2}{\sigma^2}\right)$$

Using change of variable, we have,

$$\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}t^2\right)\sigma \exp(\mu+\sigma t)dt \tag{6}$$

Equation (36) is further simplified as

$$\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}t^2 - \left(2\sigma t+\sigma^2\right)\right)\exp\left(\mu+\frac{1}{2}\sigma^2\right)dt$$

$$= \exp\left(\mu+\frac{1}{2}\sigma^2\right) \tag{7}$$

Where in Equation (6), $t = \dfrac{\ln(x)-\mu}{\sigma}, dt = \dfrac{1}{\sigma x}dx \Rightarrow \sigma x dx = dx \Rightarrow \sigma \exp(\mu+\sigma t)dt = dx$

$x \to 0 \Rightarrow t \to -\infty, x \to \infty \Rightarrow t \to \infty$

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(ii) The Variance

The variance of a lognormal random variable is

$$Var[X] = \exp\left(2\mu + 2\sigma^2\right) - \exp\left(2\mu + \sigma^2\right)$$

Proof:

Let us first derive the second moment as

$$E\left[X^2\right] = \int_{-\infty}^{\infty} x^2 f(x)\,dx \tag{8}$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}t^2\right) \sigma \exp\left(2\sigma t + 2\mu\right) dt \tag{9}$$

Further simplification of Equation (9) yields,

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}\left(t^2 - 4\sigma t + 4\sigma^2\right)\right) \exp\left(2\sigma^2 + 2\mu\right) dt$$

$$= \exp\left(2\sigma^2 + 2\mu\right) \tag{10}$$

Applying the variance formula, we then have

$$Var(X) = E\left[X^2\right] - \left[E[X]\right]^2$$

$$= \exp\left(2\sigma^2 + 2\mu\right) - \left[\exp\left(\mu + \frac{1}{2}\sigma^2\right)\right]^2$$

$$= \exp\left(2\sigma^2 + 2\mu\right) - \exp\left(\sigma^2 + 2\mu\right) \tag{11}$$

(iii) Higher Moments

The n^{th} moment of a lognormal random variable X is

$$E\left[X^n\right] = \exp\left(n\mu + \frac{1}{2}n^2\sigma^2\right)$$

Proof:

$$E\left[X^n\right] = \int_{-\infty}^{\infty} x^n f(x)\,dx \tag{12}$$

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$$\int_0^\infty x^n \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\frac{\left(\ln(x)-\mu^2\right)}{\sigma^2}\right)dx$$

$$\frac{1}{\sqrt{2\pi\sigma^2}}\int_0^\infty x^{n-1} \exp\left(-\frac{1}{2}\frac{\left(\ln(x)-\mu^2\right)}{\sigma^2}\right)dx$$

Using change of variable, we obtain,

$$=\frac{1}{\sqrt{2\pi\sigma^2}}\int_\infty^\infty \exp\left(-\frac{1}{2}t^2\right)\sigma \exp\left(n\sigma t+n\mu\right)dt \quad (13)$$

And further simplification of Equation (13) we obtain

$$=\frac{1}{\sqrt{2\pi}}\int_\infty^\infty \exp\left(-\frac{1}{2}\left(t^2-2n\sigma t+n^2\sigma^2\right)\right)\exp\left(\frac{1}{2}n^2\sigma^2+n\mu\right)dt$$

$$=\exp\left(\frac{1}{2}n^2\sigma^2+n\mu\right) \quad (14)$$

where ,in Equation(13), we made the change of variable as $t=\frac{\ln(x)-\mu}{\sigma}$,

$$dt=\frac{1}{\sigma x}dx \Rightarrow ax^n dt = x^{n-1}dx \Rightarrow \sigma\exp\left(n\sigma t+n\mu\right)dt = x^{n-1}dx, x\to 0 = t\to -\infty, x\to\infty \Rightarrow t\to\infty$$

Therefore, the first four moments of the lognormal distribution are obtained as follows:

(i) $$\mathrm{E}\left(X_{Log}\right)=\exp\left(\mu+\frac{1}{2}\sigma^2\right) \quad (15)$$

(ii) $$Var\left(X_{Log}\right)=\exp\left(2\sigma^2+2\mu\right)-\exp\left(\sigma^2+2\mu\right) \quad (16)$$

(iii) $$\mathrm{Skew}\left(X_{Log}\right)=\left[\left(\exp\left(\sigma^2\right)+2\right)\sqrt{\left(\exp\left(\sigma^2\right)-1\right)}\right] \quad (17)$$

