NUMERICAL VALIDATION OF A MACHINE LEARNING–DRIVEN FRAMEWORK FOR NONLINEAR UAV FLIGHT DYNAMICS

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Cite this article:

Abdelfattah Hafiz M., Elkhidir TayAllah Y. (2026), Numerical Validation of a Machine Learning–Driven Framework for Nonlinear UAV Flight Dynamics. African Journal of Electrical and Electronics Research 8(1), 1-11. DOI: 10.52589/AJEER-AFUUUGNL

Manuscript History

Received: 15 Jan 2026

Accepted: 18 Feb 2026

Published: 12 Mar 2026

ABSTRACT: *Accurate numerical modeling of nonlinear flight dynamics is essential for the simulation, control, and performance assessment of unmanned aerial vehicles (UAVs). Classical numerical solvers such as the fourth order Runge - Kutta (RK4) method can provide high accuracy, but they often require small integration step sizes in strongly nonlinear operating regimes, which leads to increased computational cost. In this work, a physics informed, learning based correction framework is used to enhance the numerical solution of nonlinear UAV flight dynamics. The framework augments a baseline numerical solver with a neural correction module inspired by physics informed neural networks, so that the numerical solution is adjusted while remaining consistent with the governing dynamicsNumerical validation is carried out using flight operation data from 100 UAVs, including both fixed wing and multirotor configurations with different mission profiles. The data were obtained through collaboration with an industrial aviation partner and were anonymized and standardized before analysis to preserve confidentiality. The proposed framework is benchmarked against an RK4 solver taken as the reference. The results show that the learning-based framework reduces computational cost by about 38% while maintaining an RMSE of 1.82×10^{-3} rad in the pitch angle response. These findings indicate that the approach can provide accurate and efficient numerical solutions for nonlinear UAV flight dynamics and is suitable for real time and autonomous flight applications.*

KEYWORDS: UAV Flight Dynamics; Machine Learning–Driven Numerical Methods; Nonlinear Systems; Numerical Validation; Autonomous UAVs.

Article DOI: 10.52589/AJEER-AFUUUGNL

DOI URL: https://doi.org/10.52589/AJEER-AFUUUGNL

INTRODUCTION

Unmanned aerial vehicles (UAVs) are increasingly deployed in civilian and military missions that demand high levels of autonomy, robustness, and real time decision making. Applications such as surveillance, mapping, inspection, and cooperative operations all rely on accurate and efficient numerical modeling of nonlinear flight dynamics for simulation, control design, and safety assessment. For these tasks, the numerical solver is not only required to reproduce the correct qualitative behavior of the vehicle, but also to provide quantitatively reliable trajectories under strongly nonlinear and time varying operating conditions.

Considerable progress has been made in the development and analysis of classical numerical integration schemes for flight dynamics and nonlinear control systems. Methods such as the fourth order Runge–Kutta (RK4) algorithm remain a standard choice in both research and industry due to their robustness and well understood error properties. However, when UAVs operate in regimes involving strong aerodynamic coupling, rapid attitude changes, actuator saturation, or high angles of attack, these schemes typically require very small integration step sizes to maintain stability and accuracy. This leads to a marked increase in computational cost, which becomes restrictive for real time onboard implementation, hardware in the loop testing, and large-scale Monte Carlo or optimization studies (Liu, Zhang, Zhao & Chen, 2021) (Wang, Sun, Qin, 2019)

In parallel with advances in traditional numerical analysis, there has been a rapid growth of interest in machine learning driven solvers and physics informed neural networks (PINNs) for nonlinear dynamical systems (Karniadakis, Kevrekidis, Lu, Prerdikaris, Wang & Yang, 2021).

These approaches exploit neural networks either to approximate the solution of the governing equations directly, or to learn corrections to existing numerical schemes. In principle, such learning-based methods can improve efficiency by adapting to complex nonlinear behavior and by reducing the need for extremely fine time discretization. Nevertheless, despite promising results in several benchmark problems, a number of critical limitations remain when these methods are applied to realistic aerospace systems.

