African Journal of Mathematics and Statistics Studies

ISSN: 2689-5323

Volume 8, Issue 3, 2025 (pp. 15-30)

www.abjournals.org

UNDERSTANDING LONG-TERM SHARE PRICE BEHAVIOR OF JUBILEE HOLDINGS LIMITED ON THE KENYA SECURITIES EXCHANGE: A MARKOV MODEL APPROACH COMPLEMENTED BY ERGODICITY AND STATIONARITY

Kamara S. M.

Cite this article:

Kamara, S. M. (2025), Understanding Long-Term Share Price Behavior of Jubilee Holdings Limited on the Kenya Securities Exchange: A Markov Model Approach complemented by Ergodicity and Stationarity. African Journal of Mathematics and Statistics Studies 8(3), 15-30. DOI: 10.52589/AJMSS-LPIDHOVK

Manuscript History

Received: 16 Apr 2025

Accepted: 22 May 2025

Published: 18 Jul 2025

ABSTRACT: *This paper investigates the long-term behavior of Jubilee Holdings Limited's share price by employing a Markov model to conduct a state-based analysis of its daily returns. The state space is defined by three states: Positive, Negative, and Zero. The Markov model results indicate long-term probabilities of 31.2% for a positive return, 30.0% for a negative return, and 38.8% for zero return. Additionally, the study incorporates the concepts of ergodicity and stationarity to assess the magnitude of the daily returns through drift analysis. The drift analysis further reveals that the ensemble mean of the daily returns of the share price is zero, as confirmed by a Z-test. These findings suggest that the share price demonstrates long-term stability, with no sustained directional movement. While this analysis focuses on capital gains, it highlights Jubilee Holdings Limited's consistent dividend payments in recent years as a consideration for income-focused investors.*

KEYWORDS: Markov Chain, Time Series Analysis, Ergodicity, Weak Stationarity, Transition Matrix, Jubilee Holdings Limited, Nairobi Securities Exchange.

INTRODUCTION

Equity investment plays a crucial role in driving economic growth by serving as a significant source of cash flow for businesses. A country's productivity is closely tied to the productive capacity of its businesses. Equity markets facilitate this by enabling companies to raise capital through mechanisms such as Initial Public Offerings (IPOs) and Secondary Public Offerings (SPOs). Share prices act as key indicators for assessing firm-specific performance, while a share index reflects the general economic outlook of a country, assuming market efficiency. Examples of such indices include the Nairobi Securities Exchange All-Share Index (NASI) and the S&P 500 of the New York Stock Exchange (NYSE).

For an investor, the return from buying shares is derived from two components: dividend payments and capital gains. The realization of capital gains, however, is contingent upon the behavior of share prices, which are influenced by company performance and broader market dynamics. This underscores the importance of predicting share prices behavior to inform investment decisions. While it is impossible to predict share prices with certainty, statistical and probabilistic models can reduce uncertainty by forecasting the direction of movement with lesser degrees of error. Various methodologies, including neural networks, data mining, moving averages, regression analysis, Autoregressive Integrated Moving Average (ARIMA) models, and Markov Chain analysis, have been employed to analyze stock market trends (Simeyo et al., 2015, p. 14713). Each of these models has unique strengths and limitations, and the choice of model depends on how well it fits the data.

The Kenyan stock market, particularly the Nairobi Securities Exchange (NSE), has witnessed increased investor confidence following favorable economic conditions, such as the end of post-election violence in 2008 and the peaceful general elections of 2013. Daily stock price movements in Kenya are influenced by a range of factors, including political forces, fuel prices, exchange rates, inflation, dividend announcements, and the introduction of new products (Kiplangat, 2024, p. 1529). These factors create a dynamic environment that necessitates robust modelling approaches to analyze and predict share price trends.

Jubilee Holdings Limited (JHL) provides an excellent case study for such an analysis. Founded in 1937 as the first incorporated insurance company in Mombasa, JHL has grown into the leading regional composite insurer. It is listed on three stock exchanges: the Nairobi Securities Exchange, the Dar-es-Salaam Stock Exchange, and the Uganda Securities Exchange. JHL's consistent performance, particularly in dividend payments and delivering shareholder value, has made it a favorable investment option among Kenyan investors. As the parent company of Jubilee Insurance, a major player in Kenya's insurance sector, its share price behavior offers valuable insights into potential capital gains and losses.

