A COMPARATIVE SOLUTION USING LAPLACE TRANSFORM METHOD ON KIRCHHOFF SECOND LAW WITH ANGULAR FREQUENCY EFFECT.

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ABSTRACT: *This study shows the unique solution to a second-order non-homogeneous linear differential equation using the Laplace transform method with two mathematical models for the sum of voltage drop across the R-L-C circuit with required boundary conditions. Solutions for the charge flowing across the circuit were obtained using the Laplace transform with parameters such as Inductance L, resistance R, capacitance C, Electromotive force (EMF) and angular frequency, all having an effect on the total charge flowing across the circuit. The results obtained showed that the increase in the capacitance and EMF increased the electric charge flowing across the circuit. The presence of the angular frequency in the second mathematical model decreased the electric charge as the resistance increased, when compared to the first Mathematical model, where the electric charge increased. The increase in the inductance increased the electric charge in the second mathematical model when compared to the first mathematical mode,l where the electric charge decreased. The results of the solution from the second mathematical model proved to be better and safer for electrical/mechanical appliances with the effect of angular frequency.*

KEYWORDS: Laplace Transform, Electric Charge, R-L-C Circuit, Electromotive Force (EMF), Kirchhoff's second law.

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INTRODUCTION

Applications of Laplace transform on ordinary differential equations with applications to LRC circuit are a major area of interest to researchers in science, engineering and technology. The study on the applications of ordinary differential equations [1], uses various methods of solutions for charges and current including Laplace on Kirchhoff's second law. Similarly, [2] improved on his study with application of higher order differential equations on LRC circuits with solutions for charges and current gotten using the required boundary conditions. A study on forced linear equations [1] was carried out using an electrical circuit experiment to verify the theoretical solutions for RLC circuits. Application of Laplace transforms on RLC circuit with the application of linear ordinary differential equation [4] was carried out to analyze transient and steady response for second order RLC circuit that was closed, Laplace transforms with engineering applications extensively studied [5] while recommendation on RLC circuit and its uses in electronic devices studied [6].

Application of Laplace transform to Kirchhoff's law, transfer functions and mechanical systems with complex systems simplified was studied [7]. Solutions to differential equations using Laplace transforms method was further studied with relevant conclusion [8-15]. Laplace transform was applied to unsteady flow problem in a reservoir [16], where a problem encountered in the fluid flow was analyzed. Laplace transform was applied [17] in nuclear physics, signal processing, control and automation in engineering. Consequently, we will/shall study and compare solutions and results from two separate mathematical models for the voltage drop across RLC circuit with application of Laplace to obtain the solutions for the electric charge flowing across the circuit with Resistor, inductor and capacitor having significant effect on the overall charge per time.

MATHEMATICAL MODELING

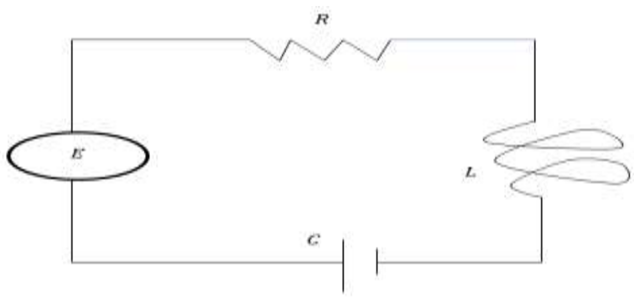

Figure 1.1 Diagram showing the RLC circuit with sum of voltage drop

From the diagram above,

First Mathematical Model from Kirchhoff's law

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = E(t) \tag{1}$$

Second Mathematical Model from Kirchhoff's law

$$L\frac{d^2q_1}{dt^2} + R\frac{dq_1}{dt} + \frac{1}{C}q_1 = E\,sin\,sin\,\omega t \tag{2}$$

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Subject to

$\{q(0) = 0\,;\; q^I(0) = I_0\, q_1(0) = 0\,;\; {q_1}^I(0) = I_0\,\}$ (3)

METHOD OF SOLUTION

The second order nonhomogeneous linear differential equation in equation (1-2) is solved using the Laplace transform with the application of theorems 1 – 3 defined by [18].