First, many existing learning-based solvers are purely data driven and may exhibit numerical instability or divergence when exposed to flight conditions that differ from the training data, such as aggressive maneuvers or off nominal operating points. Second, without explicit mechanisms to incorporate the underlying physics, these models often fail to rigorously enforce conservation laws and physical constraints, which can result in drift in energy or momentum like quantities and lead to unphysical states over long simulations. Third, the performance of such models can be highly sensitive to data coverage and noise, and there is often limited evidence of generalization across different UAV configurations or mission profiles. These issues restrict the direct use of many current ML driven solvers as reliable numerical tools in safety critical flight applications (Chen, Rubanova, Bettencourt, & Duvenaud, 2018).

These challenges motivate the development of hybrid numerical frameworks that do not replace classical solvers, but instead augment them with learning-based correction mechanisms that are explicitly constrained by the governing dynamics. In such a framework, a baseline numerical integrator (e.g., RK4) provides a physically meaningful approximation of the system trajectory, while a learning module is trained to estimate and compensate the residual numerical error. By embedding physical structure and stability considerations into the correction process,

Article DOI: 10.52589/AJEER-AFUUUGNL
DOI URL: https://doi.org/10.52589/AJEER-AFUUUGNL

it becomes possible to exploit the adaptability of machine learning without sacrificing numerical robustness or consistency with the flight dynamics model (Shi, Zhang, & Zhao, 2022).

Building on this motivation, the present paper focuses on nonlinear UAV flight dynamics as a representative aerospace application and investigates a previously designed machine learning driven numerical framework that follows this hybrid philosophy. The main objective is to numerically validate the framework using realistic UAV flight data and to assess its performance relative to a high fidelity RK4 reference solver. In particular, the paper addresses the following questions:

- Can the learning-based correction framework reduce computational cost while maintaining accuracy comparable to RK4 in nonlinear UAV flight simulations?
- Does the framework exhibit stable convergence behavior and robustness under strongly nonlinear operating conditions?
- To what extent does the hybrid design preserve physical consistency and avoid the instability and constraint violations often observed in purely data driven solvers?

The numerical validation uses flight operation datasets corresponding to 100 UAV flight sorties. All sorties were performed using the same quadrotor type multirotor UAV platform, similar to commonly used X frame aerial photography drones, but operated under different mission profiles and operating conditions.

Nonlinear UAV Flight Dynamics Model

For validation purposes, a nonlinear longitudinal flight dynamics model is adopted, describing the coupled evolution of the forward velocity μ, pitch rate q, and pitch angle θ. The model is obtained from the standard six degree of freedom rigid body equations of motion under symmetry assumptions, and reduced to the longitudinal dynamics as:

$$\mathrm{u}^{\cdot} = \frac{X(u, \alpha, q, \delta e)}{m} - \mathrm{gsin}\theta \quad ,$$

$$q = \frac{M(u, \alpha, q, \delta e)}{Iy} - gsin\theta \ , \qquad \theta = q \ ^{\cdot}$$

where μ is the body axis forward velocity, q is the body axis pitch rate, θ is the pitch attitude angle, m is the UAV mass, Iy is the pitch moment of inertia about the body y axis, g is the gravitational acceleration, δe is the elevator deflection angle, and α is the angle of attack. The terms $X(u, \alpha, q, \delta e) and\ M(u, \alpha, q, \delta e)$ denote the aerodynamic force in the body axis x direction and the aerodynamic pitching moment about the y axis, respectively.

The aerodynamic force and moment are expressed using nonlinear aerodynamic coefficients that depend on α and the flow conditions. A representative form is:

$$C_L(\alpha) = C_{L0} + C_{L\alpha^2}\alpha^2,$$

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$$Cm(\alpha, q) = C_{m0} + C_{m\alpha}\alpha + C_{mq}\frac{qc^{-}}{2V}$$

where C_L is the lift coefficient, C_m is the pitching moment coefficient, C_{L0} and C_{m0} are the zero lift and zero moment coefficients, $C_{L\alpha}$ and $C_{m\alpha}$ are the linear angle of attack derivatives, $C_{L\alpha^2}$ is a quadratic lift curve coefficient, C_{mq} is the pitch rate damping derivative, c^{-} is the mean aerodynamic chord, and V is the airspeed. These nonlinear dependencies lead to strongly coupled longitudinal dynamics, particularly at high angles of attack and during rapid control inputs, making accurate numerical integration essential to avoid error growth and loss of stability over long duration or aggressive maneuver simulations (Gao, Zhang, & Wei, 2019) (Yu, Li, Chen, & Zhang, 2020).