This paper provides a comprehensive analysis of the long-term behavior of Jubilee Holdings Limited's share price by combining a Markov model with the concepts of ergodicity and stationarity. The Markov model is used for state-based analysis of the daily returns, while the concepts of ergodicity and stationarity address its limitations. Although the Markov model effectively identifies the probabilities of positive or negative returns based on the relative frequency of positive and negative price changes, it does not account for the magnitude of those returns. For instance, a share price may increase more frequently than it decreases, but if the decreases are of greater magnitude than the increases, the overall trend could still be bearish. Therefore, relying solely on the Markov model may lead to misleading conclusions.

Article DOI: 10.52589/AJMSS-LPIDHOVK
DOI URL: https://doi.org/10.52589/AJMSS-LPIDHOVK

To overcome this limitation, the study incorporates ergodicity and stationarity to estimate the ensemble mean (drift) of JHL's share price daily returns.

In short, this study integrates a Markov model with the concepts of ergodicity and stationarity to examine the long-term share price behavior of Jubilee Holdings Limited. By addressing the limitations of traditional Markov models in equity analysis, it provides a more precise and nuanced understanding of JHL's share price trends, offering valuable insights for academic research and informed investment decisions.

Problem statement

The ultimate aim of every investor is to generate profit. Among the various investment options available, the stock market remains a popular choice. However, the daily volatility of share prices for listed companies introduces significant risks, making investment decisions increasingly challenging. Understanding the stochastic behavior of a company's share price over the long run is essential for fostering investor confidence and enabling sound decision-making.

This paper seeks to address these challenges by providing a detailed analysis of the long-term prospects of Jubilee Holdings Limited's share price. By offering insights into its expected behavior, this study aims to empower investors to make informed decisions aligned with their risk tolerance, ultimately shielding them from the pitfalls of unsound investments.

LITERATURE REVIEW

Due to the inherent randomness in share price data, various statistical methods, including Markov chains, have been employed to analyze the future dynamics of share prices for investment purposes. The overarching goal is to enhance the quality of decision-making in the stock market.

Simeyo et al. (2014) used a Markov model to predict the trend of Safaricom's share price on the Nairobi Securities Exchange. Their study utilized three states: "Decrease," "Increase," and "Unchanged," forming an irreducible transition matrix that converges to a stationary distribution after 104 trading days. The Markov model generated from their study indicated that Safaricom's share price would depreciate, remain unchanged, or appreciate in value with probabilities of 0.3, 0.1, and 0.5, respectively. Their findings demonstrated that the Markov model is effective in predicting the daily closing price of stocks while emphasizing that combining the Markov chain with other methods enhances its predictive power.

Kiplangat (2024) applied the Markov model to analyze the returns of stock prices for four companies listed on the Nairobi Securities Exchange: Centum, Equity Bank, EABL, and Kenya Airways. The study concluded that the returns of these companies satisfy the first-order Markov property, as verified by statistical tests. Additionally, the transition matrices for all four companies converged to stationary distributions. The mean returns were approximately zero, with distributions exhibiting excess kurtosis. The study predicted that, in the long run, irrespective of the current share price state, the shares would transition among five distinct ranges with specific probabilities. For instance, the probabilities for Centum were approximately 0.0044, 0.3788, 0.2130, 0.3950, and 0.0088 for transitions to

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minimum and lower average ranges, lower average and zero ranges, remaining unchanged, zero and upper average ranges, and upper average to maximum ranges, respectively.

Osu et al. (2024) analyzed the share price movements of Fidelity Bank and Access Bank on the Nigerian Stock Exchange using a Markov chain model. The study classified share price changes into three states: “Increase,” “Decrease,” and “Steady.” Separate and merged analyses were conducted to examine the prospects of each bank individually and their combined prospects. The study, valid under the assumption that share prices exhibit the Markov property, found that the transition matrices for both banks converged to stationary distributions. The findings provided insights into the long-term dynamics of share prices for these banks.