Theorem 1

$f(t)$ be defined $\forall\, t \geq 0$, then

$L\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)dt$, where $t \in [0, \infty]$

Theorem 2

For a continuous function $f(t)$ its derivative $f^I(t)$ and $f^{II}(t)$in all finite interval $0 \leq t \leq T$, will be defined in Laplace transform as

$L\{f^I(t)\} = sF(s) - f(0)\,;\, L\{f^{II}(t)\} = s^2F(s) - sf(0) - f^I(0).$

Theorem 3

This theorem is called the convolution theorem defined by two piecewise functions $f(t)$ and $g(t)$ $\forall\, t \geq 0$ in exponential order defined as

$$L^{-1}\{F(s)G(s)\} = \int_0^t f(\tau)g(t-\tau)d\tau = L\{f(t)\} \times L\{g(t)\}$$

Solution to the first mathematical model

Apply theorem 2 into equation (1)

$$L\left\{L\frac{d^2q}{dt^2}\right\} + L\left\{R\frac{dq}{dt}\right\} + L\left\{\frac{1}{C}q\right\} = L\{E(t)\} \quad (4)$$

$$L\{s^2q(s) - sq(0) - q^I(0)\} + R[sq(s) - q(0)] + \frac{1}{C}q(s) = \frac{E}{s} \quad (5)$$

Apply the boundary conditions in equation (3), equation (5) is expressed as

$$\left(Ls^2 + Rs + \frac{1}{C}\right)q(s) = \frac{E}{s} + I_0 \quad (6)$$

$$q(s) = \frac{E}{s\left(Ls^2+Rs+\frac{1}{C}\right)} + \frac{I_0}{\left(Ls^2+Rs+\frac{1}{C}\right)} \quad (7)$$

Taking the inverse Laplace transforms of equation (7)

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$$L^{-1}\{q(s)\} = q(t) = L^{-1}\left\{\frac{E}{s\left(Ls^2+Rs+\frac{1}{C}\right)}\right\} + L^{-1}\left\{\frac{I_0}{Ls^2+Rs+\frac{1}{C}}\right\} \quad (8)$$

$$L^{-1}\{q(s)\} = q(t) = L^{-1}\left\{\frac{E}{s(s-\alpha)(s-\beta)}\right\} + L^{-1}\left\{\frac{I_0}{(s-\alpha)(s-\beta)}\right\} \quad (9)$$

Where α and β are the roots of the equation $Ls^2 + Rs + \frac{1}{C}$.

$$\alpha = \frac{-R+\sqrt{R^2-\frac{4L}{C}}}{2L}\ ; \beta = \frac{-R-\sqrt{R^2-\frac{4L}{C}}}{2L}$$

Equation (9) is solved further to get

$$L^{-1}\{q(s)\} = q(t) = \frac{E}{\alpha\beta} L^{-1}\left\{\frac{1}{s}\right\} + \frac{E(\alpha+\beta)}{\alpha\beta} L^{-1}\left\{\frac{1}{(s-\alpha)(s-\beta)}\right\} + L^{-1}\left\{\frac{I_0}{(s-\alpha)(s-\beta)}\right\} \quad (10)$$

Taking the inverse Laplace for equation (10) and applying theorem 3, the solution to the non-homogeneous linear differential equation for the first mathematical model for the charge of the sum of voltage drop across the R-L-C circuit will be expressed as

$$q(t) = \frac{E}{\alpha\beta} + \left(I_0 + \frac{E(\alpha+\beta)}{\alpha\beta}\right)\left(\frac{e^{(\alpha+\beta)t} - e^{(\beta)t}}{\alpha-\beta}\right) \quad (11)$$

Solution to the second mathematical model

Apply theorem 2 into equation (2)

$$L\left\{L\frac{d^2q_1}{dt^2}\right\} + L\left\{R\frac{dq_1}{dt}\right\} + L\left\{\frac{1}{C}q_1\right\} = L\{E \sin \sin \omega t\} \quad (12)$$

$$L\{s^2q_1(s) - sq_1(0) - {q_1}^I(0)\} + R[sq_1(s) - q_1(0)] + \frac{1}{C}q_1(s) = \frac{E\ \omega}{s^2+\omega^2} \quad (13)$$