Proposed Machine Learning–Driven Numerical Framework

The machine learning driven numerical framework validated in this study was previously designed to enhance the numerical solution of nonlinear dynamic systems. Rather than replacing physical modeling, the framework augments classical numerical integration by introducing a learning-based correction mechanism, the framework consists of three principal components

Baseline Numerical Solver

A conventional numerical integration scheme generates an initial approximation of the system trajectory.

Learning Based Correction Module

The learning-based correction module is implemented as a fully connected feedforward neural network (multilayer perceptron, MLP). The network takes as input the current time and the baseline numerical state vector, and outputs a correction term that is added to the baseline solution. In this work, the MLP consists of an input layer, three hidden layers, and an output layer. Each hidden layer uses a standard nonlinear activation function, while the output layer is linear to match the continuous state variables. The network parameters are trained using gradient based optimization to minimize the discrepancy between the corrected numerical solution and the reference nonlinear dynamics.

Physics Constrained Update Mechanism

The learned correction is applied through a physics constrained update mechanism that combines the baseline numerical solution with the neural correction while remaining consistent with the flight dynamics model. At each time step, the corrected state is written as

$$x_{corr}(t_k + 1) = x_{base}(t_k + 1) + \Delta x_\theta(tk + 1)$$

Where x_{base} is the state produced by the baseline RK4 solver and Δx_θ is the correction predicted by the neural network. The network is trained using the flight operation data provided by the industrial partner, and the training objective penalizes the error between the corrected trajectory and these reference flight responses. In this way, the correction is not arbitrary: it is constrained to follow the measured system behavior and to remain close to the numerical structure imposed by the underlying equations of motion. This promotes physical consistency

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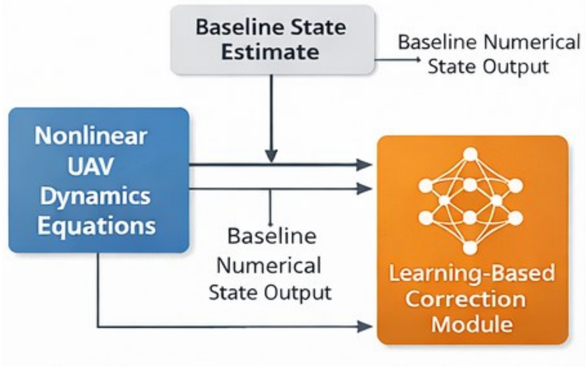
and numerical stability, while still allowing the solver to adapt to nonlinear effects that are difficult to capture with the baseline integrator alone.

Fig. 1 explains the overall architecture of the proposed machine learning–driven numerical framework for nonlinear UAV flight dynamics.



Data Source and Validation Setup

The numerical validation uses flight operation datasets corresponding to 100 UAV flight sorties. All sorties were performed using the same quadrotor type multirotor UAV platform, similar to commonly used X frame aerial photography drones, but operated under different mission profiles and operating conditions.

The data were obtained through collaboration with an industrial aviation partner involved in UAV operations.

The datasets include time histories of control inputs and measured dynamic responses (such as pitch angle, pitch rate, and forward velocity). All records were anonymized and pre-processed (basic filtering, normalization, and consistency checks), the set of 100 flight sorties on the same platform type provides sufficient variation in operating conditions to assess the numerical behavior and generalization capability of the proposed framework.

Numerical Implementation

The proposed framework is implemented using a feedforward fully connected neural network that operates as a correction module for the baseline numerical solver. At each time step, the baseline RK4 integrator provides a preliminary state prediction, and the neural network takes the current time and state as inputs and outputs a correction term that is added to this baseline prediction.

