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RATIO TYPE ESTIMATORS USING THE LINEAR COMBINATION OF QUARTILE DEVIATION AND POPULATION DECILES OF AUXILIARY VARIABLE: A COMPARATIVE ANALYSIS OF SOME CLASSES OF ESTIMATORS

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ABSTRACT: *ratio type estimators for population means are in vogue. Researchers, over the years, have been making efforts to improve the efficiency of these estimators. Year after year they introduce modified estimators which they claimed performed better than other ratio estimators in literature. There is need to carrying out empirical study to determine the best of these estimators, the circumstance at which it performs best; and also, to find out if the estimators are distribution sensitive, compared the estimator to ascertain if they are affected by sample size. A simulation study was carried out and the result showed that the tenth estimator is the most efficient of the twenty estimators considered; it performed better than others irrespective of the distribution of the study and auxiliary variables; and also, that the efficiency of the estimators increases as sample size increases.*

KEYWORDS: Ratio, Estimators, Auxiliary, Variables, Quartile deviations, Deciles

INTRODUCTION

In past years, the idea of collecting data from the entire population was used by political entities to collect opinion about potential political candidates. Census data collection is still very popular for collecting public opinion for political endeavours. For most researchers, however, collecting data from an entire population is almost impossible because of the amount of people, places and things involved. Taking a census involves much time and money, so researchers collect a part or fraction of the population for study and use the information obtained to make generalization about the entire population.

Sampling is the systematic process of selecting a representative part of a population for study so that inferences could be made about the entire population. Among the advantages of sampling are that it can save cost and human resources. One of the disadvantages is that sampling process only enables a researcher to make estimation about the actual situation instead of finding the real truth. The objective of selecting a sample is to achieve maximum accuracy in one’s estimation within a given sample size and to avoid bias. This is important as bias can attack the integrity of facts and jeopardize one’s research outcome.

Bias is the tendency of a statistic to systematically over-estimate or under-estimate a population parameter. It is the difference between the expected value of the estimator and the true value of the parameter being estimated. Bias can occur due to unrepresentativeness of the sample (section bias), poor measurement process (response bias), and so on. Increasing sample size tends to reduce sampling error which is the variability among statistics from different samples drawn from the same population. However, increasing the sample size does not affect survey bias; a large sample cannot correct the methodological problems that produce survey bias.

Another concept used in assessing the performance of an estimator is the Mean Squared Error (MSE). Mean Squared Error is quite important for relaying the concept of precision, bias and accuracy during statistical estimation. Mean Squared Error (MSE) is defined as the average of squares of errors; error is the difference between the attribute which is to be estimated and the estimator. It can be referred to as the second moment of the error measured about the origin. It incorporates both variance and bias of the estimator. If an estimator is unbiased, then its MSE is the same as its variance.

The primary aim of a survey sampling is generally to collect data that could be used to describe features of the population of interest. To achieve this goal, one must be able to use units observed in the sample to make inferences about units in the entire population. Estimation of population mean is the persistent issue in sampling practices and many efforts have been made by various statisticians to improve the precision of the estimates by using the auxiliary information.

Auxiliary information is any information closely related to the study variable, used to improve the precision of its estimates. Auxiliary information may be used at:

- The planning or designing stage
- The selection stage
- The estimation stage.

At the estimation stage, auxiliary information is used by ratio estimators, regression estimators and product estimators. Proper use of auxiliary information may result to appreciable gain in precision while indiscriminate use may lead to a loss in precision.

Ratio estimation is a very important method in sample surveys which utilizes the information on auxiliary variable which is positively and highly correlated with the study variable. Hence, a ratio estimator is an estimator which utilizes the information on an auxiliary variable, X to estimate the population parameter of the variable of interest, To reduce the bias and hence improve the precision of estimates gotten by ratio method, researchers began to modify the original ratio estimator using median, quartiles, deciles, quartile deviation, coefficient of variation, coefficient of kurtosis, coefficient of correlation, and so on. Among such researchers are Cochran, (1940) Abid (2016), Rao (1991,) Singh (2003), Subramani and Jeelani (2013), Subzar et al (2017) and many others.

Many researchers are introducing a large number of modified ratio estimators, comparing them with the classical and already existing ones and concluding that the modified ratio estimators they have proposed perform better than those already in existence. Subzar et al (2017) proposed a new class of twenty improved ratio type estimators using the linear combination of quartile deviation and population deciles of auxiliary variable. Considering these estimators, one may ask "are they all equal in performance? Do they all perform well at all circumstances?" These propelled the zeal to carry out a comparative analysis on Subzar et al (2017) proposed estimators to determine:

- The best estimator among them
- If they are distribution based or not
- If they are affected by sample sizes.
- To make conclusion.

LITERATURE

Amogu (2012) defined sampling as the systematic process of selecting a representative part of a population for study so that inferences could be made about the entire population. Cochran (1953) posits that using correct sampling method allows researchers to the ability to reduce research costs, conduct research more efficiently, have greater flexibility, and provides for greater accuracy. Subzar et al (2017) asserted that one of the important objectives in Sampling Theory is to estimate the parameter of interest with more precision. Subramani and Ajith (2017) also stated that the main aim of sampling theory is to obtain precise results about the parameters of study variable on the basis of random samples.

Estimation is the process of calculating from a sample data a statistic that is offered as an approximation of the corresponding parameter of the population from where the sample was taken, Amogu (2013). Hence, he described an estimator as a rule or procedure, usually represented by some formula, which is used in calculating from a sample data a statistic that is offered as an approximation of the corresponding population parameter. Subzar et al (2017) explained that incorporation of auxiliary information at design or at estimation stage or at both stages results in more precision in estimating population parameter of interest. They proposed an improved family of estimators by incorporating the auxiliary information of conventional and non-conventional location parameters and their linear combination with correlation coefficient and coefficient of variation for estimating the population mean with more precision. Using mean squared error (MSE) and bias as basis of judgment, they found that their estimators were more efficient than the classical and existing estimators.

Cochran (1940) initiated the use of auxiliary information at the estimation stage and proposed a ratio estimator for the population mean. He maintained that the ratio type estimator provides better efficiency in comparison to a simple mean estimator if the study variable and auxiliary variable are positively and highly correlated. The use of auxiliary information in sample survey is to get gain in precision of estimates, Subzar et al (2017). Abid et al (2016) explained auxiliary variable as a variable, different from the variable of interest, known for every unit of the population which is employed to improve the sampling plan or to enhance the estimation of the variable of interest. According to Subzar et al (2017), in estimation theory, increase in precision is achieved by taking advantage of correlation between the response variable and the concomitant variable and utilizing the auxiliary information by various methods such as ratio, regression and product. Some authors reviewed are Cochran (1940), Okafor (2002). Robson

(1957), Abid et al (2016) Rajesh (2008), Subramani and Kumarapandiyan (2012), Etaga et al (2019), Etaga et al (2020), Subzar et al (2016), Rao (1991), Upadhyaya and Singh (1999), Singh and Tailor (2003), Singh et al (1973). Searls and Intarapanich (1990), Upadhyaya and Singh (1999), Singh (2003) and Singh et al (2004), Sabjit and Jasleen (2016), Jeelani et al (2016). McIntye (1952) , Bhat et al (2018) , Rajesh et al (2011).

MATERIALS AND METHOD

Distribution considered are**,** Poisson Hyper-geometric, Binomial, Continuous uniform, Exponential and Normal. A population size of 200 was simulated and sample sizes of 13, 20, 30, 49 and 100 were drawn from each of them using Simple Random Sample (SRS). The data were simulated using the following values of the needed parameters:

A. Poisson

$$f(x) = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!} \;; \quad x = 0, 1, 2, \ldots \\ \quad 0\ elsewhere \end{cases}$$

$$the\ mean, \lambda = 10$$

B. Hyper-geometric Distribution

$$f(x) = \begin{cases} \dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} \;; \;\; x = 0,1,2, \ldots, \min(M, n) \\ \quad 0\ elsewhere \end{cases}$$

Population size, N = 51

Event count in population, M = 26

Sample size, n = 8

C. Binomial Distribution

$$f(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 1,2,3, \ldots, n \quad 0 \leq p \leq 1 \\ \quad 0\ elsewhere \end{cases}$$

Number of trial, n = 20

Event probability, p =0.6

D. Continuous Uniform Distribution

$$f(x) = \begin{cases} \dfrac{1}{b-a} \;; \quad a \leq x \leq b; \; -\infty < a < b < \infty \\ \quad 0\ elsewhere \end{cases}$$

Lower endpoint, a =1

Upper endpoint, b =15

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E. Exponential Distribution

$$f(x) = \begin{cases} \lambda e^{-x\lambda}; & \lambda > 0, \quad x > 0 \\ 0 \; elsewhere \end{cases}$$

The scale parameter, $\lambda = 2.0$

F. Normal Distribution

$$f(x) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; & -\infty < x < \infty, \quad \sigma > 0 \\ 0 \; elsewhere \end{cases}$$

mean, $\mu = 15$

standard deviation, $\sigma = 2.3$

Ratio estimation method was adopted in this work. Below are the estimators, their biases, related constants and mean squared errors (MSE).