Dar et al. (2022) applied a Markov model to analyze the growth prospects of Tata Consultancy Services Limited (TCS Ltd.) on the Indian Stock Market. Using secondary data for daily closing prices, the study defined five states: Higher Gain (Hh), Lower Gain (Lg), No Change (Nc), Higher Loss (Hl), and Lower Loss (Ll). The process converged to a stationary distribution with probabilities of Hh = 0.4207, Hl = 0.1106, Nc = 0.0291, Ll = 0.0892, and Hl = 0.3504. The stationary distribution was further used to compute the expected return time to each state. The study highlighted the Markov chain model as an ideal tool for understanding the long-term dynamics of TCS Ltd.’s share prices.

Ersen et al. (2023) used Markov chains to predict the behavior of stock prices for companies in the paper and paper products industry listed on the Borsa Istanbul (BIST). The study considered three states: “Decrease,” “Constant,” and “Increase.” Empirical analysis revealed that six stocks were likely to decrease in the long term, while the stock of TEZOL was predicted to increase. Among the companies, VIKING exhibited the highest growth potential, whereas KARTIN had the lowest growth prospects.

Mettle and Laryea (2014) utilized a Markov chain to model equity price changes on the Ghana Stock Exchange using weekly closing prices. Their study assumed Markov dependency with a state space of increase, stable, and decrease. The constructed chain was found to be aperiodic and ergodic, thus approaching a limiting distribution. They developed a methodology to determine the expected mean return time for stock price increases and established criteria for improving investment decisions based on the highest transition probabilities, the lowest mean return times, and the highest limiting distributions. Additionally, they developed an R algorithm to implement the methodology introduced in their study.

The reviewed literature highlights the widespread use of Markov chains in modeling the long-term dynamics of share prices for companies listed on various stock exchanges, including the Nairobi Securities Exchange. However, to date, no study has specifically analyzed the share price of Jubilee Holdings Limited using a Markov model. This paper seeks to address this gap by using a Markov model to provide a detailed empirical analysis of Jubilee Holdings Limited’s (JUB) share price on the Nairobi Securities Exchange for investment purposes. Furthermore, while most of the referenced studies rely solely on the stationary distribution, this paper enhances the Markov model by incorporating an analysis of the magnitude of returns. This approach ensures that the findings are based not only on the frequency of occurrences but also on the magnitude of those occurrences.

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METHODOLOGY

A stochastic process is a collection of random variables indexed by time, or a process that evolves randomly over time. Mathematically, it is represented as $\{X_t\}_{t\geq 0}$, where t represents time. The state space, usually denoted as $\mathbf{S}$ ($\mathbf{S} \subseteq R$), is the set of possible values or states the process can occupy at any given point in time. The time domain, denoted as $\mathbf{T}$ ($\mathbf{T} \subseteq R$), refers to the set of time points over which the process evolves. Both the state space and time domain can be discrete or continuous. Share price is an example of a stochastic process, and the difference between the daily closing prices is what this text refers to as the daily return.

Many real-world data exhibit autocorrelations, which quantify the relationship between the past and future of a stochastic process, measuring the dependency between X_{t-1} and X_t for all $t \in \mathbf{T}$. This property is fundamental for forecasting a time series (stochastic process).

Stationarity, a key characteristic of a time series, comes in two forms: strong and weak. Strong stationarity requires the joint distribution of the process to be invariant under time shifts, which is often too strict for real-world data. It is mathematically represented as:

$$F(X_{t_1+k}, X_{t_2+k}, \dots, X_{t_n+k}) = F(X_{t_1}, X_{t_2}, \dots, X_{t_n}), \quad \forall (t_1, t_2, \dots, t_n) \in \mathbf{T} \text{ and } \quad \forall k \in \mathbf{T}.$$

Weak stationarity assumes that a process has a constant mean and variance over time, and its autocovariance depends only on the time lag, not specific time points. It is denoted as:

$$E(X_t) = \mu, \quad \mathrm{Var}(X_t) = \sigma^2, \quad \mathrm{Cov}(X_t, X_{t+k}) = f(k), \quad \forall t, k \in \mathbf{T}.$$

When stationarity is referred to in this text, it specifically means weak stationarity.

The ensemble mean is the expected value of the process at a given time . If we had all possible paths (the population of paths) at time t, the average of the values yielded by each path would be the ensemble mean. It is denoted as:

$$A_t = \frac{1}{N}\left(\sum_{k=1}^{N} z_k\right)$$

where z_k is the value yielded at the end of the k-th path up to time t, and A_t is the population mean of X_t for $t \in \mathbf{T}$. A similar approach can be applied to compute the ensemble variance.