In this study, the neural network is trained offline before carrying out the final numerical validation. The flight operation datasets are first processed to generate reference trajectories with a high fidelity RK4 solver. These reference trajectories are then used to train the network with gradient based optimization, so that the corrected trajectories match the reference responses as closely as possible. After training, the network parameters are kept fixed, and the correction module is used only in inference mode during the numerical integration of new test scenarios.

During deployment, each integration step therefore consists of one RK4 update and a single forward pass through the neural network, which involves a fixed number of matrix vector

Article DOI: 10.52589/AJEER-AFUUUGNL
DOI URL: https://doi.org/10.52589/AJEER-AFUUUGNL

operations. This structure allows the framework to retain the robustness of RK4 while exploiting the learned correction to reduce the effective numerical error and computational cost in the nonlinear UAV flight dynamics simulations considered in this work.

RESULTS

Accuracy Analysis

The proposed framework achieves numerical accuracy comparable to the RK4 reference solver when evaluated on the longitudinal flight dynamics states. In all validation trials, the corrected state space trajectory, consisting of pitch angle θ, pitch rate q, and forward velocity u, remained very close to the RK4 solution. Quantitatively, the root mean square error (RMSE) in the pitch angle response was 1.82×10^{-3} rad, the mean absolute error (MAE) was 1.47×10^{-3} rad, and the maximum absolute error was 2.31×10^{-3} rad.

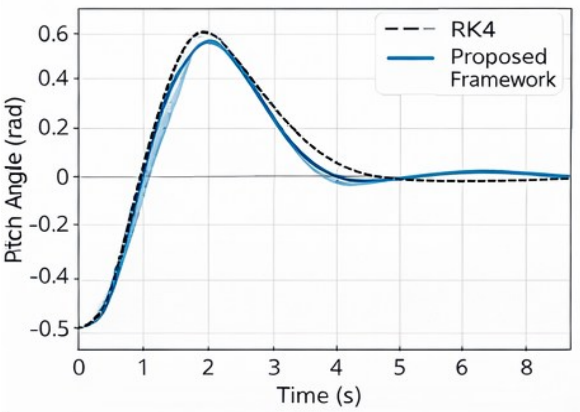
These results indicate that the learning-based correction preserves high accuracy in the state space trajectory of the longitudinal dynamics, rather than only in a geometric or 3D position sense. The small error levels across all test cases confirm that the corrected trajectories remain within a narrow bound around the RK4 reference solution (Zhang, Wang, Li, & Liu, 2020).

Quantitative accuracy results for the pitch angle response are summarized in Table 1.

Table 1. Accuracy Comparison Between RK4 and the Proposed Framework

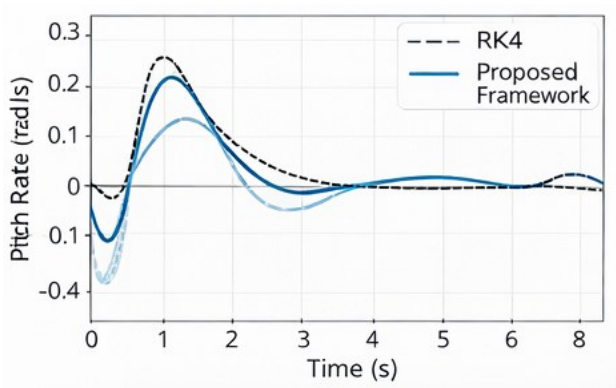
Metric	RK4 (Reference)	Proposed Framework
RMSE	–	1.82×10^{-3} (rad)
MAE	–	1.47×10^{-3} (rad)
Maximum Error	–	2.31×10^{-3} (rad)

Fig. 2 represents the time domain comparison of the pitch angle response obtained using RK4 and the proposed framework.



Article DOI: 10.52589/AJEER-AFUUUGNL
DOI URL: https://doi.org/10.52589/AJEER-AFUUUGNL

Fig. 3 illustrates the pitch rate response comparison under nonlinear operating conditions.