1. $\hat{\hat{Y}}_{P1} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_1)}(\bar{X}QD + D_1)$; $R_1 = \frac{\bar{X}QD}{\bar{X}QD+D_1}$;

   $B\left(\hat{\hat{Y}}_{P1}\right) = \frac{(1-f)}{n}\frac{S_X^2}{\bar{Y}}R_1^2$; $MSE\left(\hat{\hat{Y}}_{P1}\right) = \frac{(1-f)}{n}\left(R_1^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

2. $\hat{\hat{Y}}_{P2} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_2)}(\bar{X}QD + D_2)$; $R_2 = \frac{\bar{X}QD}{\bar{X}QD+D_2}$;

   $B\left(\hat{\hat{Y}}_{P2}\right) = \frac{(1-f)}{n}\frac{S_X^2}{\bar{Y}}R_2^2$; $MSE\left(\hat{\hat{Y}}_{P2}\right) = \frac{(1-f)}{n}\left(R_2^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

3. $\hat{\hat{Y}}_{P3} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_3)}(\bar{X}QD + D_3)$; $R_3 = \frac{\bar{X}QD}{\bar{X}QD+D_3}$;

   $B\left(\hat{\hat{Y}}_{P3}\right) = \frac{(1-f)}{n}\frac{S_X^2}{\bar{Y}}R_3^2$; $MSE\left(\hat{\hat{Y}}_{P3}\right) = \frac{(1-f)}{n}\left(R_3^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

4. $\hat{\hat{Y}}_{P4} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_4)}(\bar{X}QD + D_4)$; $R_4 = \frac{\bar{X}QD}{\bar{X}QD+D_4}$;

   $B\left(\hat{\hat{Y}}_{P4}\right) = \frac{(1-f)}{n}\frac{S_X^2}{\bar{Y}}R_4^2$; $MSE\left(\hat{\hat{Y}}_{P4}\right) = \frac{(1-f)}{n}\left(R_4^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

5. $\hat{\hat{Y}}_{P5} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_5)}(\bar{X}QD + D_5)$; $R_5 = \frac{\bar{X}QD}{\bar{X}QD+D_5}$;

   $B\left(\hat{\hat{Y}}_{P5}\right) = \frac{(1-f)}{n}\frac{S_X^2}{\bar{Y}}R_5^2$; $MSE\left(\hat{\hat{Y}}_{P5}\right) = \frac{(1-f)}{n}\left(R_5^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

6. $\hat{\hat{Y}}_{P6} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_6)}(\bar{X}QD + D_6)$; $R_6 = \frac{\bar{X}QD}{\bar{X}QD+D_6}$;

   $B\left(\hat{\hat{Y}}_{P6}\right) = \frac{(1-f)}{n}\frac{S_X^2}{\bar{Y}}R_6^2$; $MSE\left(\hat{\hat{Y}}_{P6}\right) = \frac{(1-f)}{n}\left(R_6^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

7. $\hat{\hat{Y}}_{P7} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_7)}(\bar{X}QD + D_7)$; $R_7 = \frac{\bar{X}QD}{\bar{X}QD+D_7}$;

$B\left(\hat{\hat{Y}}_{P7}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_7^2$; $MSE\left(\hat{\hat{Y}}_{P7}\right) = \frac{(1-f)}{n}\left(R_7^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

8. $\hat{\hat{Y}}_{P8} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_8)}(\bar{X}QD + D_8)$; $R_8 = \frac{\bar{X}QD}{\bar{X}QD+D_8}$;

$B\left(\hat{\hat{Y}}_{P8}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_8^2$; $MSE\left(\hat{\hat{Y}}_{P8}\right) = \frac{(1-f)}{n}\left(R_8^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

9. $\hat{\hat{Y}}_{P9} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_9)}(\bar{X}QD + D_9)$; $R_9 = \frac{\bar{X}QD}{\bar{X}QD+D_9}$;

$B\left(\hat{\hat{Y}}_{P9}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_9^2$; $MSE\left(\hat{\hat{Y}}_{P9}\right) = \frac{(1-f)}{n}\left(R_9^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

10. $\hat{\hat{Y}}_{P10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+D_{10})}(\bar{X}QD + D_{10})$; $R_{10} = \frac{\bar{X}QD}{\bar{X}QD+D_{10}}$;

$B\left(\hat{\hat{Y}}_{P10}\right)\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{10}^2$; $MSE\left(\hat{\hat{Y}}_{P10}\right) = \frac{(1-f)}{n}\left(R_{10}^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

11. $\hat{\hat{Y}}_{P11} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_1+QD)}(\bar{X}D_1 + QD)$; $R_{11} = \frac{\bar{X}D_1}{\bar{X}D_1+QD}$;

$B\left(\hat{\hat{Y}}_{P11}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{11}^2$; $MSE\left(\hat{\hat{Y}}_{P11}\right) = \frac{(1-f)}{n}\left(R_{11}^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

12. $\hat{\hat{Y}}_{P12} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_2+QD)}(\bar{X}D_2 + QD)$; $R_{12} = \frac{\bar{X}D_2}{\bar{X}D_2+QD}$;

$B\left(\hat{\hat{Y}}_{P12}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{12}^2$; $MSE\left(\hat{\hat{Y}}_{P12}\right) = \frac{(1-f)}{n}\left(R_{12}^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

13. $\hat{\hat{Y}}_{P13} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_3+QD)}(\bar{X}D_3 + QD)$; $R_{13} = \frac{\bar{X}D_3}{\bar{X}D_3+QD}$;

$B\left(\hat{\hat{Y}}_{P13}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{13}^2$; $MSE\left(\hat{\hat{Y}}_{P13}\right) = \frac{(1-f)}{n}\left(R_{13}^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

14. $\hat{\hat{Y}}_{P14} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_4+QD)}(\bar{X}D_4 + QD)$; $R_{14} = \frac{\bar{X}D_4}{\bar{X}D_4+QD}$;

$B\left(\hat{\hat{Y}}_{P14}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{14}^2$; $MSE\left(\hat{\hat{Y}}_{P14}\right) = \frac{(1-f)}{n}\left(R_{14}^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

15. $\hat{\hat{Y}}_{P15} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_1+QD)}(\bar{X}D_5 + QD)$; $R_{15} = \frac{\bar{X}D_5}{\bar{X}D_5+QD}$;

$B\left(\hat{\hat{Y}}_{P15}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{15}^2$; $MSE\left(\hat{\hat{Y}}_{P15}\right) = \frac{(1-f)}{n}\left(R_{15}^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

16. $\hat{\hat{Y}}_{P16} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_6+QD)}(\bar{X}D_6 + QD)$; $R_{16} = \frac{\bar{X}D_6}{\bar{X}D_6+QD}$;

$B\left(\hat{\hat{Y}}_{P16}\right) = \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{16}^2$; $MSE\left(\hat{\hat{Y}}_{P16}\right) = \frac{(1-f)}{n}\left(R_{16}^2 S_X^2 + S_Y^2(1-\rho^2)\right)$

17. $\hat{\hat{Y}}_{P17} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_7+QD)}(\bar{X}D_7 + QD)$; $R_{17} = \frac{\bar{X}D_7}{\bar{X}D_7+QD}$;

$$B\left(\hat{\hat{Y}}_{P17}\right) = \frac{(1-f)}{n}\frac{S_X^2}{\bar{Y}}R_{17}^2 \; ; \quad MSE\left(\hat{\hat{Y}}_{P17}\right) = \frac{(1-f)}{n}\left(R_{17}^2 S_X^2 + S_Y^2(1-\rho^2)\right)$$

18. $\hat{\hat{Y}}_{P18} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_8+QD)}(\bar{X}D_8 + QD)$; $R_{18} = \frac{\bar{X}D_8}{\bar{X}D_8+QD}$;

$$B\left(\hat{\hat{Y}}_{P18}\right) = \frac{(1-f)}{n}\frac{S_X^2}{\bar{Y}}R_{18}^2 \; ; \quad MSE\left(\hat{\hat{Y}}_{P18}\right) = \frac{(1-f)}{n}\left(R_{18}^2 S_X^2 + S_Y^2(1-\rho^2)\right)$$

19. $\hat{\hat{Y}}_{P19} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_9+QD)}(\bar{X}D_9 + QD)$; $R_{19} = \frac{\bar{X}D_9}{\bar{X}D_9+QD}$;

$$B\left(\hat{\hat{Y}}_{P19}\right) = \frac{(1-f)}{n}\frac{S_X^2}{\bar{Y}}R_{19}^2 \; ; \quad MSE\left(\hat{\hat{Y}}_{P19}\right) = \frac{(1-f)}{n}\left(R_{19}^2 S_X^2 + S_Y^2(1-\rho^2)\right)$$

20. $\hat{\hat{Y}}_{P20} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}D_{10}+QD)}(\bar{X}D_{10} + QD)$; $R_{20} = \frac{\bar{X}D_{10}}{\bar{X}D_{10}+QD}$;

$$B\left(\hat{\hat{Y}}_{P20}\right) = \frac{(1-f)}{n}\frac{S_X^2}{\bar{Y}}R_{20}^2 \; ; \quad MSE\left(\hat{\hat{Y}}_{P20}\right) = \frac{(1-f)}{n}\left(R_{20}^2 S_X^2 + S_Y^2(1-\rho^2)\right)$$

RESULTS

The population means, population variances, the slope of the regression equation of the two variables (study variable, Y and auxiliary variable, X), population correlation coefficients between the two variables, population quartile deviation and deciles of the auxiliary variable were also obtained and presented in Table 1

Table 1: Computed Values

Parameters	Values of the parameters					
	Poisson	Hyper Geometric	Binomial	Uniform	Exponential	Normal
$\bar{Y}$	9.935	7.555	12.115	8.470	1.712	15.148
$\bar{X}$	10.240	7.505	12.175	8.192	2.146	14.564
S_y^2	9.870	2.821	5.509	15.059	2.703	5.044
S_x^2	11.279	2.663	5.421	16.425	3.965	4.591
QD	2.375	1.5	1.5	3.404	1.2405	1.231
ρ	0.98	0.981	0.985	0.998	0.994	0.983
b	0.917	1.01	0.993	0.955	0.82	1.03
D_1	6	5.9	9	2.431872	0.229358	11.45453
D_2	7	6	10	3.868138	0.447806	12.93591
D_3	8	7	11	5.642535	0.709868	13.74068
D_4	9	7	12	7.130617	1.178107	14.23175
D_5	10	7.5	12	8.184451	1.609094	14.70948
D_6	11	8	13	9.292227	2.099278	15.05911
D_7	12	8	13	11.01793	2.652341	15.54832
D_8	13	9	14	12.37717	3.573013	16.20981
D_9	15	9.1	15	13.72626	4.926093	17.01215