(iv) $$\mathrm{Kurt}\left(X_{Log}\right)=\left[\exp\left(4\sigma^2\right)+2\exp\left(\sigma^4\right)+3\exp\left(\sigma^4\right)-3\right] \quad (18)$$

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Maximum Likelihood Estimate of the Parameters of Lognormal Distribution

The likelihood function of the lognormal distribution for a series of $X_{i's}\left(i=1.2,...,n\right)$ is derived by taking the product of the probability densities of the individual $X_{i's}$;

$$L\left(x_i,\mu,\sigma^2\right)=\prod_{i=1}^{n} f\left(x_i,\mu,\sigma^2\right)$$

$$=\prod_{i=1}^{n}\left(\left(2\pi\sigma^2\right)^{-\frac{1}{2}}\frac{1}{x}\exp\left(-\frac{\ln\left(x\right)-\mu}{2\sigma^2}\right)^2\right]$$

$$=\left(2\pi\sigma^2\right)^{-\frac{n}{2}}\prod_{i=1}^{n}\frac{1}{x}\exp\left(\sum_{i=1}^{n}-\frac{\left(\ln\left(x\right)-\mu\right)^2}{2\sigma^2}\right)$$

(19)

The log – likelihood function of the lognormal for the series of $X_{i's}\left(i=1,2,...,n\right)$ is then derived by taking the natural log of the likelihood function.

$$\ell\left(x_i,\mu,\sigma^2\right)=\ln\left(2\pi\sigma^2\right)^{-\frac{n}{2}}\prod_{i=1}^{n}\frac{1}{x}\exp\left(\sum_{i=1}^{n}\left[\frac{\left(\ln\left(x\right)-\mu\right)^2}{2\sigma^2}\right]\right)$$

$$=-\frac{n}{2}\ln\left(2\pi\sigma^2\right)-\sum_{i=1}^{n}\ln\left(x_i\right)-\sum_{i=1}^{n}\left(\frac{\left(\ln\left(x\right)-\mu\right)^2}{2\sigma^2}\right)$$

$$=-\frac{n}{2}\ln\left(2\pi\sigma^2\right)-\sum_{i=1}^{n}\ln\left(x_i\right)-\sum_{i=1}^{n}\left(\frac{\ln\left(x_i\right)^2}{2\sigma^2}\right)+\sum_{i=1}^{n}\left(\frac{2\ln\left(x_i\right)\mu}{2\sigma^2}\right)-\frac{n\mu}{2\sigma^2} \qquad (20)$$

To find the estimate of μ and σ,that maximizes the likelihood function $\ell\left(x_i,\mu,\sigma^2\right)$, we differentiate partially with respect to μ and σ and set it to equal to zero.

Now let $\theta=\ell\left(x_i,\mu,\sigma^2\right)$ such that,

$$\frac{\partial\theta}{\mu}=\sum_{i=1}^{n}\frac{\ln\left(x_i\right)}{\hat{\sigma}^2}-\frac{2n\hat{\mu}}{2\sigma^2}=0$$

(21)

$$\frac{n\hat{\mu}}{\sigma^2}=\sum_{i=1}^{n}\frac{\ln\left(x_i\right)}{\hat{\sigma}^2}$$

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$$\hat{\mu}=\frac{\sum_{i=1}^{n}\ln\left(x_i\right)}{n} \tag{22}$$

$$\frac{\partial\theta}{\partial\sigma^2}=-\frac{n}{2}\frac{1}{\sigma^2}-\sum_{i=1}^{n}\frac{\left(\ln\left(x_i\right)-\mu\right)^2}{2}\left(-\hat{\sigma}^2\right)^{-2}$$

$$=-\frac{n}{2\hat{\sigma}^2}+\sum_{i=1}^{n}\frac{\left(\ln\left(x_i\right)-\mu\right)^2}{2\hat{\sigma}^4}=0$$
(23)

Hence,

$$\hat{\sigma}^2=\sum_{i=1}^{n}\frac{\left(\ln\left(x_i\right)-\sum_{i=1}^{n}\frac{\ln\left(x_i\right)}{n}\right)^2}{n}$$
(24)

So that,

$$\hat{\sigma}=\sqrt{\sum_{i=1}^{n}\frac{\left(\ln\left(x_i\right)-\sum_{i=1}^{n}\frac{\ln\left(x_i\right)}{n}\right)^2}{n}}$$
(25)

Weibull Distribution and its Properties

The Weibull Distribution developed by Wallodi Weibull (1951) is widely used in Engineering, medicine, energy, the social sciences, finance, insurance and others. As an extreme value distribution, the Weibull distribution has proven itself reasonably successful in predicting the occurrence of extreme phenomena like floods, earthquake, high wind speed and others. Also,because, the Weibull distribution is derived from the assumption of monomial hazard function. It is very good in describing survival statistics, such as survival after a diagnosis of cancer, light bulbs failure times and divorce rate and among others.