Apply the boundary conditions in equation (3), equation (13) is expressed as

$$\left(Ls^2 + Rs + \frac{1}{C}\right)q_1(s) = \frac{E\ \omega}{s^2+\omega^2} + I_0 \quad (14)$$

$$q_1(s) = \frac{E\ \omega}{(s^2+\omega^2)\left(Ls^2+Rs+\frac{1}{C}\right)} + \frac{I_0}{\left(Ls^2+Rs+\frac{1}{C}\right)} \quad (15)$$

Taking the inverse Laplace transforms of equation (15)

$$L^{-1}\{q_1(s)\} = q_1(t) = L^{-1}\left\{\frac{E\ \omega}{(s^2+\omega^2)\left(Ls^2+Rs+\frac{1}{C}\right)}\right\} + L^{-1}\left\{\frac{I_0}{\left(Ls^2+Rs+\frac{1}{C}\right)}\right\} \quad (16)$$

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$$L^{-1}\{q_1(s)\} = q_1(t) = L^{-1}\left\{\frac{E\,\omega}{(s^2+\omega^2)((s-\alpha)(s-\beta))}\right\} + L^{-1}\left\{\frac{I_0}{((s-\alpha)(s-\beta))}\right\} \quad (17)$$

Where α and β are the roots of the equation $Ls^2 + Rs + \frac{1}{C}$.

$$\alpha = \frac{-R+\sqrt{R^2-\frac{4L}{C}}}{2L}\ ; \beta = \frac{-R-\sqrt{R^2-\frac{4L}{C}}}{2L}$$

Equation (17) is solved further obtain

$$L^{-1}\{q_1(s)\} = q_1(t) = \left(\frac{E\omega}{(\alpha-\beta)(1+\omega^2)}\right)\left\{L^{-1}\left(\frac{1}{(s-\alpha)}\right) - L^{-1}\left(\frac{1}{(s-\beta)}\right)\right\} - \left(\frac{E\omega}{(\alpha-\beta)(1+\omega^2)}\right)L^{-1}\left(\frac{1}{s^2+\omega^2}\right) + +I_0L^{-1}\left\{\frac{1}{((s-\alpha)(s-\beta))}\right\} \quad (18)$$

$$q_1(t) = I_0\left(\frac{e^{(\alpha+\beta)t} - e^{(\beta)t}}{\alpha-\beta}\right) + \left(\frac{E\omega}{(\alpha-\beta)(1+\omega^2)}\right)\left(e^{\alpha t} - e^{\beta t}\right) - \left(\frac{E\omega coscos\ \omega t}{(1+\omega^2)}\right) \quad (17)$$

RESULTS

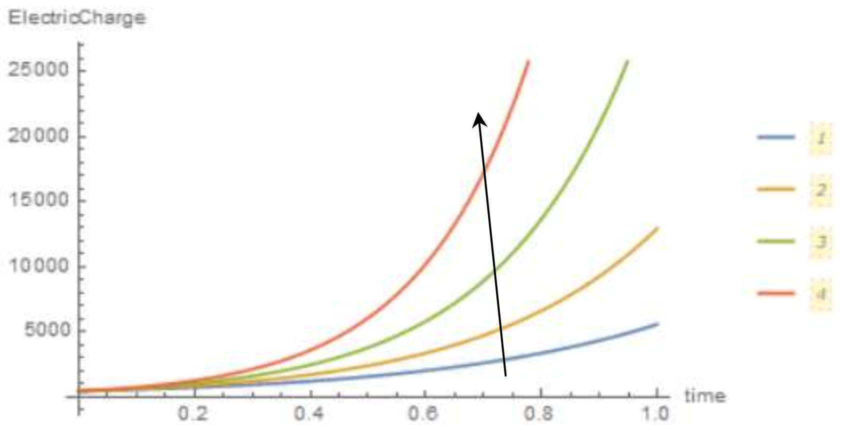

Figure 1.2 Effect of varying resistance R (10, 20, 30 and 40) on the electric charge in the first consideration