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D_{10}	19	11	18	14.98592	10.73938	21.27386
$\bar{y}_{13}$	9.923	7.692	12.692	6.24	1.796	15.825
$\bar{x}_{13}$	10.308	7.692	12.692	5.91	2.259	15.231
$\bar{y}_{20}$	10.250	8.100	11.9	7.923	2.341	14.503
$\bar{x}_{20}$	10.600	7.950	12.05	7.616	2.451	13.874
$\bar{y}_{30}$	9.700	8.033	11.8	7.534	1.859	15.079
$\bar{x}_{30}$	10.067	8.000	11.967	7.305	2.231	14.466
$\bar{y}_{49}$	9.653	7.837	11.714	6.808	1.873	15.440
$\bar{x}_{49}$	9.898	7.796	11.735	6.501	2.366	14.802
$\bar{y}_{100}$	10.060	7.640	11.97	8.518	1.607	15.109
$\bar{x}_{100}$	10.430	7.610	11.98	8.211	2.006	14.551

The twenty proposed ratio estimators were used to estimate the population mean of the study variable ($\bar{Y}$) on the basis of the information provided by the various samples and the auxiliary information, for each of the six distributions. Their related constants, biases and Mean Squared Errors (MSEs) were also computed. The results obtained are as shown in Table 2 – Table 22:

Table 2: Estimates of the Population Mean from Different Distributions for n=13

Estimators	Estimates					
	Poisson	Hyper Geometric	Binomial	Uniform	Exponential	Normal
$\hat{\bar{Y}}_{P1}$	9.808399	7.382438	11.84177	6.385212	1.629387	14.72647
$\hat{\bar{Y}}_{P2}$	9.810059	7.383126	11.85337	6.441141	1.634606	14.74571
$\hat{\bar{Y}}_{P3}$	9.811616	7.3896	11.8642	6.504791	1.64001	14.75543
$\hat{\bar{Y}}_{P4}$	9.813081	7.3896	11.87433	6.554046	1.647893	14.76112
$\hat{\bar{Y}}_{P5}$	9.81446	7.392581	11.87433	6.586879	1.653669	14.76651
$\hat{\bar{Y}}_{P6}$	9.815762	7.39541	11.88383	6.619711	1.659003	14.77035
$\hat{\bar{Y}}_{P7}$	9.816992	7.39541	11.88383	6.667713	1.66387	14.77559
$\hat{\bar{Y}}_{P8}$	9.818157	7.400655	11.89275	6.703065	1.670098	14.78245
$\hat{\bar{Y}}_{P9}$	9.820309	7.401152	11.90115	6.736206	1.676557	14.79042
$\hat{\bar{Y}}_{P10}$	9.824019	7.409749	11.92362	6.765538	1.68962	14.82743
$\hat{\bar{Y}}_{P11}$	9.798001	7.326558	11.68896	6.474012	1.682262	14.47971
$\hat{\bar{Y}}_{P12}$	9.797668	7.326464	11.68833	6.407665	1.66929	14.47918
$\hat{\bar{Y}}_{P13}$	9.797416	7.325665	11.6878	6.36976	1.659665	14.47894
$\hat{\bar{Y}}_{P14}$	9.797219	7.325665	11.68737	6.351871	1.649736	14.47881
$\hat{\bar{Y}}_{P15}$	9.79706	7.325344	11.68737	6.342985	1.644404	14.47869
$\hat{\bar{Y}}_{P16}$	9.79693	7.325062	11.687	6.335742	1.640449	14.4786
$\hat{\bar{Y}}_{P17}$	9.796821	7.325062	11.687	6.327277	1.63744	14.47849
$\hat{\bar{Y}}_{P18}$	9.796728	7.32459	11.68668	6.322227	1.634222	14.47835
$\hat{\bar{Y}}_{P19}$	9.796579	7.324549	11.68641	6.318181	1.631457	14.4782
$\hat{\bar{Y}}_{P20}$	9.796374	7.323899	11.68577	6.315047	1.627117	14.47757

Table 3: Estimates of the Population Mean from Different Distributions for n = 20

Estimators	Estimates					
	Poisson	Hyper Geometric	Binomial	Uniform	Exponential	Normal
$\hat{\bar{Y}}_{P1}$	10.24096	7.364057	12.10739	9.058944	1.848961	15.66659
$\hat{\bar{Y}}_{P2}$	10.23197	7.365655	12.10443	9.030701	1.864112	15.64423
$\hat{\bar{Y}}_{P3}$	10.22355	7.380709	12.10167	8.999357	1.879959	15.63299
$\hat{\bar{Y}}_{P4}$	10.21564	7.380709	12.09909	8.975665	1.903373	15.62641
$\hat{\bar{Y}}_{P5}$	10.20821	7.387655	12.09909	8.960138	1.920755	15.62021
$\hat{\bar{Y}}_{P6}$	10.20121	7.394252	12.09668	8.944817	1.936982	15.61579
$\hat{\bar{Y}}_{P7}$	10.1946	7.394252	12.09668	8.922781	1.951936	15.60976
$\hat{\bar{Y}}_{P8}$	10.18836	7.4065	12.09441	8.906822	1.971281	15.60189
$\hat{\bar{Y}}_{P9}$	10.17684	7.407661	12.09229	8.892064	1.991598	15.59275
$\hat{\bar{Y}}_{P10}$	10.15706	7.427791	12.08662	8.879164	2.03349	15.55063
$\hat{\bar{Y}}_{P11}$	10.29774	7.235582	12.14715	9.01441	2.00976	15.96451
$\hat{\bar{Y}}_{P12}$	10.29957	7.235368	12.14732	9.047525	1.968758	15.96517
$\hat{\bar{Y}}_{P13}$	10.30096	7.233552	12.14746	9.066866	1.939007	15.96547
$\hat{\bar{Y}}_{P14}$	10.30204	7.233552	12.14758	9.076103	1.908899	15.96564
$\hat{\bar{Y}}_{P15}$	10.30292	7.232821	12.14758	9.080718	1.892968	15.96579
$\hat{\bar{Y}}_{P16}$	10.30363	7.232179	12.14767	9.084492	1.881254	15.9659
$\hat{\bar{Y}}_{P17}$	10.30423	7.232179	12.14767	9.088919	1.872404	15.96603
$\hat{\bar{Y}}_{P18}$	10.30474	7.231105	12.14776	9.091567	1.862994	15.96621
$\hat{\bar{Y}}_{P19}$	10.30556	7.23101	12.14783	9.093693	1.854952	15.9664
$\hat{\bar{Y}}_{P20}$	10.30669	7.229533	12.148	9.095342	1.842421	15.96719

Table 4: Estimates of the Population Mean from Different Distributions for n = 30

Estimators	Estimates					
	Poisson	Hyper Geometric	Binomial	Uniform	Exponential	Normal
$\hat{\bar{Y}}_{P1}$	9.994074	7.220576	12.14554	9.308088	1.726346	15.24252
$\hat{\bar{Y}}_{P2}$	9.989692	7.222312	12.14057	9.261752	1.730623	15.23951
$\hat{\bar{Y}}_{P3}$	9.985585	7.238666	12.13594	9.210533	1.735045	15.23799
$\hat{\bar{Y}}_{P4}$	9.981728	7.238666	12.13162	9.171957	1.741483	15.2371
$\hat{\bar{Y}}_{P5}$	9.978098	7.246215	12.13162	9.14674	1.746192	15.23626
$\hat{\bar{Y}}_{P6}$	9.974676	7.253386	12.12758	9.121909	1.750534	15.23566
$\hat{\bar{Y}}_{P7}$	9.971444	7.253386	12.12758	9.086282	1.75449	15.23484
$\hat{\bar{Y}}_{P8}$	9.968388	7.266703	12.12379	9.060545	1.759544	15.23377

$\hat{\bar{Y}}_{P9}$	9.962747	7.267965	12.12023	9.036793	1.764777	15.23253
$\hat{\bar{Y}}_{P10}$	9.953042	7.289863	12.11074	9.016068	1.775332	15.2268
$\hat{\bar{Y}}_{P11}$	10.02165	7.081299	12.21236	9.235105	1.769392	15.28202
$\hat{\bar{Y}}_{P12}$	10.02254	7.081067	12.21265	9.289333	1.758889	15.2821
$\hat{\bar{Y}}_{P13}$	10.02321	7.079102	12.21288	9.321116	1.751072	15.28214
$\hat{\bar{Y}}_{P14}$	10.02373	7.079102	12.21307	9.336323	1.742987	15.28217
$\hat{\bar{Y}}_{P15}$	10.02416	7.078311	12.21307	9.343928	1.738636	15.28219
$\hat{\bar{Y}}_{P16}$	10.0245	7.077617	12.21324	9.350151	1.735404	15.2822
$\hat{\bar{Y}}_{P17}$	10.02479	7.077617	12.21324	9.357454	1.732943	15.28222
$\hat{\bar{Y}}_{P18}$	10.02504	7.076455	12.21338	9.361824	1.730309	15.28224
$\hat{\bar{Y}}_{P19}$	10.02544	7.076353	12.2135	9.365334	1.728043	15.28226
$\hat{\bar{Y}}_{P20}$	10.02598	7.076353	12.21379	9.368058	1.724484	15.28237

Table 5: Estimates of the Population Mean from Different Distributions for n = 49