A random variable, X, is Weibull distributed if its probability density function is given as,

$$f\left(x\right)=\begin{cases}\frac{\beta}{\alpha}\left(\frac{x-\nu}{\alpha}\right)^{\beta-1}\exp\left\{-\left(\frac{x-\nu}{\alpha}\right)^{\beta}\right\}, x_t\geq 0,\alpha>0,\beta>0 \text{ and } \nu=0\\ 0\end{cases} \tag{26}$$

where, α and β are positive constants, while ν is nonnegative. The two constants α and β are the parameters of the Weibull distribution; when α , β and ν are known, the distribution

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whose probability density function (pdf) is given by Equation (26) and it is referred to as the three – parameter Weibull Distribution, see for example, Nwobi and Ugomma (2014)). When $\nu = 0$, the Weibull distribution model in Equation (26) turns into two-parameter model given as,

$$f(x) = \left\{ \left(\frac{\beta}{\alpha}\right)\left(\frac{X_t}{\alpha}\right)^{\beta-1} \exp\left\{-\left(\frac{X_t}{\alpha}\right)^{\beta} \quad x > 0, \beta > 0, \alpha > 0, \nu = 0 \right.\right. \tag{27}$$

The rth moments of the Weibull distribution are obtained as in the properties of Weibull Distribution as follows:

(i) The Mean is obtained as follows:

$$E(X) = \mu \; E(X^r) = \int_0^\infty \beta x^{\beta+r-1} \exp\{-x^\beta\} dx \tag{28}$$

$$= \int_0^\infty \beta U \left(1 + \frac{r}{\beta}\right)^{-1} - \frac{1}{\beta} \exp(-u) \left(\mu^{\frac{1}{\beta}-1}\right) \frac{du}{\beta}$$

$$\int u \left(1 + \frac{r}{\beta}\right) - \exp(-u) du = \Gamma\left(1 + \frac{r}{\beta}\right)$$

$$E(X) = \alpha \Gamma\left(1 + \frac{1}{\beta}\right) \tag{29}$$

Where, (Γ) is a gamma function

(ii) The variance is also obtained as

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \alpha \Gamma\left(1 + \frac{1}{\beta}\right)^2 - \left[\left(\Gamma\left(1 + \frac{1}{\beta}\right)\right)\right]^2$$

$$= \alpha^2 \left[\Gamma\left(1 + \frac{1}{\beta}\right)\Gamma\left(1 + \frac{1}{\beta}\right)\right] - \left[\left(\Gamma\left(1 + \frac{1}{\beta}\right)\right)\right]^2 = \Gamma\left(1 + \frac{2}{\beta}\right) - \left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2$$

$$\therefore Var(X) = \sigma^2 \Gamma\left(\frac{2}{\beta} + 1\right) - \Gamma^2\left(\frac{1}{\beta} + 1\right) \tag{30}$$

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(iii) Skewness of Weibull Distribution

$$Skew(X) = E\left[(X) - E(X)\right]^3$$

$$= 2\frac{1}{\alpha^{\frac{3}{\beta}}}\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^3 - 3\frac{1}{\alpha^{\frac{2}{\beta}}}\Gamma\left(1+\frac{2}{\beta}\right)\frac{1}{\alpha^{\frac{1}{\beta}}}\Gamma + \left(1+\frac{1}{\beta}\right)\frac{1}{\alpha^{\frac{3}{\beta}}}\Gamma\left(1+\frac{3}{\beta}\right)$$

$$= \frac{1}{\left(\alpha^{\beta}\right)^{-1}}\left[\left[2\Gamma\left(1+\frac{1}{\beta}\right)\right]^3 - 3\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1+\frac{2}{\beta}\right) + \Gamma\left(1+\frac{3}{\beta}\right)\right] \quad (31)$$