It is impossible to calculate the population parameters because we only observe a single realization of the process. This limitation leads to the finding of unbiased estimates, the time estimates which are calculated from a single realized path. However, this approach assumes ergodicity. Ergodicity is a fundamental property in time series analysis, particularly in stochastic processes. It ensures that the statistical properties of a process can be inferred from a single realization of the series over time. For a process to be ergodic, it must be stationary and the autocorrelation between variables should diminish and eventually become statistically insignificant as the lag increases ($\mathrm{Cov}(X_t, X_{t+k}) \to 0 \quad \text{as} \quad k \to \infty$). Under this property, the law of large numbers applies, as the process demonstrates weak autocorrelation and approaches independence over extended lags.

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The time average is given by

$$\bar{X}_t = \frac{1}{n}\sum_{t=1}^{n} X_t\,,\quad \text{where } t \in \mathbf{T} \text{ and } n \text{ is the number of observations in the realized path.}$$

A similar approach can be applied to compute the time variance.

A Markov chain is a specific type of stochastic process characterized by a discrete time domain and a finite state space. A process satisfies the Markov property if the future state of the process depends only on the present state and not on its past states, given the history of the process. Mathematically, this property is represented as

$$P(X_t = j \mid X_1 = i_1, X_2 = i_2, \dots, X_{t-2} = i_{t-2}, X_{t-1} = i_{t-1}) = P(X_t = j \mid X_{t-1} = i_{t-1}),\quad \text{for } t \geq 0$$

Transition probabilities are fundamental to a Markov chain, representing the likelihood of transitioning from one state to another. These probabilities are expressed as $P_{ij}^{n} \geq 0$, where $i, j \in \mathbf{S}$, and $n \in \mathbb{N}$, indicating the probability of moving from state i to state j in n-steps. For a time-homogeneous Markov chain, the one-step transition probability, which is central to this study, is calculated as $P_{ij} = \frac{N_{ij}}{N_i}$, where$N_{ij}$ represents the number of times the process transitions from state i to state j, and N_i is the total number of transitions from state i. When referring to a Markov chain in this text, it specifically denotes a homogeneous Markov chain, where transition probabilities do not depend on time.

The transition matrix, denoted as $\mathbf{P^n}$, is a matrix where each entry represents a transition probability over n-steps. Each row corresponds to a transition probability distribution, given the process is in a specific state. This ensures that each row sums to 1. The transition matrix is a square matrix with dimensions $m \times m$, where m is the cardinality of the state space. It is generally represented as:

$$\mathbf{P^n} = \left(\begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1m} \\ P_{21} & P_{22} & \cdots & P_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ P_{m1} & P_{m2} & \cdots & P_{mm} \end{pmatrix}\right)^{n}$$

where n represents the number of steps.

The initial probability distribution represents the probabilities associated with the initial state of the process. It is denoted as: $\pi = (\pi_1, \pi_2, \dots, \pi_n)$, where π_r for $r \in (1,2,\dots,n)$ is the probability of the process starting in the $r\text{-}th$ state. The initial probability distribution estimate from a sample can be calculated as the relative frequency of visits to each state (t_j) during the studied period. It is denoted as:

$$\hat{\pi}_j = \frac{t_j}{t},\quad \text{where } t \text{ here is the total number of states that make up the process and } j \in \mathbf{S}.$$

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A Markov model is defined by its initial probability distribution and one-step transition matrix. These two elements are sufficient to describe the entire behavior of the chain and derive any other probability distributions of the process.

The stationary probability distribution describes the long-term behavior of the process. For a Markov model, the unconditional probability distribution indexed by time converges to a stationary distribution, providing insights into the process's long-term behavior. It is denoted as: $\pi = \pi\mathbf{P^n}$, where π is a row vector of unconditional stationary probabilities, and $\mathbf{P^n}$ is the n-steps transition matrix. Not all Markov chains achieve their stationary distribution. For a Markov chain to reach its stationary distribution, it must be irreducible and aperiodic (defined below).

- Irreducible Chain: All states communicate, meaning it is possible to reach any state from any other state with non-zero probability. ($P_{ij}^n > 0, \forall i, j \in \mathbf{S},$ and $n \in \mathbb{N}$)

- Aperiodic Chain: A state is aperiodic if the number of steps required to return to it has the greatest common divisor of 1. This prevents the chain from being trapped in a periodic cycle. A Markov chain made entirely of aperiodic states is aperiodic as well.