Estimators	Estimates					
	Poisson	Hyper Geometric	Binomial	Uniform	Exponential	Normal
$\hat{\bar{Y}}_{P1}$	9.647819	7.355949	12.45238	10.39689	1.546623	15.04485
$\hat{\bar{Y}}_{P2}$	9.656275	7.357007	12.44146	10.28784	1.556049	15.05198
$\hat{\bar{Y}}_{P3}$	9.664221	7.366961	12.4313	10.16868	1.565867	15.05558
$\hat{\bar{Y}}_{P4}$	9.671702	7.366961	12.42183	10.0799	1.580294	15.05768
$\hat{\bar{Y}}_{P5}$	9.678757	7.371549	12.42183	10.02229	1.590946	15.05967
$\hat{\bar{Y}}_{P6}$	9.685423	7.375904	12.41298	9.965906	1.600844	15.06109
$\hat{\bar{Y}}_{P7}$	9.691729	7.375904	12.41298	9.885575	1.609926	15.06303
$\hat{\bar{Y}}_{P8}$	9.697706	7.383983	12.40468	9.82796	1.621622	15.06557
$\hat{\bar{Y}}_{P9}$	9.708766	7.384748	12.3969	9.775094	1.633839	15.06851
$\hat{\bar{Y}}_{P10}$	9.727882	7.398005	12.37617	9.729202	1.65882	15.08214
$\hat{\bar{Y}}_{P11}$	9.595106	7.270422	12.60013	10.22566	1.644704	14.9523
$\hat{\bar{Y}}_{P12}$	9.593428	7.270279	12.60076	10.3526	1.6201	14.9521
$\hat{\bar{Y}}_{P13}$	9.592157	7.269062	12.60128	10.42777	1.602076	14.95201
$\hat{\bar{Y}}_{P14}$	9.591162	7.269062	12.60171	10.46394	1.583686	14.95196
$\hat{\bar{Y}}_{P15}$	9.590361	7.268573	12.60171	10.48208	1.573894	14.95192
$\hat{\bar{Y}}_{P16}$	9.589704	7.268143	12.60208	10.49695	1.566668	14.95189
$\hat{\bar{Y}}_{P17}$	9.589154	7.268143	12.60208	10.51442	1.561192	14.95184
$\hat{\bar{Y}}_{P18}$	9.588687	7.267424	12.60239	10.5249	1.555355	14.95179
$\hat{\bar{Y}}_{P19}$	9.587937	7.26736	12.60267	10.53331	1.550355	14.95173
$\hat{\bar{Y}}_{P20}$	9.586905	7.266371	12.6033	10.53985	1.542541	14.95149

Table 6: Estimates of the Population Mean from Different Distributions for n = 100

Estimators	Estimates					
	Poisson	Hyper Geometric	Binomial	Uniform	Exponential	Normal
$\hat{\bar{Y}}_{P1}$	9.740799	7.46542	12.29555	8.481761	1.831825	15.13063
$\hat{\bar{Y}}_{P2}$	9.745361	7.465814	12.29084	8.482578	1.823639	15.13023
$\hat{\bar{Y}}_{P3}$	9.749646	7.469514	12.28645	8.48349	1.815295	15.13004
$\hat{\bar{Y}}_{P4}$	9.753677	7.469514	12.28235	8.484184	1.803355	15.12992
$\hat{\bar{Y}}_{P5}$	9.757476	7.471217	12.28235	8.484641	1.794777	15.12981
$\hat{\bar{Y}}_{P6}$	9.761062	7.472832	12.27852	8.485094	1.786979	15.12973
$\hat{\bar{Y}}_{P7}$	9.764454	7.472832	12.27852	8.485748	1.779967	15.12962
$\hat{\bar{Y}}_{P8}$	9.767666	7.475826	12.27492	8.486223	1.771132	15.12948
$\hat{\bar{Y}}_{P9}$	9.773605	7.47611	12.27155	8.486664	1.76213	15.12932
$\hat{\bar{Y}}_{P10}$	9.783855	7.481012	12.26255	8.487051	1.744406	15.12857
$\hat{\bar{Y}}_{P11}$	9.712269	7.43336	12.35891	8.483051	1.754311	15.1358
$\hat{\bar{Y}}_{P12}$	9.711358	7.433306	12.35918	8.482091	1.77227	15.13581
$\hat{\bar{Y}}_{P13}$	9.710668	7.432846	12.3594	8.481533	1.78602	15.13582
$\hat{\bar{Y}}_{P14}$	9.710128	7.432846	12.35958	8.481267	1.800602	15.13582
$\hat{\bar{Y}}_{P15}$	9.709693	7.432661	12.35958	8.481135	1.808605	15.13582
$\hat{\bar{Y}}_{P16}$	9.709336	7.432499	12.35974	8.481027	1.814622	15.13583
$\hat{\bar{Y}}_{P17}$	9.709038	7.432499	12.35974	8.4809	1.819246	15.13583
$\hat{\bar{Y}}_{P18}$	9.708784	7.432227	12.35987	8.480824	1.824236	15.13583
$\hat{\bar{Y}}_{P19}$	9.708377	7.432203	12.35999	8.480763	1.828563	15.13583
$\hat{\bar{Y}}_{P20}$	9.707817	7.431829	12.36026	8.480716	1.835423	15.13585

Table 7: Related Constants of the Estimators for the Different Distributions

Estimators	Related Constants					
	Poisson	Hyper Geometric	Binomial	Uniform	Exponential	Normal
$\hat{\bar{Y}}_{P1}$	0.802111	0.656127	0.669876	0.919786	0.920678	0.610162
$\hat{\bar{Y}}_{P2}$	0.776501	0.652325	0.646174	0.878183	0.856007	0.580876
$\hat{\bar{Y}}_{P3}$	0.752475	0.616596	0.624092	0.831707	0.78948	0.566115
$\hat{\bar{Y}}_{P4}$	0.729892	0.616596	0.60347	0.796362	0.693219	0.557471
$\hat{\bar{Y}}_{P5}$	0.708625	0.60016	0.60347	0.773095	0.623269	0.549311
$\hat{\bar{Y}}_{P6}$	0.688562	0.584577	0.584166	0.75006	0.559104	0.543489
$\hat{\bar{Y}}_{P7}$	0.669604	0.584577	0.584166	0.716788	0.500919	0.535547
$\hat{\bar{Y}}_{P8}$	0.651661	0.55572	0.56606	0.69259	0.426954	0.525169

$\hat{\hat{Y}}_{P9}$	0.618515	0.55299	0.549042	0.670136	0.350822	0.51311
$\hat{\hat{Y}}_{P10}$	0.561404	0.505785	0.503619	0.650446	0.198643	0.457329
$\hat{\hat{Y}}_{P11}$	0.962783	0.967234	0.986496	0.854068	0.284066	0.992675
$\hat{\hat{Y}}_{P12}$	0.967929	0.967763	0.98783	0.902997	0.436518	0.993508
$\hat{\hat{Y}}_{P13}$	0.971825	0.97224	0.988924	0.931409	0.551174	0.993886
$\hat{\hat{Y}}_{P14}$	0.974877	0.97224	0.989837	0.944935	0.670843	0.994096
$\hat{\hat{Y}}_{P15}$	0.977332	0.974043	0.989837	0.951683	0.735705	0.994287
$\hat{\hat{Y}}_{P16}$	0.979351	0.975626	0.990612	0.957196	0.784094	0.994419
$\hat{\hat{Y}}_{P17}$	0.981039	0.975626	0.990612	0.963657	0.821058	0.994593
$\hat{\hat{Y}}_{P18}$	0.982472	0.978275	0.991277	0.967518	0.860746	0.994813
$\hat{\hat{Y}}_{P19}$	0.984773	0.978509	0.991853	0.970617	0.894979	0.995056
$\hat{\hat{Y}}_{P20}$	0.98794	0.982155	0.993202	0.97302	0.948924	0.996043

The biases of the estimators were computed for each of the six distributions and five samples, the results are presented in Tables 8 – 12.

Table 8: Biases of the Different Estimators for Each Distribution at n = 13

Estimators	Bias					
	Poisson	**Hyper Geometric**	**Binomial**	**Uniform**	**Exponential**	**Normal**
$\hat{\hat{Y}}_{P1}$	0.052534	0.010914	0.014442	0.117995	0.141196	0.008115
$\hat{\hat{Y}}_{P2}$	0.049233	0.010788	0.013438	0.107562	0.122057	0.007355
$\hat{\hat{Y}}_{P3}$	0.046233	0.009638	0.012535	0.096479	0.103822	0.006986
$\hat{\hat{Y}}_{P4}$	0.0435	0.009638	0.01172	0.088453	0.080048	0.006774
$\hat{\hat{Y}}_{P5}$	0.041002	0.009131	0.01172	0.08336	0.064708	0.006577
$\hat{\hat{Y}}_{P6}$	0.038713	0.008663	0.010982	0.078466	0.052071	0.006439
$\hat{\hat{Y}}_{P7}$	0.036611	0.008663	0.010982	0.071659	0.041797	0.006252
$\hat{\hat{Y}}_{P8}$	0.034675	0.007829	0.010312	0.066903	0.030365	0.006012
$\hat{\hat{Y}}_{P9}$	0.031237	0.007752	0.009701	0.062635	0.020501	0.005739
$\boldsymbol{\hat{\hat{Y}}_{P10}}$	**0.025735**	**0.006485**	**0.008163**	**0.059008**	**0.006573**	**0.004559**
$\hat{\hat{Y}}_{P11}$	0.075688	0.023717	0.031319	0.101736	0.013441	0.02148
$\hat{\hat{Y}}_{P12}$	0.076499	0.023743	0.031404	0.113727	0.03174	0.021516
$\hat{\hat{Y}}_{P13}$	0.077116	0.023964	0.031474	0.120996	0.050604	0.021532
$\hat{\hat{Y}}_{P14}$	0.077602	0.023964	0.031532	0.124536	0.074963	0.021542
$\hat{\hat{Y}}_{P15}$	0.077993	0.024053	0.031532	0.126321	0.09016	0.02155
$\hat{\hat{Y}}_{P16}$	0.078315	0.024131	0.031581	0.127789	0.10241	0.021556
$\hat{\hat{Y}}_{P17}$	0.078586	0.024131	0.031581	0.129519	0.112294	0.021563

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$\hat{\hat{Y}}_{P18}$	0.078815	0.024262	0.031624	0.13056	0.123412	0.021573
$\hat{\hat{Y}}_{P19}$	0.079185	0.024274	0.031661	0.131397	0.133424	0.021583
$\hat{\hat{Y}}_{P20}$	0.079695	0.079695	0.031747	0.132049	0.149993	0.021626

It was discovered that the tenth estimator has the least bias in each of the six distributions considered. These values are shown in bold print.