(iv) Kurtosis of Weibull Distribution

$$kurt(X) = \frac{\Gamma\left(1+\frac{4}{\beta}\right) - 4\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1+\frac{3}{\beta}\right) + 6\Gamma^2\left(1+\frac{1}{\beta}\right)\Gamma\left(1+\frac{2}{\beta}\right) - 3\Gamma^4\left(1+\frac{1}{\beta}\right)}{\left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)^2\right]}$$

$$\frac{\Gamma\left(1+\frac{4}{\beta}\right) - 4\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1+\frac{3}{\beta}\right) + 6\Gamma\left(1+\frac{1}{\beta}\right)^2\Gamma\left(1+\frac{2}{\beta}\right) - 3\Gamma\left(1+\frac{1}{\beta}\right)^4}{\left[\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{2}{\beta}\right)^2\right]} \quad (32)$$

Maximum Likelihood Estimate of Weibull Parameters

This method has been widely used to estimate the parameters of Weibull Distribution. For detail information about its use in the estimation of Weibull Parameter, see for example, Harter and Moore (1965), Cohen (1965), Cheng and Chen (1988), Al-fawazan (2000) and Nwobi and Ugomma (2014)).

Let $X_1, X_2, ..., X_n$ be a random sample of size, $n,$ drawn from a population with probability density function, $f(x,\lambda)$; where $\underset{\sim}{\lambda} = (\beta, \alpha)$ is an unknown vector of parameters. The likelihood function is defined by;

$$L = f(x_i, \beta, \alpha) = \prod_{i=1}^{n} f(x_i, \beta, \alpha) \quad (33)$$

The maximum likelihood of $\lambda = (\beta, \alpha)$, maximizes L or equivalently, the logarithm of L when

$$\frac{\partial \ln L}{\partial \theta} = 0 \quad (34)$$

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Then, the likelihood function of Equation (3.27) is given as;

$$L\left(x_1, x_2, ..., x_n; \beta, \alpha\right) = \prod_{i=1}^{n}\left(\frac{\beta}{\alpha}\right)\left(\frac{X}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{X}{\alpha}\right)^{\beta}\right]$$

$$= \left(\frac{\beta}{\alpha}\right)\left(\frac{X}{\alpha}\right)^{n\beta-n} \sum_{i=1}^{n} x_i^{(\beta-1)} - \ln\left(\alpha^{\beta-1}\right)\exp\left[-\sum_{i=1}^{n}\left(\frac{X}{\alpha}\right)^{\beta}\right] \tag{35}$$

Taking the natural logarithm of Equation (35) yields,

$$\ln L = n\ln\left(\frac{\beta}{\alpha}\right) + (\beta-1)\sum_{i=1}^{n} x_i - \ln\left(\alpha^{\beta-1}\right) - \sum_{i=1}^{n}\left(\frac{X}{\alpha}\right)^{\beta} \tag{36}$$

Taking the partial differentiation of Equation (36) with respect to β and α, and in turn equating to zero, the following equations are obtained:

$$\frac{\partial}{\partial\beta}\ln L = \tfrac{n}{\beta} + \sum_{i=1}^{n}\ln x_i - \frac{1}{\alpha}\sum_{i=1}^{n} x_i^{\beta}\ln x_i = 0 \tag{37}$$

and

$$\frac{\partial}{\partial\alpha}\ln L = -\tfrac{n}{\alpha} + \frac{1}{\alpha^2}\sum_{i=1}^{n} x_i^{\beta} = 0 \tag{38}$$

Simplifying Equation (38), we obtain an estimator of α as

$$\hat{\alpha} = \frac{1}{n}\sum_{i=1}^{n} x_i^{\beta} \tag{39}$$

Substituting Equation (39) into Equation (40) yields,

$$\frac{1}{\beta} + \frac{1}{\alpha}\sum_{i=1}^{n}\ln x_i - \frac{\sum_{i=1}^{n} x_i^{\beta}\ln x_i}{\sum_{i=1}^{n} x_i^{\beta}} = 0 \tag{40}$$

$$\frac{1}{\beta} + \frac{1}{n}\sum_{i=1}^{n} x_i = \ln x_i$$

and

$$\hat{\beta} = \left(\ln x_i - \frac{\sum_{i=1}^{n} x_i}{n}\right)^{-1} \tag{41}$$

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Method of Data Analysis

The method of analysis adopted for this study will estimate the absolute returns of closed stock prices of BUA cement with the help of Microsoft Excel software as demonstrated as follows:

Let $X_i = ABS\ln\left(\frac{P_t}{P_{t-1}}\right)$, P_t is the closed stock price at time t.

$$\mu = \bar{r} = \frac{1}{n}\sum_{i-1}^{n} ABS\ln\left(\frac{P_t}{P_{t-1}}\right) \qquad (42)$$

and

$$\sigma_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_t - \bar{r}_t\right)^2$$

(43)

Selection Criterion

To select the best among the two distributions, we would apply the Minimum Mean Squared Error (MSE) and Akaike Information criterion (AIC) test criteria such that the distribution with the smallest MSE and AIC out performs the other distribution. The minimum Mean Squared Error for the test is given by

$$MSE_{(\min)} = \frac{\sum_{i=1}^{n}\left(\hat{F}(X_t) - F(X\ )\right)^2}{N} \qquad (44)$$

Where

$\hat{F}(X_t)$ is the cumulative distribution function of both distribution obtained as

$$\hat{F}(X_W) = 1 - \exp\left(-\frac{x}{\beta}\right)^{\beta} \qquad (45)$$

and

$$\hat{F}(X_L) = \Phi\left(\frac{\ln(x) - \mu}{\sigma}\right) \qquad (46)$$

Where, Equations (45) and (46) are cumulative functions of Weibull and lognormal distributions respectively

and

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$$F\left(X_t\right)=\sum_{i=1}^{n} i = 1+2+3+...+57 \text{ (N-ordered sample size)}$$

Akaike Information Criterion (AIC) (Akaike, 1973) is a measure of how well a distribution is able to make accurate prediction taking into account the complexity of the distribution.

AIC test criterion is given as:

$$2\left(K\right)-2\ln\left(\hat{L}\right) \tag{47}$$

Where,

$\hat{L}$ is the maximum value of the likelihood estimate

k is the number of parameters of the model

Goodness of Fit – Tests for Lognormal and Weibull Distributions

Here, we would compare the goodness of fit test for the Weibull and lognormal distributions in order to select the best distribution that fits our dataset. The goodness of fit test procedure for the Weibull and lognormal distributions can be generally described as:

H_{01} : The data does not follow a lognormal distribution

H_{02} :The data does not follow a Weibull distribution.

In the case of our work we would apply one sample test of Kolmogorov-Smirnov test of goodness – of – fit. To perform the Kolmogorov-Smirnov test (See for example, Chakravarti, 1967; Lawless, 1982), the maximum distance between the cumulative frequency of the failure times and the theoretical cumulative frequency provided by the estimated model would be required. If this distance is large enough, the hypothesis that the chosen distribution(s) fit(s) the dataset would be rejected. The distance between theoretical frequency and observed frequency are expressed as:

$$D_n^+ = Max_{(1,\alpha)}\left(\frac{1}{n}-F\left(X_t\right)\right), \text{ for right tailed test} \tag{48}$$

$$D_n^- = Max_{(1,\alpha)}\left(F\left(X_t\right)-\frac{i-1}{n}\right), \text{ for left tailed test} \tag{49}$$

and

$$\hat{D} = Max_{(1,\alpha)}\left(D_n^+, D_n^-\right), \text{ for two tailed test} \tag{50}$$

where,

$F\left(X_t\right)$ is the cumulative distribution function of the distributions

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The Null hypothesis for lognormal and Weibull distributions would be rejected if the maximum distance in Equation (50) is greater than or equal to critical value, otherwise we accept the null hypothesis.

EMPIRICAL EVIDENCE AND DISCUSSIONS

3.1 Presentation of Descriptive Statistics of BUA Cement Data

Table 1: Summary Statistics of BUACEM Original Data

Sample Size	Sample Mean $(\bar{X})$	Sample Std. Dev (S)	Skewness $Skew(X)$	Kurtosis $Kurt(X)$
58	83.0991	32.7105	0.8548	32.6

Table 1 shows the descriptive statistics of the original data of BUA cement for the period under study. From the statistics, we observed that the original data showed normality since the skewness is positive indicating that the right tail of the distribution is longer than the left and the kurtosis also suggested that the distribution is perfectly peaked.

Table 2: Descriptive Statistics of Absolute Returns BUACEM Dataset

Sample Size	Sample Mean $(\bar{X})$	Sample Std. Dev (S)	Sample variance (S^2)	Skewness $Skew(X)$	Kurtosis $Kurt(X)$
57	0.0866	0.1141	0.0130	2.6705	9.5733

Table 2 shows the descriptive statistics of the absolute returns of BUA cement data. The result further indicates positive skewness, thus, showing the right tail of the distribution is longer than left tail. Hence, this result proved that the absolute returns of BUACEM data for the period of study are normally distributed and the kurtosis also suggested that the distribution is perfectly peaked.

