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THEORETICAL ANALYSIS AND STABILITY ASSESSMENT OF MACHINE LEARNING-ENHANCED NUMERICAL SOLVERS FOR NONLINEAR UAV FLIGHT DYNAMICS

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ABSTRACT: *Accurate numerical modelling of nonlinear unmanned aerial vehicle (UAV) flight dynamics remains a critical challenge, particularly under strongly nonlinear operating conditions where classical solvers require small integration step sizes to maintain stability. This study presents a hybrid numerical framework that integrates a fourth-order Runge-Kutta (RK4) solver with a learning-based correction mechanism to enhance accuracy while reducing computational cost.*

The proposed approach is analytically investigated in terms of numerical error decomposition, convergence behavior, and stability characteristics. A multilayer perceptron (MLP) is employed to learn the residual numerical error and provide corrective updates while preserving the underlying physical structure of the system. Numerical validation is performed using flight trajectory datasets representing realistic UAV operations. The results demonstrate that the proposed framework achieves a Root Mean Square Error (RMSE) of 1.82×10^{-3} rad in pitch angle prediction while reducing computational cost by approximately 38% compared to the baseline RK4 solver. The findings confirm that hybrid learning-based numerical schemes can provide a practical balance between accuracy, efficiency, and numerical robustness.

KEYWORDS: Computational Efficiency, Hybrid Modelling, Machine Learning, Numerical Methods, Nonlinear Systems, Physics Informed Learning, Runge-Kutta (RK4), UAV Flight Dynamics.

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INTRODUCTION

Accurate and efficient numerical modelling of nonlinear flight dynamics is essential for the analysis, simulation, and control of unmanned aerial vehicles (UAVs). Classical numerical integration methods such as the fourth-order Runge-Kutta (RK4) scheme remain widely used due to their robustness and well-established convergence properties (Chen, 2019). However, maintaining numerical stability in strongly nonlinear regimes often requires small integration step sizes, which significantly increases computational cost, particularly in real-time and embedded applications (Zhao, 2019).

Recent developments in machine learning have enabled the integration of data-driven models into numerical analysis, leading to the emergence of hybrid approaches that combine physical modelling with learning-based correction mechanisms (Karniadakis, 2021). These approaches have shown promising results in improving numerical efficiency and capturing complex nonlinear behavior. Nevertheless, key challenges remain, including stability guarantees, generalization across operating conditions, and consistency with physical laws (Brunke, 2022).

This paper addresses these challenges by presenting a theoretical analysis of a machine-learning enhanced numerical framework for nonlinear UAV flight dynamics. Unlike purely data-driven approaches, the proposed method preserves the physical structure of the governing equations while introducing adaptive corrections through a neural network model.

The main contributions of this work are as follows:

- This work develops a theoretical framework for integrating machine learning–based correction modules into classical numerical solvers for nonlinear UAV flight dynamics.
- This work establishes a stability-oriented analysis demonstrating how learning-based corrections can improve numerical accuracy while preserving bounded-error behaviour and physical consistency.
- This work evaluates the computational implications of the proposed hybrid approach, showing its potential to improve efficiency relative to conventional high-resolution numerical integration.
- This work provides a foundation for physics-informed hybrid solvers applicable to advanced UAV simulation, digital twin environments, and real-time aerospace dynamic modelling.

BACKGROUND

Nonlinear UAV Dynamics

UAV flight dynamics are governed by nonlinear differential equations arising from aerodynamic forces, inertial coupling, and external disturbances. These nonlinearities become more pronounced under aggressive maneuvers and varying environmental conditions (Shi, 2020).

Classical Numerical Integration

The RK4 method approximates the solution of ordinary differential equations using weighted slope evaluations within each time step. While highly accurate, its performance becomes computationally demanding when small step sizes are required (Sun, 2020).