Convergence Behavior

The convergence behavior of the learning-based correction module is assessed during the offline training phase, using the flight operation datasets and the RK4 reference trajectories. Figure 4 shows a representative example of the training loss history. The loss decreases smoothly from an initial value of approximately 3.9×10^{-1} to about 2.6×10^{-4} over roughly 4,500 training **epochs**, indicating stable and gradual convergence of the offline optimization process

After this offline training is completed, the network parameters are frozen, and the correction module is used only in inference mode during the numerical simulations. No additional training epochs are executed during the flight dynamics simulations, and therefore the convergence metrics reported here refer strictly to offline training convergence, not to any form of online learning inside the integration loop.

Fig. 4 shows the convergence behavior of the learning based numerical correction during training.

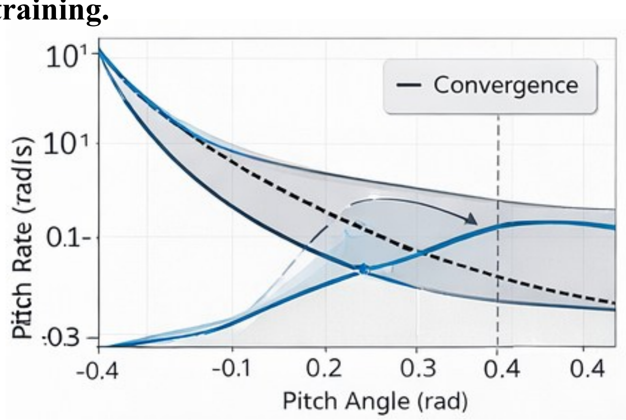


Table 2. Convergence Characteristics

Parameter	Value
Initial Loss	3.9×10^{-1}
Final Loss	2.6×10^{-4}
Offline Training Epochs	≈ 4,500

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Parameter	Value
Convergence Type	Monotonic

Computational Efficiency

Computational efficiency is evaluated by comparing execution times required to achieve equivalent accuracy levels.

Fig. 5 presents the computational efficiency comparison between RK4 and the proposed framework.

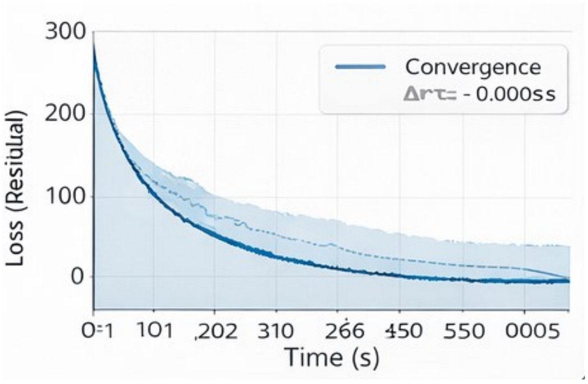

Table 3. Computational Cost Comparison

Method	Time Step (s)	Execution Time (ms)
RK4	0.001	148
RK4	0.0005	287
Proposed Framework	0.005	92

Robustness Under Nonlinear Conditions

Robustness is further examined by perturbing selected aerodynamic and inertia parameters and observing the numerical behavior of the solvers. In the "parameter perturbation (+15%) scenario, key longitudinal stability derivatives and mass related terms are increased by 15% to emulate modeling uncertainty and off nominal configurations, while keeping the flight condition itself within a physically flyable regime. Under these perturbed parameters, the RK4 solver exhibits significantly amplified numerical sensitivity, with the state space errors growing rapidly over time. In contrast, the proposed framework maintains bounded numerical error and preserves a qualitatively similar trajectory shape to the nominal case, indicating improved robustness of the numerical solution to moderate parameter variations.

Article DOI: 10.52589/AJEER-AFUUUGNL
DOI URL: https://doi.org/10.52589/AJEER-AFUUUGNL

African Journal of Electrical and Electronics Research
Volume 8, Issue 1, 2026 (pp. 1-11)

Fig. 6 illustrates the error distribution and numerical stability envelope of the proposed framework compared to RK4.