Analysis

The focus of this study is Jubilee Holdings Limited (JHL), a company listed on the Kenya Securities Exchange. The objective was to model JHL's share price behavior to forecast its long-run dynamics. The share price was analyzed in two distinct forms:

1. Markov Chain (State-Based Analysis)

The transition distribution of the state-based daily returns was modeled using a Markov chain. This approach provided insights into the likelihood of the price moving up, down, or remaining unchanged. The state space consisted of three states: Positive, Negative, and Zero. For instance, if the share price closed at 150 yesterday and 160 today, the daily return would fall into the Positive state. A closing price of 140 tomorrow would shift it to the Negative state, while an unchanged price the following day would place it in the Zero state. The Markov chain used to model the daily closing price is both irreducible and aperiodic.

2. Drift Analysis

To address the limitation of the Markov chain model, which does not account for the magnitude of the daily returns, the second part of the study focused on analyzing the drift in the share price. This analysis utilized the concepts of ergodicity and stationarity, as the daily returns of the share price exhibited these properties.

The data for this study was obtained from the official Jubilee Holdings Limited website, ensuring its credibility and quality. It included daily closing prices from January 1, 2010, to December 24, 2024, totaling 3,732 observations. The 3,731 states were derived from the changes in the daily closing prices.

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Transitions Frequency

	Positive	Negative	Zero	Total
Positive	316	488	359	1163
Negative	505	252	362	1119
Zero	342	379	727	1448

Transition Matrix

	Positive	Negative	Zero
Positive	0.272	0.420	0.309
Negative	0.451	0.225	0.324
Zero	0.236	0.262	0.502

Table 1: Transition Frequencies (n_{ij})

Table 1 presents the sample transition frequencies. For example, the entry in the first row and second column (n_{posneg}) indicates that Jubilee Holdings Limited experienced a negative daily return 488 times after previously obtaining a positive return, out of a total of 1,163 transitions from the Positive state, between January 1, 2010, and December 24, 2024.

Table 2: Transition Probabilities ($\widehat{P_{ij}}$)

Table 2 presents the sample transition probabilities. In other words, the entries in Table 2 represent the likelihood of the share price transitioning from one state to another. For instance, the entry in the first row and second column ($\hat{P}_{posneg}$) indicates that 42% of the time, when the daily return of the share price was previously positive, it transitions to a negative return. This quantifies the tendencies in share price movements.

The live share price of JHL on the Nairobi Securities Exchange allows us to observe the current state of the daily return directly. In such cases, the initial probability distribution will be one of the vectors in the list, $[(1,0,0), (0,1,0), (0,0,1)]$, indicating that the daily return will be in one of the states when observed. However, for this text, it is computed from the observed sample data as follows.

$$t_j = (1163,1120,1448), \quad t = 3731$$

From these values, the initial probability distribution is computed as:

$$\hat{\pi} = \left(\frac{1163}{3731}, \frac{1120}{3731}, \frac{1448}{3731}\right) = (0.312,0.300,0.388)$$

Thus, the initial probability distribution is:

$$\hat{\pi} = (0.312,0.300,0.388)$$

To use the Markov model, we need to ensure that the process satisfies the Markov property. One of the tests used is the triplet test, which has a chi-square statistic:

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African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 8, Issue 3, 2025 (pp. 15-30)
www.abjournals.org

$$X^2 = \sum_{i,j,k} \frac{N_{ijk} - N_{ij}P_{jk}}{N_{ij}P_{jk}}, \quad \text{where } i,j,k \in \mathbf{S}, \quad \text{df} = m^3 - m, \quad \text{where } m \text{ is the cardinality of the state space.}$$

The idea is that $N_{jk} \sim \text{Binomial}(N_j, P_{jk})$, where N_j is the number of times the process transitions from state j. If the process is Markovian, then $N_{ijk} \sim \text{Binomial}(N_{ij}, P_{jk})$, where N_{ij} is the number of times the process transitions from state i to j, and then to another state. In other words, if the process is Markovian, then given a transition from state i to j, the probability of transitioning to state k should be described by P_{jk}, that is $P_{ijk} = P_{jk}$.