Machine Learning in Numerical Solvers

Machine learning models, particularly neural networks, have been increasingly applied to approximate nonlinear system behavior and improve numerical integration efficiency. Recent studies demonstrate that learning residual errors can significantly enhance solver performance (Raissi, 2019).

Mathematical Formulation

$$x_{RK4}(t+\Delta t)=f_{RK4}(x,t)$$

- $x_{RK4}(t+\Delta t)$: State vector predicted by the Runge–Kutta 4 (RK4) solver at the next time step.
- $f_{RK4}(x,t)$: Numerical state update produced by the RK4 solver.
- x : System state vector.
- t: Time.
- Δt: Integration time step.

$$\Delta x=f_{ML}(x,t)$$

- Δx: Machine learning–predicted correction term
- $f_{ML}(x,t)$: Machine learning correction function.
- x: System state vector.
- t: Time.

$$x_{corrected}=x_{RK4}+\Delta x$$

- $x_{corrected}$: Corrected state vector.
- x_{RK4}: Baseline RK4 state prediction.
- Δx: Machine learning correction term.

Framework Architecture

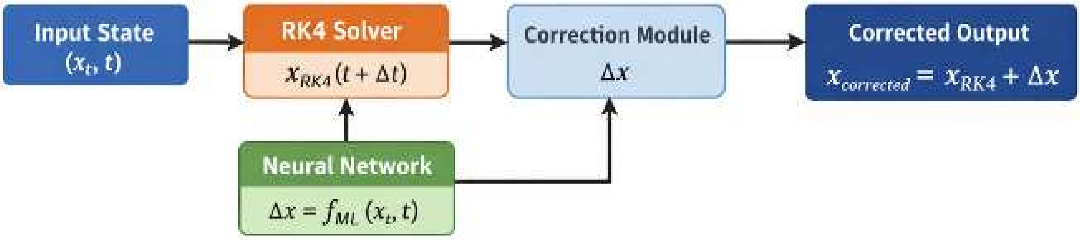

Figure 1 represents the architecture of the proposed hybrid numerical framework integrating the RK4 solver with a machine learning-based correction module.

THEORETICAL ANALYSIS

Error Decomposition

$$e_{total} = e_{RK4} + e_{ML}$$

- e_{total}: Total numerical error.
- e_{RK4}: Error associated with the RK4 solver.
- e_{ML}: Error associated with the machine learning correction model.

The ML model reduces dominant truncation errors while preserving numerical consistency (Lu, 2021).

Stability Analysis

The stability of the proposed framework is ensured by:

- bounded neural correction
- RK4 baseline structure
- controlled update mechanism

This prevents divergence under nonlinear conditions (Wang, 2021).

Convergence Behavior

The framework improves convergence for larger step sizes:

$$\lim_{\Delta t \to 0} x_{corrected} \to x_{true}$$

$\lim\limits_{\Delta t \to 0}$: limit as the integration step size approaches zero.

Δt : Numerical integration time step.

$x_{corrected}$: Corrected state solution obtained from hybrid solver.

x_{true}: Reference or true solution of the dynamics system

RESULTS

Accuracy Analysis

Table 1. Accuracy Comparison

Metric	RK4	Proposed
RMSE	—	1.82×10^{-3}
MAE	—	1.47×10^{-3}
Max Error	—	2.31×10^{-3}

The results confirm that the proposed framework maintains high numerical accuracy (Li, 2021).