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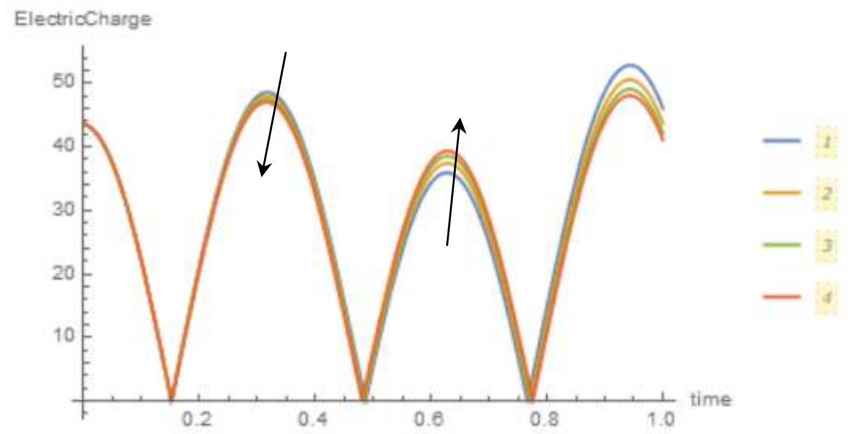

Figure 1.3 Effect of varying resistance R (10, 20, 30 and 40) on the electric charge in the second consideration

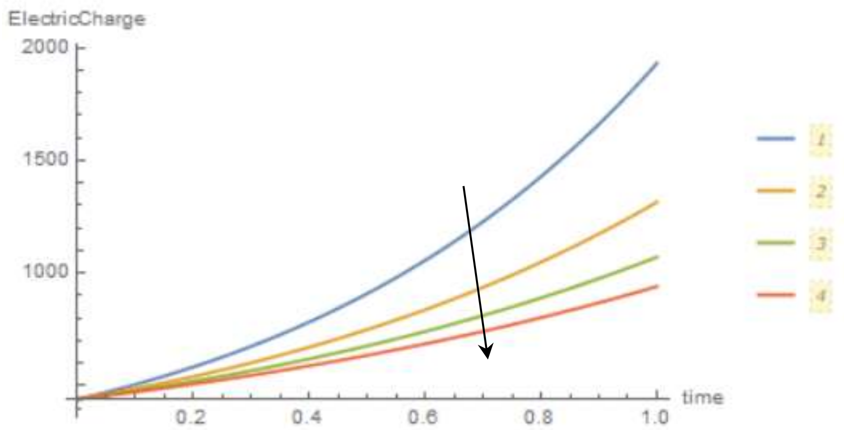

Figure 1.4 Effect of varying inductance L (0.1, 0.2, 0.3 and 0.4) on the electric charge in the first consideration

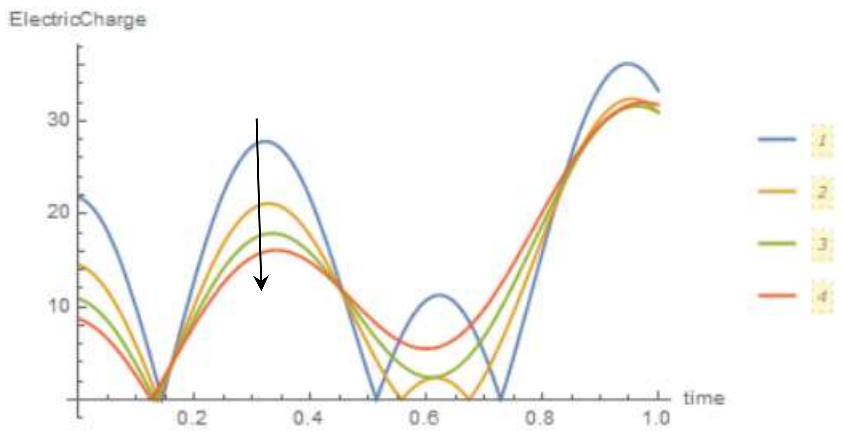

Figure 1.5 Effect of varying inductance L (0.1, 0.2, 0.3, and 0.4) on the electric charge in the second consideration

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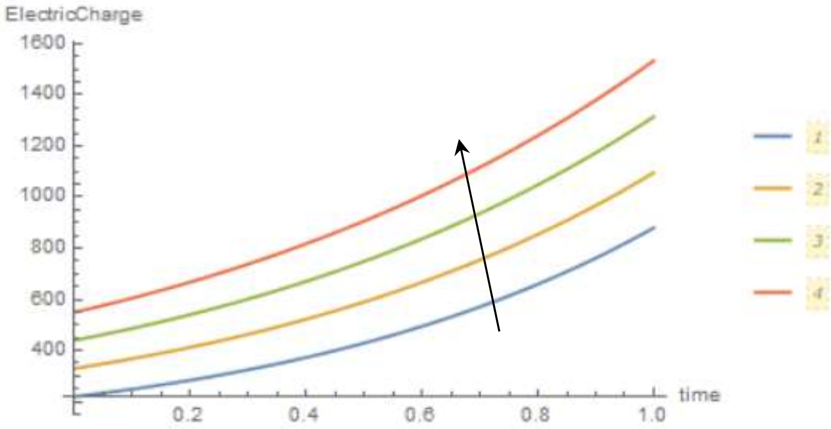