This can be expressed as:

$$P(X_{t+3} = k \mid X_{t+1} = i, X_{t+2} = j) = P(X_{t+3} = k \mid X_{t+2} = j) \quad \text{for } t \in \mathbf{T}$$

Observed and Expected Frequencies

	Observed-Positive	Observed-Negative	Observed-Zero	Expected-Positive	Expected-Negative	Expected-Zero	Total
PositivePositive	92	140	84	85.952	132.720	97.644	316
PositiveNegative	211	112	164	219.637	109.575	157.788	487
PositiveZero	84	103	172	84.724	94.058	180.218	359
NegativePositive	143	188	174	137.360	212.100	156.045	505
NegativeNegative	125	53	74	113.652	56.700	81.648	252
NegativeZero	102	89	171	85.432	94.844	181.724	362
ZeroPositive	81	160	101	93.024	143.640	105.678	342
ZeroNegative	168	87	124	170.929	85.275	122.796	379
ZeroZero	156	187	384	171.572	190.474	364.954	727

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Table 3 presents the observed and expected frequencies of transitions between the three states (i, j, k), where $i, j, k \in \mathbf{S}$. For example, the entry in the first row and first column indicates that there were 92 observed transitions from "Positive" to "Positive" to "Positive," while the entry in the first row and fourth column indicates that we expected 85.952 transitions. The purpose of this triplet test is to assess whether the difference between the observed and expected frequencies is statistically significant.

Null Hypothesis: The data fit the Markov model.
Alternative Hypothesis: The data do not fit the Markov model.

- P-value: 0.572
- Chi-square statistic: 22.125
- Chi-square critical value: 36.415

Test result: Since the P-value (0.572) is greater than the significance level (typically 0.05), we fail to reject the null hypothesis. This suggests that there is no significant evidence to conclude that the state-based daily returns do not fit the Markov model.

Long-term Distribution

	x
Up	0.312
Down	0.300
No-change	0.388

Table 4 presents the long-term distribution. The table shows that, in the long run, there is a 31.2% probability of a positive return, a 30% probability of a negative return, and a 38% probability of a zero return.

As observed, the calculated initial distribution and the stationary distribution are the same, which is quite interesting. This suggests that the share price has an unconditional stationary distribution, even from the initial state at $t = 0$. However, the calculated initial distribution is not ideal in this case, as the share price of JHL will always be in one of the three states. The stationary distribution will be reached 12 trading days from today, as shown below:

12-steps Transitions Matrix

	Positive	Negative	Zero
Positive	0.3117962	0.3	0.3882038
Negative	0.3117962	0.3	0.3882038
Zero	0.3117962	0.3	0.3882038

Since the Markov model does not provide information about the magnitude of the daily returns, I employed the concepts of ergodicity and stationarity to analyze whether there is a drift in the share price. This section evaluates the tendency of the share price to move in one direction, complementing the insights gained from the Markov model.

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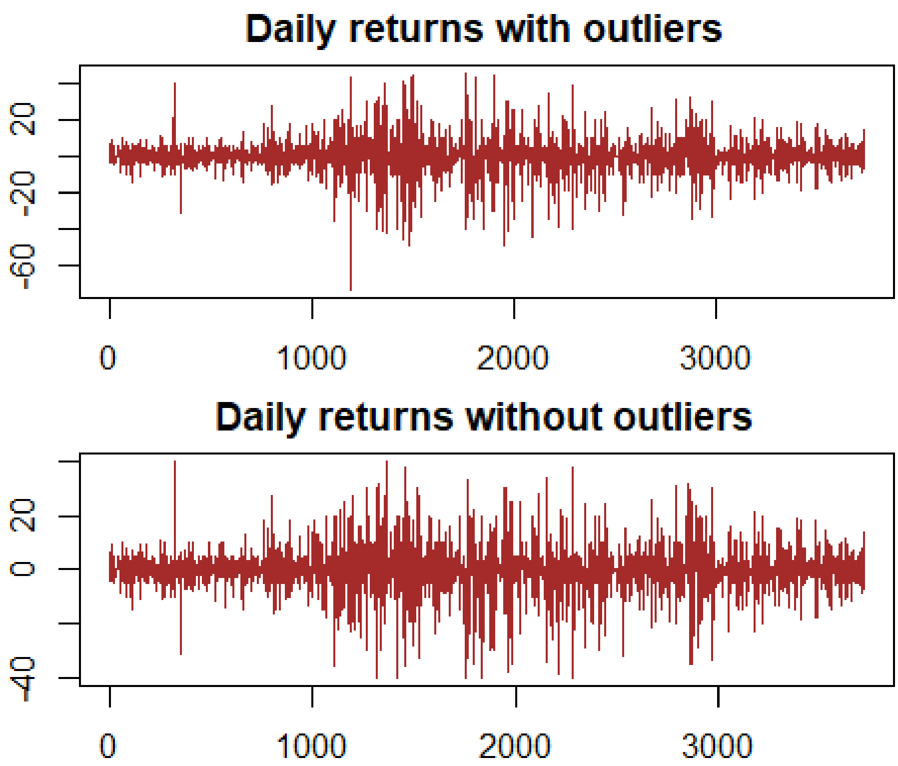
African Journal of Mathematics and Statistics Studies
ISSN: 2689-5323
Volume 8, Issue 3, 2025 (pp. 15-30) www.abjournals.org