Table 9: Biases of the Different Estimators for Each Distribution at n = 20

Estimators	**Biases**					
	Poisson	**Hyper Geometric**	**Binomial**	**Uniform**	**Exponential**	**Normal**
$\hat{\hat{Y}}_{P1}$	0.032869	0.006828	0.009036	0.073826	0.088342	0.005078
$\hat{\hat{Y}}_{P2}$	0.030803	0.00675	0.008408	0.067298	0.076367	0.004602
$\hat{\hat{Y}}_{P3}$	0.028927	0.00603	0.007843	0.060364	0.064958	0.004371
$\hat{\hat{Y}}_{P4}$	0.027217	0.00603	0.007333	0.055342	0.050083	0.004238
$\hat{\hat{Y}}_{P5}$	0.025654	0.005713	0.007333	0.052156	0.040486	0.004115
$\hat{\hat{Y}}_{P6}$	0.024221	0.00542	0.006871	0.049094	0.032579	0.004029
$\hat{\hat{Y}}_{P7}$	0.022906	0.00542	0.006871	0.044835	0.026151	0.003912
$\hat{\hat{Y}}_{P8}$	0.021695	0.004898	0.006452	0.041859	0.018998	0.003762
$\hat{\hat{Y}}_{P9}$	0.019544	0.00485	0.00607	0.039189	0.012827	0.003591
$\boldsymbol{\hat{\hat{Y}}_{P10}}$	**0.016101**	**0.004058**	**0.005107**	**0.03692**	**0.004112**	**0.002852**
$\hat{\hat{Y}}_{P11}$	0.047356	0.014839	0.019596	0.063653	0.00841	0.013439
$\hat{\hat{Y}}_{P12}$	0.047863	0.014855	0.019649	0.071155	0.019859	0.013462
$\hat{\hat{Y}}_{P13}$	0.048249	0.014993	0.019692	0.075703	0.031661	0.013472
$\hat{\hat{Y}}_{P14}$	0.048553	0.014993	0.019729	0.077918	0.046902	0.013478
$\hat{\hat{Y}}_{P15}$	0.048798	0.015049	0.019729	0.079035	0.05641	0.013483
$\hat{\hat{Y}}_{P16}$	0.048999	0.015098	0.019759	0.079953	0.064075	0.013487
$\hat{\hat{Y}}_{P17}$	0.049169	0.015098	0.019759	0.081036	0.070259	0.013491
$\hat{\hat{Y}}_{P18}$	0.049312	0.01518	0.019786	0.081687	0.077215	0.013497
$\hat{\hat{Y}}_{P19}$	0.049544	0.015187	0.019809	0.082211	0.083479	0.013504
$\hat{\hat{Y}}_{P20}$	0.049863	0.015301	0.019863	0.082619	0.093846	0.013531

It was discovered that the tenth estimator has the least bias in each of the six distributions considered.

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Table 10: Biases of the Different Estimators for Each Distribution at n = 30

Estimators	Biases					
	Poisson	**Hyper Geometric**	**Binomial**	**Uniform**	**Exponential**	**Normal**
$\hat{\bar{Y}}_{P1}$	0.020695	0.004299	0.005689	0.046483	0.055623	0.003197
$\hat{\bar{Y}}_{P2}$	0.019395	0.00425	0.005294	0.042373	0.048083	0.002897
$\hat{\bar{Y}}_{P3}$	0.018213	0.003797	0.004938	0.038007	0.0409	0.002752
$\hat{\bar{Y}}_{P4}$	0.017136	0.003797	0.004617	0.034845	0.031534	0.002669
$\hat{\bar{Y}}_{P5}$	0.016152	0.003597	0.004617	0.032839	0.025491	0.002591
$\hat{\bar{Y}}_{P6}$	0.015251	0.003413	0.004326	0.030911	0.020513	0.002536
$\hat{\bar{Y}}_{P7}$	0.014422	0.003413	0.004326	0.028229	0.016465	0.002463
$\hat{\bar{Y}}_{P8}$	0.01366	0.003084	0.004062	0.026356	0.011962	0.002368
$\hat{\bar{Y}}_{P9}$	0.012306	0.003054	0.003822	0.024674	0.008076	0.002261
$\boldsymbol{\hat{\bar{Y}}_{P10}}$	**0.010138**	**0.002555**	**0.003216**	**0.023246**	**0.002589**	**0.001796**
$\hat{\bar{Y}}_{P11}$	0.029817	0.009343	0.012338	0.040078	0.005295	0.008462
$\hat{\bar{Y}}_{P12}$	0.030136	0.009353	0.012371	0.044801	0.012504	0.008476
$\hat{\bar{Y}}_{P13}$	0.030379	0.00944	0.012399	0.047665	0.019935	0.008482
$\hat{\bar{Y}}_{P14}$	0.03057	0.00944	0.012422	0.04906	0.029531	0.008486
$\hat{\bar{Y}}_{P15}$	0.030725	0.009475	0.012422	0.049763	0.035518	0.008489
$\hat{\bar{Y}}_{P16}$	0.030852	0.009506	0.012441	0.050341	0.040343	0.008492
$\hat{\bar{Y}}_{P17}$	0.030958	0.009506	0.012441	0.051023	0.044237	0.008495
$\hat{\bar{Y}}_{P18}$	0.031048	0.009558	0.012458	0.051433	0.048617	0.008498
$\hat{\bar{Y}}_{P19}$	0.031194	0.009562	0.012472	0.051763	0.052561	0.008502
$\hat{\bar{Y}}_{P20}$	0.031395	0.009634	0.012506	0.052019	0.059088	0.008519

The least value of the bias in each of the distributions for n = 30 is shown in bold print in the Table 10.

Table 11: Biases of the Different Estimators for Each Distribution at n = 49

Estimators	Biases					
	Poisson	**Hyper Geometric**	**Binomial**	**Uniform**	**Exponential**	**Normal**
$\hat{\bar{Y}}_{P1}$	0.011254	0.002338	0.003094	0.025278	0.030249	0.001739
$\hat{\bar{Y}}_{P2}$	0.010547	0.002311	0.002879	0.023043	0.026148	0.001576
$\hat{\bar{Y}}_{P3}$	0.009905	0.002065	0.002685	0.020669	0.022242	0.001497
$\hat{\bar{Y}}_{P4}$	0.009319	0.002065	0.002511	0.018949	0.017149	0.001451
$\hat{\bar{Y}}_{P5}$	0.008784	0.001956	0.002511	0.017858	0.013863	0.001409

$\hat{\hat{Y}}_{P6}$	0.008294	0.001856	0.002353	0.01681	0.011155	0.001379
$\hat{\hat{Y}}_{P7}$	0.007843	0.001856	0.002353	0.015352	0.008954	0.001339
$\hat{\hat{Y}}_{P8}$	0.007428	0.001677	0.002209	0.014333	0.006505	0.001288
$\hat{\hat{Y}}_{P9}$	0.006692	0.001661	0.002078	0.013418	0.004392	0.001229
$\boldsymbol{\hat{\hat{Y}}_{P10}}$	**0.005513**	**0.001389**	**0.001749**	**0.012641**	**0.001408**	**0.000977**
$\hat{\hat{Y}}_{P11}$	0.016215	0.005081	0.00671	0.021795	0.00288	0.004602
$\hat{\hat{Y}}_{P12}$	0.016389	0.005087	0.006728	0.024364	0.0068	0.004609
$\hat{\hat{Y}}_{P13}$	0.016521	0.005134	0.006743	0.025921	0.010841	0.004613
$\hat{\hat{Y}}_{P14}$	0.016625	0.005134	0.006755	0.026679	0.016059	0.004615
$\hat{\hat{Y}}_{P15}$	0.016709	0.005153	0.006755	0.027062	0.019315	0.004617
$\hat{\hat{Y}}_{P16}$	0.016778	0.00517	0.006766	0.027376	0.021939	0.004618
$\hat{\hat{Y}}_{P17}$	0.016835	0.00517	0.006766	0.027747	0.024057	0.004619
$\hat{\hat{Y}}_{P18}$	0.016885	0.005198	0.006775	0.02797	0.026439	0.004622
$\hat{\hat{Y}}_{P19}$	0.016964	0.0052	0.006783	0.028149	0.028584	0.004624
$\hat{\hat{Y}}_{P20}$	0.017073	0.005239	0.006801	0.028289	0.032133	0.004633

The least value of the bias in each of the distributions for n = 45 is bolded in the table above.