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Presentation of Statistics of Parameters of Lognormal and Weibull Distributions

Table 3: Parameters of Lognormal and Weibull Distributions

Distribution	$\hat{\mu}$	$\hat{\sigma}^2$	$\hat{\alpha}$	$\hat{\beta}$
Lognormal	1.0976	0.0128		
Weibull			6307.53	-3.5764

Table 3 shows the results of the 2-parameters of both lognormal and Weibull distributions. The mean of the log-transformed stock price of BUACEM is 1.0976 meaning that the average of the natural logarithm of the dataset is 1% around the BUACEM stock price returns, hence has one unit change in the stock returns while the standard deviation of the log-transformed dataset of the stock price of BUACEM is 0.0113 meaning the log-transformed stock price are relatively close to the stock returns. Table 3 also shows that the failure rate of the BUACEM stock price is decreasing overtime $(\beta = -3.6)$ and scale parameter of the BUACEM stock price $(\alpha = 6307.53)$ stretches far to the right thereby decreasing the height of the distribution, meaning that the scale parameter represents 63.1 percentile of BUACEM stock price. Based on the results of the parameters, investors are advised to invest more in BUACEM since the risk of investing with company is relatively low.

Presentation of Statistics of Properties of Lognormal and Weibull Distributions

Table 4: Properties of Lognormal and Weibull Distributions

Distribution	Mean	Variance	Skewness	Kurtosis
Lognormal	1.0976	0.0158	0.3445	3.0534
Weibull	2.43	9.45	1.26	-1.62

From the results of the properties of both lognormal and Weibull distributions in Table 4 shows their skewness and kurtosis were acceptable since they fall into the acceptance regions of (-2 to +2) and (-7 to +7) for both skewness and kurtosis respectively. This means that both distributions are normally distributed. Hence, the stocks of BUACEM for the period of study are normally distributed. In terms of investment, the lognormal distribution with 3.1 as its kurtosis indicates a high risk of investing than the Weibull which shows a moderate risk level of investing in the BUACEM. This means that the lognormal distribution has high probability of yielding extremely large or small returns while that of the probability of Weibull distribution has extremely relative low returns in investment.

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Comparison of the performance of Lognormal and Weibull Distributions.

Table 5 Summary of Minimum Squared Errors of Estimated Weibull Parameters

Distribution	MSE	AIC
Lognormal	1.09×10^{3}	1.23×10^{1}
Weibull	1.11×10^{3}	-4.92×10^{-1}

From Table.5, we observed that lognormal distribution has the minimum mean squared error (1085.20) and the highest AIC (12.20) respectively while Weibull distribution has the mean squared error of (1107), a little variation of 0.2 higher than that of the lognormal but maintains lower values of AIC (-0.49). Comparing the two distributions based on MSE and AIC, the Weibull distribution is selected to be the best fit on the BUACEM log-transformed stock price under the period of study, hence, the goodness-of-fit test for the Weibull would be conducted.

Table 6 The Summary of Kolmogorove-Smirnov Test for Weibull Distribution

Distribution	D^{+}	D^{-}	$\hat{D}$	$WEI(\alpha,\beta)=\sqrt{n\hat{D}}$	Critical value at 5%	Decision
Weibull	0.0175	0.1887	0.1887	3.2796	1.36	Reject

Table 6 shows that the computed test statistic for Weibull distribution is greater than the critical value at 5% level of significance, we therefore have enough evidence to reject the null hypothesis and conclude that the stock price of the BUACEM stock price follows the Weibull Distribution.

CONCLUSION

In this article, the Maximum Likelihood Estimator (MLE) was used to obtain the parameters of both Lognormal and Weibull distributions in a view of comparing and selecting the best distribution that fits our dataset. The Minimum Mean Squared Error (MSE) and Akaike Information Criterion (AIC) were used as selection criteria and the Weibull distribution was found to outperform the Lognormal distribution since it exhibited the least MSE and AIC. Also, the selected Weibull distribution was subjected to goodness-fit-test using Kolmogorov-Smirnov test and the empirical shows that BUACEM stock price follows a Weibull distribution with 5% level of significance.

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