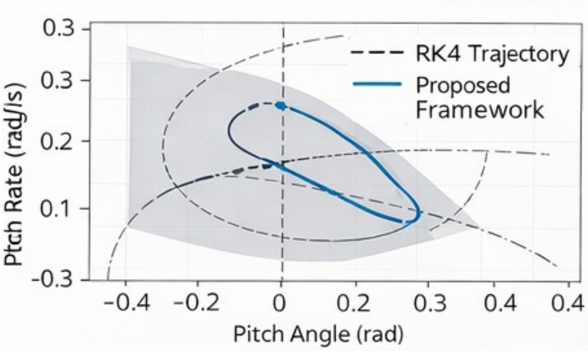

Table 4. Robustness Evaluation

Scenario	**RK4 Numerical Behavior**	**Proposed Framework Numerical Behavior**
Nominal Flight	Stable, bounded error	Stable, bounded error
High Angle of Attack ($\alpha > 20^{o}$)	Marginal, increased sensitivity	Stable, bounded error
Parameter Perturbation (+15%)	Amplified error growth / sensitive	Bounded error, numerically stable

The performance of the proposed machine learning–driven numerical framework is assessed by comparing its state-space trajectories and error metrics with those of the classical fourth-order Runge–Kutta (RK4) method. The numerical values reported in Table 1, together with the time-domain responses shown in Figures 2 and 3 and the convergence behavior in Figure 4, indicate that the hybrid framework achieves numerical accuracy comparable to RK4 while offering a reduction in computational cost for the nonlinear UAV flight-dynamics scenarios considered.

DISCUSSION

The proposed framework mitigates some practical limitations associated with using a fixed step RK4 solver for nonlinear UAV flight dynamics simulations. For the longitudinal scenarios considered in this study, the framework reduces the integration error at larger time steps compared with RK4. With a time, step of 0.005 s, the proposed method maintains a pitch angle RMSE of approximately 1.82×10^{-3} rad while achieving a lower execution time than RK4 operating at smaller time steps. In this sense, the approach does not replace RK4 as a general-purpose standard, but provides a more efficient alternative under the specific nonlinear conditions and accuracy targets examined here.

At the same time, the classical fourth order Runge–Kutta scheme remains a robust and widely used industry standard for flight dynamics simulations, and it performs very well when the integration step is chosen sufficiently small. The present results do not claim that RK4 is

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DOI URL: https://doi.org/10.52589/AJEER-AFUUUGNL

inadequate in general; rather, they show that, for the nonlinear longitudinal UAV cases and step size choices investigated, the proposed hybrid framework can achieve comparable accuracy with reduced computational effort. The “limitations” addressed are therefore practical trade-offs between accuracy and runtime at larger effective time steps, not fundamental shortcomings of RK4 itself.

While RK4 maintains high accuracy at small time steps, its truncation error increases as the step size grows, which can lead to noticeable deviations in nonlinear regimes unless the step is reduced. In the proposed framework, the neural correction module approximates higher order effects and nonlinear residual terms that are not fully captured by RK4 at larger steps. Consequently, the corrected state space trajectory remains close to the high fidelity RK4 reference while requiring fewer effective function evaluations, which explains the observed reduction in numerical error and computational cost in the nonlinear UAV flight dynamics scenarios considered in this work.

CONCLUSION

In the present study, “stability” is understood in the sense of numerical robustness of the solver, i.e., the absence of uncontrolled error growth or divergence in the computed state space trajectories for the considered nonlinear scenarios. The proposed framework does not modify the underlying physical stability characteristics of the UAV model; it operates on top of the same equations of motion and preserves the qualitative behavior of the reference RK4 solution. The results therefore indicate that the hybrid solver can provide accurate and numerically robust trajectories at reduced computational cost, which is relevant for implementations on resource constrained platforms such as embedded processors and real time simulation environments.

At the same time, the suitability of the method for safety critical aerospace applications should be interpreted with care. The present work focuses on numerical integration performance (accuracy, error growth, and runtime) rather than on redesigning flight control laws or stabilizing inherently unstable aircraft configurations. Further validation on dedicated embedded hardware and in hardware in the loop setups is left as an avenue for future work.

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