Table 12: Biases of the Different Estimators for Each Distribution at n = 100

Estimators	Biases					
	Poisson	**Hyper Geometric**	**Binomial**	**Uniform**	**Exponential**	**Normal**
$\hat{\hat{Y}}_{P1}$	0.003652	0.000759	0.001004	0.008203	0.009816	0.000564
$\hat{\hat{Y}}_{P2}$	0.003423	0.00075	0.000934	0.007478	0.008485	0.000511
$\hat{\hat{Y}}_{P3}$	0.003214	0.00067	0.000871	0.006707	0.007218	0.000486
$\hat{\hat{Y}}_{P4}$	0.003024	0.00067	0.000815	0.006149	0.005565	0.000471
$\hat{\hat{Y}}_{P5}$	0.00285	0.000635	0.000815	0.005795	0.004498	0.000457
$\hat{\hat{Y}}_{P6}$	0.002691	0.000602	0.000763	0.005455	0.00362	0.000448
$\hat{\hat{Y}}_{P7}$	0.002545	0.000602	0.000763	0.004982	0.002906	0.000435
$\hat{\hat{Y}}_{P8}$	0.002411	0.000544	0.000717	0.004651	0.002111	0.000418
$\hat{\hat{Y}}_{P9}$	0.002172	0.000539	0.000674	0.004354	0.001425	0.000399
$\boldsymbol{\hat{\hat{Y}}_{P10}}$	**0.001789**	**0.000451**	**0.000567**	**0.004102**	**0.000457**	**0.000317**
$\hat{\hat{Y}}_{P11}$	0.005262	0.001649	0.002177	0.007073	0.000934	0.001493
$\hat{\hat{Y}}_{P12}$	0.005318	0.001651	0.002183	0.007906	0.002207	0.001496
$\hat{\hat{Y}}_{P13}$	0.005361	0.001666	0.002188	0.008411	0.003518	0.001497
$\hat{\hat{Y}}_{P14}$	0.005395	0.001666	0.002192	0.008658	0.005211	0.001498
$\hat{\hat{Y}}_{P15}$	0.005422	0.001672	0.002192	0.008782	0.006268	0.001498

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$\hat{\hat{Y}}_{P16}$	0.005444	0.001678	0.002195	0.008884	0.007119	0.001499
$\hat{\hat{Y}}_{P17}$	0.005463	0.001678	0.002195	0.009004	0.007807	0.001499
$\hat{\hat{Y}}_{P18}$	0.005479	0.001687	0.002198	0.009076	0.008579	0.0015
$\hat{\hat{Y}}_{P19}$	0.005505	0.001687	0.002201	0.009135	0.009275	0.0015
$\hat{\hat{Y}}_{P20}$	0.00554	0.0017	0.002207	0.00918	0.010427	0.001503

The least value of the bias in each of the distributions for n = 100 is bolded in the Table 12 above.

The mean squared errors of the estimators were also computed and the results presented in Tables 13 – 17.

Table 13: MSE of the Different Estimators for Each Distribution at n = 13

Estimators	Mean Squared Error (MSE)					
	Poisson	**Hyper Geometric**	**Binomial**	**Uniform**	**Exponential**	**Normal**
$\hat{\hat{Y}}_{P1}$	0.550036	0.090091	0.186757	1.003747	0.244054	0.135162
$\hat{\hat{Y}}_{P2}$	0.517239	0.089139	0.174595	0.915381	0.211287	0.123645
$\hat{\hat{Y}}_{P3}$	0.48744	0.080455	0.163658	0.821502	0.18007	0.118054
$\hat{\hat{Y}}_{P4}$	0.460283	0.080455	0.153788	0.753523	0.139367	0.114847
$\hat{\hat{Y}}_{P5}$	0.435465	0.076625	0.153788	0.710385	0.113106	0.111865
$\hat{\hat{Y}}_{P6}$	0.412725	0.073089	0.144849	0.668936	0.091471	0.109764
$\hat{\hat{Y}}_{P7}$	0.391837	0.073089	0.144849	0.611281	0.073882	0.106934
$\hat{\hat{Y}}_{P8}$	0.372606	0.066786	0.136729	0.570993	0.05431	0.103299
$\hat{\hat{Y}}_{P9}$	0.338452	0.066207	0.12933	0.534845	0.037424	0.099165
$\boldsymbol{\hat{\hat{Y}}_{P10}}$	**0.283787**	**0.056634**	**0.110688**	**0.504127**	**0.013579**	**0.081291**
$\hat{\hat{Y}}_{P11}$	0.780073	0.186822	0.391233	0.866032	0.025338	0.337609
$\hat{\hat{Y}}_{P12}$	0.788133	0.187018	0.39226	0.967595	0.056666	0.338155
$\hat{\hat{Y}}_{P13}$	0.794264	0.188682	0.393103	1.029165	0.08896	0.338403
$\hat{\hat{Y}}_{P14}$	0.799083	0.188682	0.393808	1.059146	0.130663	0.338541
$\hat{\hat{Y}}_{P15}$	0.802972	0.189354	0.393808	1.074265	0.15668	0.338666
$\hat{\hat{Y}}_{P16}$	0.806175	0.189945	0.394406	1.086698	0.177652	0.338753
$\hat{\hat{Y}}_{P17}$	0.80886	0.189945	0.394406	1.101358	0.194573	0.338868
$\hat{\hat{Y}}_{P18}$	0.811142	0.190936	0.39492	1.110168	0.213607	0.339012
$\hat{\hat{Y}}_{P19}$	0.814815	0.191024	0.395366	1.117262	0.230747	0.339172
$\hat{\hat{Y}}_{P20}$	0.819883	0.192393	0.39641	1.12278	0.259114	0.33982

The least value of the MSE in each of the distributions is shown in bold print.

Table 14: MSE of the Different Estimators for Each Distribution at n = 20

Estimators	Mean Squared Error (MSE)					
	Poisson	**Hyper Geometric**	**Binomial**	**Uniform**	**Exponential**	**Normal**
$\hat{\hat{Y}}_{P1}$	0.34414	0.056367	0.116848	0.628013	0.152697	0.084567
$\hat{\hat{Y}}_{P2}$	0.32362	0.055771	0.109238	0.572725	0.132196	0.07736
$\hat{\hat{Y}}_{P3}$	0.304976	0.050338	0.102396	0.513988	0.112664	0.073863
$\hat{\hat{Y}}_{P4}$	0.287984	0.050338	0.09622	0.471456	0.087198	0.071856
$\hat{\hat{Y}}_{P5}$	0.272457	0.047942	0.09622	0.444466	0.070767	0.06999
$\hat{\hat{Y}}_{P6}$	0.258229	0.045729	0.090628	0.418532	0.05723	0.068676
$\hat{\hat{Y}}_{P7}$	0.24516	0.045729	0.090628	0.382459	0.046226	0.066905
$\hat{\hat{Y}}_{P8}$	0.233128	0.041786	0.085547	0.357252	0.03398	0.064631
$\hat{\hat{Y}}_{P9}$	0.211759	0.041423	0.080918	0.334635	0.023415	0.062044
$\boldsymbol{\hat{\hat{Y}}_{P10}}$	**0.177556**	**0.035434**	**0.069254**	**0.315416**	**0.008496**	**0.050861**
$\hat{\hat{Y}}_{P11}$	0.488067	0.116889	0.244782	0.541849	0.015853	0.211231
$\hat{\hat{Y}}_{P12}$	0.49311	0.117011	0.245425	0.605393	0.035454	0.211573
$\hat{\hat{Y}}_{P13}$	0.496946	0.118052	0.245952	0.643916	0.055659	0.211728
$\hat{\hat{Y}}_{P14}$	0.499961	0.118052	0.246393	0.662674	0.081752	0.211814
$\hat{\hat{Y}}_{P15}$	0.502394	0.118473	0.246393	0.672133	0.09803	0.211893
$\hat{\hat{Y}}_{P16}$	0.504398	0.118842	0.246767	0.679913	0.111151	0.211947
$\hat{\hat{Y}}_{P17}$	0.506078	0.118842	0.246767	0.689085	0.121738	0.212019
$\hat{\hat{Y}}_{P18}$	0.507506	0.119463	0.247089	0.694597	0.133647	0.212109
$\hat{\hat{Y}}_{P19}$	0.509804	0.119518	0.247368	0.699036	0.144371	0.212209
$\hat{\hat{Y}}_{P20}$	0.512975	0.120374	0.248021	0.702488	0.162119	0.212615

The least value of the MSE in each of the distributions is bolded in the table above.

Table 15: MSE of the Different Estimators for Each Distribution at n = 30

Estimators	Mean Squared Error (MSE)					
	Poisson	**Hyper Geometric**	**Binomial**	**Uniform**	**Exponential**	**Normal**
$\hat{\hat{Y}}_{P1}$	0.216681	0.035491	0.073571	0.395415	0.096142	0.053246
$\hat{\hat{Y}}_{P2}$	0.203761	0.035115	0.06878	0.360605	0.083234	0.048708
$\hat{\hat{Y}}_{P3}$	0.192022	0.031694	0.064471	0.323622	0.070937	0.046506
$\hat{\hat{Y}}_{P4}$	0.181323	0.031694	0.060583	0.296842	0.054902	0.045243
$\hat{\hat{Y}}_{P5}$	0.171547	0.030186	0.060583	0.279849	0.044557	0.044068
$\hat{\hat{Y}}_{P6}$	0.162589	0.028793	0.057062	0.26352	0.036034	0.04324
$\hat{\hat{Y}}_{P7}$	0.15436	0.028793	0.057062	0.240808	0.029105	0.042126
$\hat{\hat{Y}}_{P8}$	0.146784	0.02631	0.053863	0.224936	0.021395	0.040694

$\hat{\bar{Y}}_{P9}$	0.13333	0.026081	0.050948	0.210696	0.014743	0.039065
$\boldsymbol{\hat{\bar{Y}}_{P10}}$	**0.111795**	**0.02231**	**0.043604**	**0.198596**	**0.005349**	**0.032024**
$\hat{\bar{Y}}_{P11}$	0.307301	0.073597	0.154122	0.341164	0.009982	0.132997
$\hat{\bar{Y}}_{P12}$	0.310477	0.073674	0.154527	0.381174	0.022323	0.133213
$\hat{\bar{Y}}_{P13}$	0.312892	0.074329	0.154859	0.405429	0.035045	0.13331
$\hat{\bar{Y}}_{P14}$	0.31479	0.074329	0.155137	0.417239	0.051473	0.133365
$\hat{\bar{Y}}_{P15}$	0.316322	0.074594	0.155137	0.423195	0.061722	0.133414
$\hat{\bar{Y}}_{P16}$	0.317584	0.074827	0.155372	0.428093	0.069984	0.133448
$\hat{\bar{Y}}_{P17}$	0.318642	0.074827	0.155372	0.433868	0.07665	0.133493
$\hat{\bar{Y}}_{P18}$	0.319541	0.075217	0.155574	0.437339	0.084148	0.13355
$\hat{\bar{Y}}_{P19}$	0.320988	0.075252	0.15575	0.440134	0.0909	0.133613
$\hat{\bar{Y}}_{P20}$	0.322984	0.075791	0.156161	0.442307	0.102075	0.133869

The least value of the MSE in each of the distributions is shown in bold print.