Figure 1.6 Effect of varying capacitance C (5×10^{-6}, 10×10^{-6}, 15×10^{-6}, and 20×10^{-6}) on the electric charge in the first consideration

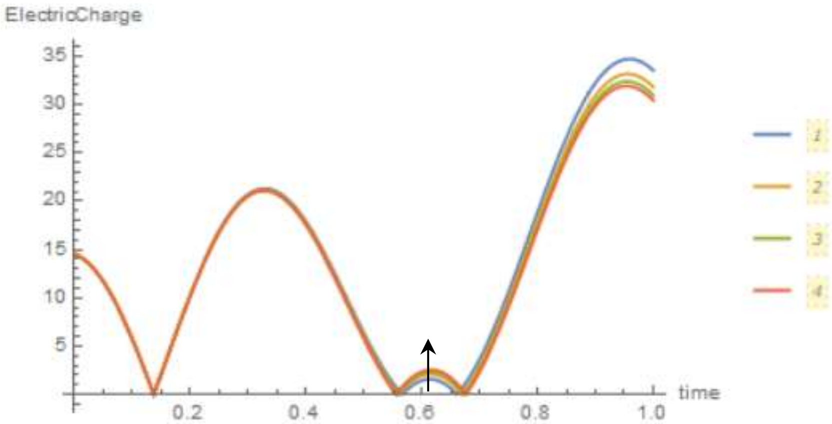

Figure 1.7 Effect of variation of capacitance C (5×10^{-6}, 10×10^{-6}, 15×10^{-6}, 20×10^{-6}) on the electric charge in the second consideration

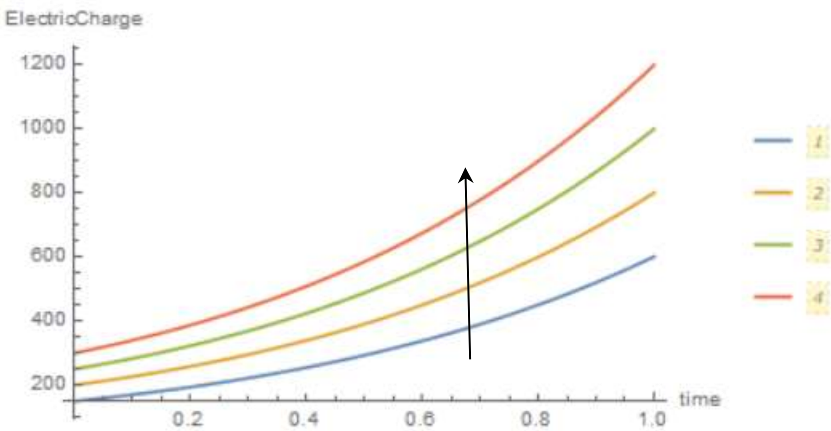

Figure 1.8 Effect of varying EMF (5, 10, 15 and 20) on the electric charge in the first consideration

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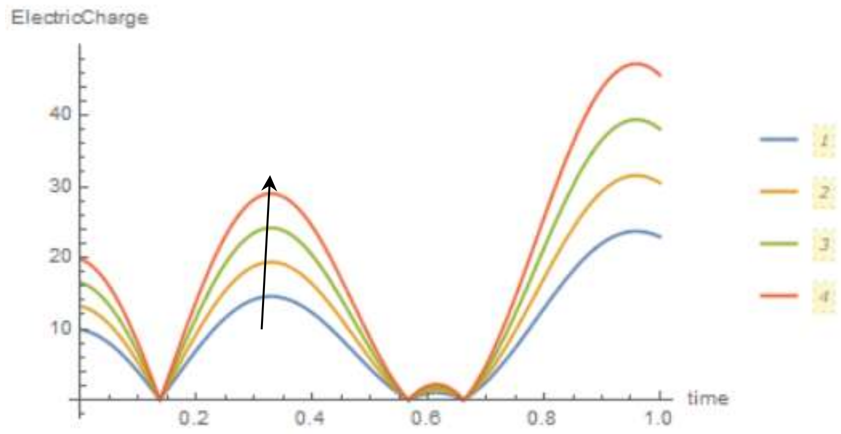