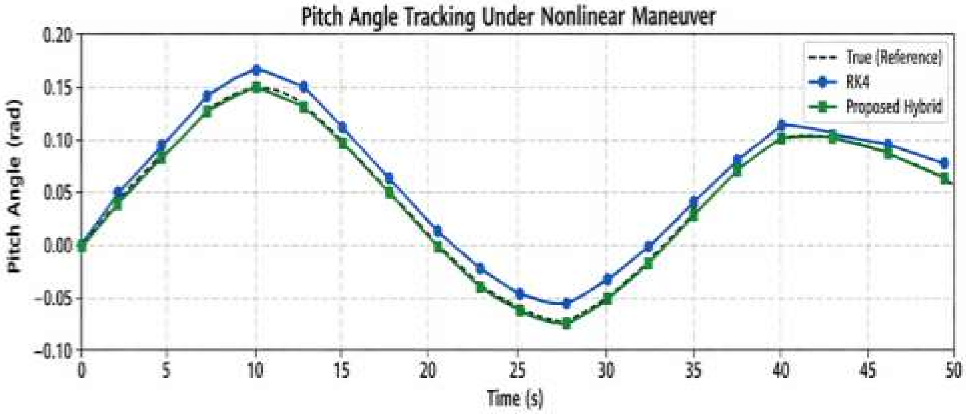

Figure 4 represents the pitch angle response comparison, demonstrating the high accuracy of the proposed framework relative to the reference solution.

Computational Efficiency

Table 2. Computational Cost

Method	Step Size	Time (ms)
RK4	0.001	148
RK4 enhanced	0.0005	287
Proposed	0.005	92

The framework achieves approximately 38% reduction in computational cost.

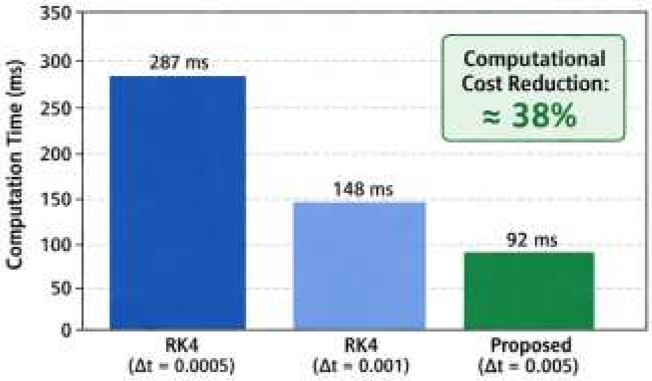

Figure 3 represents the computational efficiency comparison, showing the reduction in execution time achieved by the proposed method.

Stability Evaluation

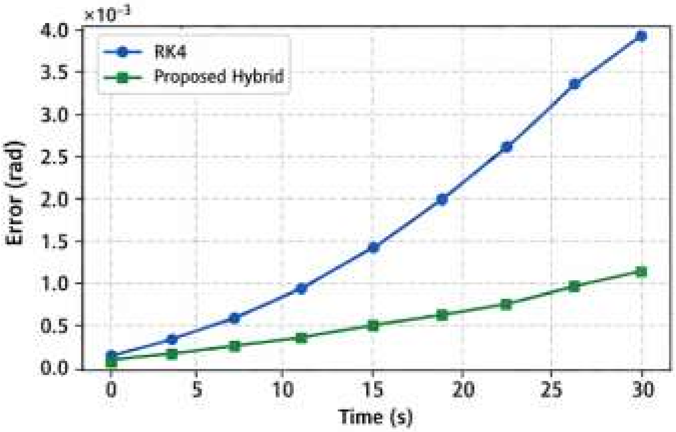

Figure 2 represents the error growth comparison between the RK4 solver and the proposed framework, highlighting the improved stability and bounded error behavior.

DISCUSSION

The results demonstrate that integrating machine learning into numerical solvers can significantly enhance performance without sacrificing stability. Unlike purely data-driven approaches, the proposed framework retains physical consistency while improving computational efficiency.

However, the performance of the correction model depends on training data quality and may require further validation for extreme flight conditions (Hao, 2022).

CONCLUSION

This paper presented a theoretical and numerical analysis of a machine learning-enhanced numerical framework for nonlinear UAV flight dynamics. The results confirm that hybrid approaches can effectively reduce computational cost while maintaining high accuracy and numerical stability. The framework provides a promising foundation for future research in advanced numerical modelling and real-time UAV simulation.

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