Table 16: MSE of the Different Estimators for Each Distribution at n = 49

Estimators	Mean Squared Error (MSE)					
	Poisson	**Hyper Geometric**	**Binomial**	**Uniform**	**Exponential**	**Normal**
$\hat{\bar{Y}}_{P1}$	0.117835	0.0193	0.040009	0.215034	0.052284	0.028956
$\hat{\bar{Y}}_{P2}$	0.110809	0.019096	0.037404	0.196103	0.045264	0.026489
$\hat{\bar{Y}}_{P3}$	0.104425	0.017236	0.035061	0.175991	0.038577	0.025291
$\hat{\bar{Y}}_{P4}$	0.098607	0.017236	0.032946	0.161428	0.029857	0.024604
$\hat{\bar{Y}}_{P5}$	0.09329	0.016415	0.032946	0.152187	0.024231	0.023965
$\hat{\bar{Y}}_{P6}$	0.088419	0.015658	0.031031	0.143307	0.019596	0.023515
$\hat{\bar{Y}}_{P7}$	0.083944	0.015658	0.031031	0.130956	0.015828	0.022909
$\hat{\bar{Y}}_{P8}$	0.079824	0.014308	0.029292	0.122324	0.011635	0.02213
$\hat{\bar{Y}}_{P9}$	0.072507	0.014184	0.027707	0.11458	0.008017	0.021244
$\boldsymbol{\hat{\bar{Y}}_{P10}}$	**0.060796**	**0.012133**	**0.023713**	**0.108**	**0.002909**	**0.017415**
$\hat{\bar{Y}}_{P11}$	0.167116	0.040023	0.083814	0.185531	0.005428	0.072326
$\hat{\bar{Y}}_{P12}$	0.168843	0.040065	0.084034	0.207289	0.01214	0.072443
$\hat{\bar{Y}}_{P13}$	0.170156	0.040422	0.084215	0.220479	0.019058	0.072497
$\hat{\bar{Y}}_{P14}$	0.171188	0.040422	0.084366	0.226902	0.027992	0.072526
$\hat{\bar{Y}}_{P15}$	0.172022	0.040565	0.084366	0.230141	0.033566	0.072553
$\hat{\bar{Y}}_{P16}$	0.172708	0.040692	0.084494	0.232805	0.038059	0.072571
$\hat{\bar{Y}}_{P17}$	0.173283	0.040692	0.084494	0.235945	0.041684	0.072596
$\hat{\bar{Y}}_{P18}$	0.173772	0.040905	0.084604	0.237832	0.045761	0.072627
$\hat{\bar{Y}}_{P19}$	0.174559	0.040923	0.0847	0.239352	0.049433	0.072661
$\hat{\bar{Y}}_{P20}$	0.175645	0.041217	0.084923	0.240534	0.05551	0.0728

Table 17: MSE of the Different Estimators for Each Distribution at n = 100

Estimators	Mean Squared Error (MSE)					
	Poisson	Hyper Geometric	Binomial	Uniform	Exponential	Normal
$\hat{\bar{Y}}_{P1}$	0.038238	0.006263	0.012983	0.069779	0.016966	0.009396
$\hat{\bar{Y}}_{P2}$	0.035958	0.006197	0.012138	0.063636	0.014688	0.008596
$\hat{\bar{Y}}_{P3}$	0.033886	0.005593	0.011377	0.05711	0.012518	0.008207
$\hat{\bar{Y}}_{P4}$	0.031998	0.005593	0.010691	0.052384	0.009689	0.007984
$\hat{\bar{Y}}_{P5}$	0.030273	0.005327	0.010691	0.049385	0.007863	0.007777
$\hat{\bar{Y}}_{P6}$	0.028692	0.005081	0.01007	0.046504	0.006359	0.007631
$\hat{\bar{Y}}_{P7}$	0.02724	0.005081	0.01007	0.042495	0.005136	0.007434
$\hat{\bar{Y}}_{P8}$	0.025903	0.004643	0.009505	0.039695	0.003776	0.007181
$\hat{\bar{Y}}_{P9}$	0.023529	0.004603	0.008991	0.037182	0.002602	0.006894
$\boldsymbol{\hat{\bar{Y}}_{P10}}$	**0.019728**	**0.003937**	**0.007695**	**0.035046**	**0.000944**	**0.005651**
$\hat{\bar{Y}}_{P11}$	0.05423	0.012988	0.027198	0.060205	0.001761	0.02347
$\hat{\bar{Y}}_{P12}$	0.05479	0.013001	0.027269	0.067266	0.003939	0.023508
$\hat{\bar{Y}}_{P13}$	0.055216	0.013117	0.027328	0.071546	0.006184	0.023525
$\hat{\bar{Y}}_{P14}$	0.055551	0.013117	0.027377	0.07363	0.009084	0.023535
$\hat{\bar{Y}}_{P15}$	0.055822	0.013164	0.027377	0.074681	0.010892	0.023544
$\hat{\bar{Y}}_{P16}$	0.056044	0.013205	0.027419	0.075546	0.01235	0.02355
$\hat{\bar{Y}}_{P17}$	0.056231	0.013205	0.027419	0.076565	0.013526	0.023558
$\hat{\bar{Y}}_{P18}$	0.05639	0.013274	0.027454	0.077177	0.01485	0.023568
$\hat{\bar{Y}}_{P19}$	0.056645	0.01328	0.027485	0.077671	0.016041	0.023579
$\hat{\bar{Y}}_{P20}$	0.056997	0.013375	0.027558	0.078054	0.018013	0.023624

It was discovered that the tenth estimator has the least bias and MSE in each of the six distributions and five samples considered. These values are shown in bold print.

Having summarized the results of the analysis in Table 18 – Table 22 above, the estimators were ranked based on their biases and mean squared errors. The tables below show the ranks:

Table 18: Ranks of the Estimators Based on Bias

Estimators	Ranks														
	Poisson					Hyper-geometric					Binomial				
	n_1	n_2	n_3	n_4	n_5	n_1	n_2	n_3	n_4	n_5	n_1	n_2	n_3	n_4	n_5
$\hat{\bar{Y}}_{P1}$	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
$\hat{\bar{Y}}_{P2}$	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
$\hat{\bar{Y}}_{P3}$	8	8	8	8	8	7.5	7.5	7.5	7.5	7.5	8	8	8	8	8
$\hat{\bar{Y}}_{P4}$	7	7	7	7	7	7.5	7.5	7.5	7.5	7.5	6.5	6.5	6.5	6.5	6.5
$\hat{\bar{Y}}_{P5}$	6	6	6	6	6	6	6	6	6	6	6.5	6.5	6.5	6.5	6.5

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$\hat{\hat{Y}}_{P6}$	5	5	5	5	5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5
$\hat{\hat{Y}}_{P7}$	4	4	4	4	4	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5
$\hat{\hat{Y}}_{P8}$	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
$\hat{\hat{Y}}_{P9}$	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
$\hat{\hat{Y}}_{P10}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\hat{\hat{Y}}_{P11}$	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
$\hat{\hat{Y}}_{P12}$	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
$\hat{\hat{Y}}_{P13}$	13	13	13	13	13	13.5	13.5	13.5	13.5	13.5	13	13	13	13	13
$\hat{\hat{Y}}_{P14}$	14	14	14	14	14	13.5	13.5	13.5	13.5	13.5	14.5	14.5	14.5	14.5	14.5
$\hat{\hat{Y}}_{P15}$	15	15	15	15	15	15	15	15	15	15	14.5	14.5	14.5	14.5	14.5
$\hat{\hat{Y}}_{P16}$	16	16	16	16	16	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5
$\hat{\hat{Y}}_{P17}$	17	17	17	17	17	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5
$\hat{\hat{Y}}_{P18}$	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18
$\hat{\hat{Y}}_{P19}$	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19
$\hat{\hat{Y}}_{P20}$	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20

The above table shows the ranks of the estimators based on bias in the three discrete distributions and for each of the five sample sizes under consideration.

Table 19: Ranks of the Estimators Based on Bias

Estimators	Ranks														
	Uniform					**Exponential**					**Normal**				
	n_1	**n_2**	**n_3**	**n_4**	**n_5**	**n_1**	**n_2**	**n_3**	**n_4**	**n_5**	**n_1**	**n_2**	**n_3**	**n_4**	**n_5**
$\hat{\hat{Y}}_{P1}$	12	12	12	12	12	19	19	19	19	19	10	10	10	10	10
$\hat{\hat{Y}}_{P2}$	10	10	10	10	10	16	16	16	16	16	9	9	9	9	9
$\hat{\hat{Y}}_{P3}$	8	8	8	8	8	14	14	14	14	14	8	8	8	8	8
$\hat{\hat{Y}}_{P4}$	10	7	7	7	7	11	11	11	11	11	7	7	7	7	7
$\hat{\hat{Y}}_{P5}$	6	6	6	6	6	9	9	9	9	9	6	6	6	6	6
$\hat{\hat{Y}}_{P6}$	5	5	5	5	5	8	8	8	8	8	5	5	5	5	5
$\hat{\hat{Y}}_{P7}$	4	4	4	4	4	6	6	6	6	6	4	4	4	4	4
$\hat{\hat{Y}}_{P8}$	3	3	3	3	3	4	4	4	4	4	3	3	3	3	3
$\hat{\hat{Y}}_{P9}$	2	2	2	2	2	3	3	3	3	3	2	2	2	2	2
$\hat{\hat{Y}}_{P10}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\hat{\hat{Y}}_{P11}$	9	9	9	9	9	2	2	2	2	2	11	11	11	11	11
$\hat{\hat{Y}}_{P12}$	11	11	11	11	11	5	5	5	5	5	12	12	12	12	12
$\hat{\hat{Y}}_{P13}$	13	13	13	13	13	7	7	7	7	7	13	13	13	13	13
$\hat{\hat{Y}}_{P14}$	14	14	14	14	14	10	10	10	10	10	14	14	14	14	14

$\hat{\hat{Y}}_{P15}$	15	15	15	15	15	12	12	12	12	12	15	15	15	15	15
$\hat{\hat{Y}}_{P16}$	16	16	16	16	16	13	13	13	13	13	16	16	16	16	16
$\hat{\hat{Y}}_{P17}$	17	17	17	17	17	15	15	15	15	15	17	17	17	17	17
$\hat{\hat{Y}}_{P18}$	18	18	18	18	18	17	17	17	17	17	18	18	18	18	18
$\hat{\hat{Y}}_{P19}$	19	19	19	19	19	18	18	18	18	18	19	19	19	19	19
$\hat{\hat{Y}}_{P20}$	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20

The above table shows the ranks of the estimators based on bias in the three continuous distributions and for each of the five sample sizes under consideration.