Figure 1.9 Effect of variation of EMF (5, 10, 15 and 20) on the electric charge in the second consideration

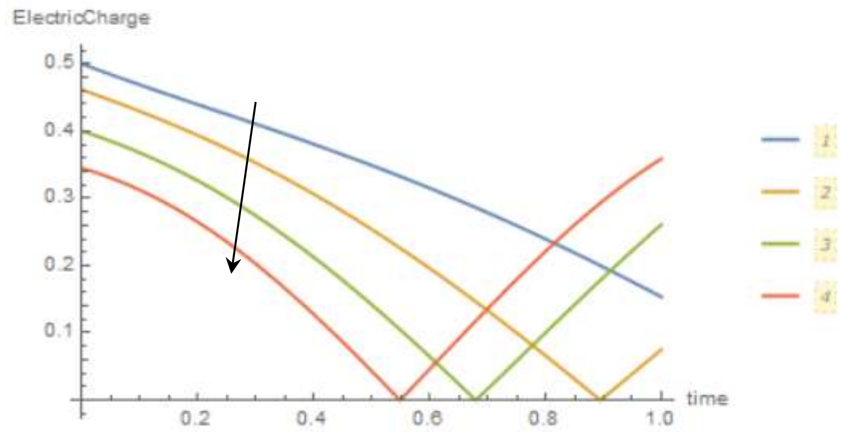

Figure 1.10 Effect of varying angular frequency ω (100, 200, 300 and 400) on the electric charge in the second consideration

Table showing the values of the Parameters from Figure 1.2-1.10

S/N	Parameters	Value/Unit
1.	Resistance R (Ω)	50
2.	Inductance L (H)	0.2
3.	Electromotive Force EMF E (V)	10
4.	Time t (s)	0.01
5.	Charge Q (C)	2×10^{-3}
6.	Angular Frequency ω (r/s)	1000

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DISCUSSION

From Figure 1.2 and Figure 1.3 it was observed that the increase in the resistance created an increase in the electric charge flowing across the circuit in the first consideration but a decrease in the electric charge in the second consideration. As the inductance increased, a contrary result was observed in Figure 1.4 and Figure 1.5 which showed a decrease in the electric charge flowing across the circuit in the first consideration with an increase in the electric charge flowing across the circuit in the second consideration. These variations in the result from Figure 1.2 to Figure 1.5 occurred as a result of the angular frequency introduced in the circuit voltage. Furthermore, in Figure 1.6 and Figure 1.7 it was observed that for both considerations, there was an increase in the flow of electric charge across the circuit as the capacitance increased. Figure 1.8 and Figure 1.9 showed an increase in the electric charge across the circuit as the EMF increased. Finally, in Figure 1.10, the increase in the angular frequency decreased the electric charge flowing across the circuit. This effect is the reason there was a conflicting result in the effect of the resistance and capacitance on the electric charge because the angular frequency altered both the effect of the resistance and the capacitance on the electric charge across the circuit.

CONCLUSION

This research shows the application of Laplace transform on the sum of voltage drop across an LRC circuit with two separate mathematical models considered to analyze the effect of resistance, inductance, capacitance, electromotive force and angular frequency on the electric charge across the circuit. The results from the solution of the second mathematical model showed a better and safer result for applicability to electrical/mechanical appliances with introduction of angular frequency in the governing equation.

Finally the research showed that;

i. An increase in the capacitance and EMF increased the electric charge across the circuit while increase in the inductance decreased the electric charges across the circuit.

ii. The presence of the angular frequency in the second consideration decreased the electric charge as the resistance increased when compared to the first consideration where the electric charge increased.

iii. The presence of the angular frequency in the second consideration increased the electric charge as the inductance increased when compared to the first consideration where the electric charge decreased.

CONFLICT OF INTEREST

There is no conflict of interest or competing interest among authors.

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