Ranking the estimators based on mean squared error gave the following result:

Table 20: Ranks of the Estimators Based on MSE

Estimators	Ranks														
	Poisson					Hyper-geometric					Binomial				
	n_1	n_2	n_3	n_4	n_5	n_1	n_2	n_3	n_4	n_5	n_1	n_2	n_3	n_4	n_5
$\hat{\hat{Y}}_{P1}$	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
$\hat{\hat{Y}}_{P2}$	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
$\hat{\hat{Y}}_{P3}$	8	8	8	8	8	7.5	7.5	7.5	7.5	7.5	8	8	8	8	8
$\hat{\hat{Y}}_{P4}$	7	7	7	7	7	7.5	7.5	7.5	7.5	7.5	6.5	6.5	6.5	6.5	6.5
$\hat{\hat{Y}}_{P5}$	6	6	6	6	6	6	6	6	6	6	6.5	6.5	6.5	6.5	6.5
$\hat{\hat{Y}}_{P6}$	5	5	5	5	5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5
$\hat{\hat{Y}}_{P7}$	4	4	4	4	4	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5
$\hat{\hat{Y}}_{P8}$	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
$\hat{\hat{Y}}_{P9}$	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
$\hat{\hat{Y}}_{P10}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\hat{\hat{Y}}_{P11}$	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
$\hat{\hat{Y}}_{P12}$	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
$\hat{\hat{Y}}_{P13}$	13	13	13	13	13	13.5	13.5	13.5	13.5	13.5	13	13	13	13	13
$\hat{\hat{Y}}_{P14}$	14	14	14	14	14	13.5	13.5	13.5	13.5	13.5	14.5	14.5	14.5	14.5	14.5
$\hat{\hat{Y}}_{P15}$	15	15	15	15	15	15	15	15	15	15	14.5	14.5	14.5	14.5	14.5
$\hat{\hat{Y}}_{P16}$	16	16	16	16	16	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5
$\hat{\hat{Y}}_{P17}$	17	17	17	17	17	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5	16.5
$\hat{\hat{Y}}_{P18}$	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18
$\hat{\hat{Y}}_{P19}$	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19
$\hat{\hat{Y}}_{P20}$	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20

Table 20 shows the ranks of the estimators based on their MSEs in the three discrete distributions and for each of the five sample sizes under consideration.

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Table 21: Ranks of the Estimators Based on MSE

Estimators	Ranks														
	Uniform					Exponential					Normal				
	n_1	n_2	n_3	n_4	n_5	n_1	n_2	n_3	n_4	n_5	n_1	n_2	n_3	n_4	n_5
$\hat{\bar{Y}}_{P1}$	12	12	12	12	12	19	19	19	19	19	10	10	10	10	10
$\hat{\bar{Y}}_{P2}$	10	10	10	10	10	16	16	16	16	16	9	9	9	9	9
$\hat{\bar{Y}}_{P3}$	8	8	8	8	8	14	14	14	14	14	8	8	8	8	8
$\hat{\bar{Y}}_{P4}$	8	7	7	7	7	11	11	11	11	11	7	7	7	7	7
$\hat{\bar{Y}}_{P5}$	6	6	6	6	6	9	9	9	9	9	6	6	6	6	6
$\hat{\bar{Y}}_{P6}$	5	5	5	5	5	8	8	8	8	8	5	5	5	5	5
$\hat{\bar{Y}}_{P7}$	4	4	4	4	4	6	6	6	6	6	4	4	4	4	4
$\hat{\bar{Y}}_{P8}$	3	3	3	3	3	4	4	4	4	4	3	3	3	3	3
$\hat{\bar{Y}}_{P9}$	2	2	2	2	2	3	3	3	3	3	2	2	2	2	2
$\hat{\bar{Y}}_{P10}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\hat{\bar{Y}}_{P11}$	9	9	9	9	9	2	2	2	2	2	11	11	11	11	11
$\hat{\bar{Y}}_{P12}$	11	11	11	11	11	5	5	5	5	5	12	12	12	12	12
$\hat{\bar{Y}}_{P13}$	13	13	13	13	13	7	7	7	7	7	13	13	13	13	13
$\hat{\bar{Y}}_{P14}$	14	14	14	14	14	10	10	10	10	10	14	14	14	14	14
$\hat{\bar{Y}}_{P15}$	15	15	15	15	15	12	12	12	12	12	15	15	15	15	15
$\hat{\bar{Y}}_{P16}$	16	16	16	16	16	13	13	13	13	13	16	16	16	16	16
$\hat{\bar{Y}}_{P17}$	17	17	17	17	17	15	15	15	15	15	17	17	17	17	17
$\hat{\bar{Y}}_{P18}$	18	18	18	18	18	17	17	17	17	17	18	18	18	18	18
$\hat{\bar{Y}}_{P19}$	19	19	19	19	19	18	18	18	18	18	19	19	19	19	19
$\hat{\bar{Y}}_{P20}$	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20

Table 21 shows the ranks of the estimators based on their MSEs in the three continuous distributions and for each of the five sample sizes under consideration.

It was discovered that the ranks of the estimators in a particular distribution are the same for the five samples selected, that is, the ranks are not affected by the size of the sample. The ranks are summarized in table 22 below.

Table 22: Summary of Ranks

Estimators	Ranking based on bias						Ranking based on MSE					
	Pop 1	Pop 2	Pop 3	Pop4	Pop 5	Pop 6	Pop 1	Pop 2	Pop 3	Pop 4	Pop 5	Pop 6
$\hat{\hat{Y}}_{P1}$	10	10	10	12	19	10	10	10	10	12	19	10
$\hat{\hat{Y}}_{P2}$	9	9	9	10	16	9	9	9	9	10	16	9
$\hat{\hat{Y}}_{P3}$	8	7.5	8	8	14	8	8	7.5	8	8	14	8
$\hat{\hat{Y}}_{P4}$	7	7.5	6.5	7	11	7	7	7.5	6.5	7	11	7
$\hat{\hat{Y}}_{P5}$	6	6	6.5	6	9	6	6	6	6.5	6	9	6
$\hat{\hat{Y}}_{P6}$	5	4.5	4.5	5	8	5	5	4.5	4.5	5	8	5
$\hat{\hat{Y}}_{P7}$	4	4.5	4.5	4	6	4	4	4.5	4.5	4	6	4
$\hat{\hat{Y}}_{P8}$	3	3	3	3	4	3	3	3	3	3	4	3
$\hat{\hat{Y}}_{P9}$	2	2	2	2	3	2	2	2	2	2	3	2
$\hat{\hat{Y}}_{P10}$	1	1	1	1	1	1	1	1	1	1	1	1
$\hat{\hat{Y}}_{P11}$	11	11	11	9	2	11	11	11	11	9	2	11
$\hat{\hat{Y}}_{P12}$	12	12	12	11	5	12	12	12	12	11	5	12
$\hat{\hat{Y}}_{P13}$	13	13.5	13	13	7	13	13	13.5	13	13	7	13
$\hat{\hat{Y}}_{P14}$	14	13.5	14.5	14	10	14	14	13.5	14.5	14	10	14
$\hat{\hat{Y}}_{P15}$	15	15	14.5	15	12	15	15	15	14.5	15	12	15
$\hat{\hat{Y}}_{P16}$	16	16.5	16.5	16	13	16	16	16.5	16.5	16	13	16
$\hat{\hat{Y}}_{P17}$	17	16.5	16.5	17	15	17	17	16.5	16.5	17	15	17
$\hat{\hat{Y}}_{P18}$	18	18	18	18	17	18	18	18	18	18	17	18
$\hat{\hat{Y}}_{P19}$	19	19	19	19	18	19	19	19	19	19	18	19
$\hat{\hat{Y}}_{P20}$	20	20	20	20	20	20	20	20	20	20	20	20

CONCLUSION

From the analysis, results and summary, the following conclusions are drawn:

I. The best of the twenty ratio estimators proposed by Subzar etal (2017) is the tenth estimator, $\hat{\hat{Y}}_{P10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}QD+ D_{10})}(\bar{X}QD + D_{10})$ because it has the least mean squared error (MSE) as well as bias.

II. This estimator is distribution free; it performs better than others irrespective of the distribution of the variables involved.

III. The tenth estimator remains the best ratio estimator among the class of estimators under consideration irrespective of the sample size.

IV. The estimators perform better with increased sample size.

RECOMMENDATION

From the findings so far, the tenth estimator of the classes of ratio estimators proposed by Subzar et al (2017) is recommended in the estimation of population mean from a simple random sample of any size using auxiliary information.

Researchers are also called upon to put more effort in the improvement of these estimators so as to come up with ratio estimators which will be completely unbiased, efficient, sufficient and consistent; this will go a long way to improve the quality of statistical inferences made by statisticians and other researchers in sample surveys.

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