To begin, it is essential to verify that the daily return of the share price is both stationary and ergodic. For the stationarity test, I used the Augmented Dickey-Fuller (ADF) test to check for the presence of a unit root in the time series. To test for ergodicity, I examined the graph of the Auto-correlation Function (ACF) in R, looking for significant auto-correlations in the daily returns and determining whether these correlations diminish over time. If the correlations become statistically insignificant after a certain lag, it would indicate that the returns lose dependence over time, thereby satisfying the ergodicity condition, provided the process is stationary.

Because this is a magnitude-based analysis, it is necessary to remove outliers that could affect the results. Below is a comparison of the daily returns with and without outliers.



Daily returns with and without outliers.

Augmented Dickey-Fuller Test

- Data: ts(Daily_returns)
- Test Statistic: −18.746
- Lag Order: 10
- P-value: 0.1
- Alternative Hypothesis: The time series is stationary.

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African Journal of Mathematics and Statistics Studies

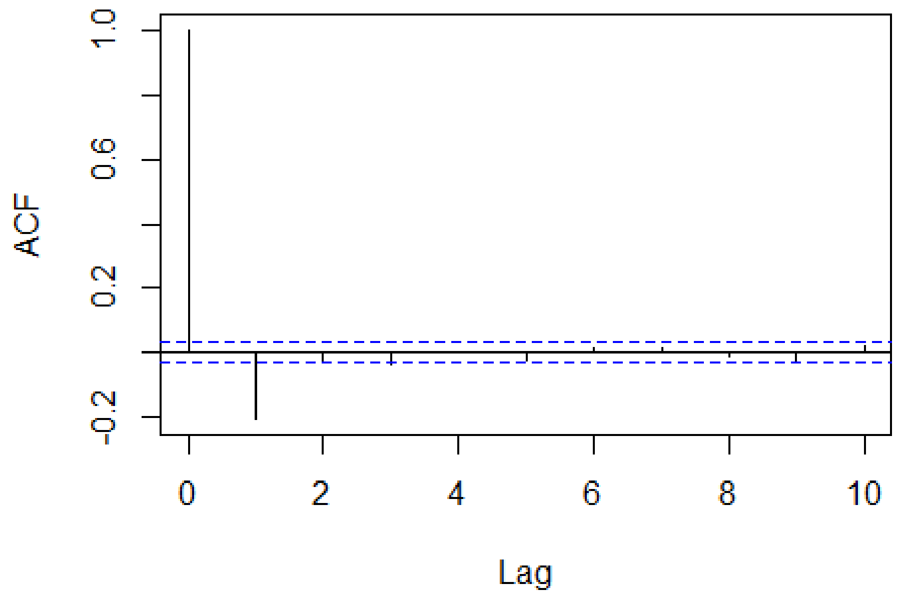
ISSN: 2689-5323

Volume 8, Issue 3, 2025 (pp. 15-30)

www.abjournals.org

Test result:
Since the p-value (0.1) is less than the commonly used significance level (0.05), we reject the null hypothesis that the time series has a unit root. This indicates that the daily return of the share price is stationary.

Below is the graph of the Autocorrelation Function (ACF) function in R.



Autocorrelation of the daily return.

The Auto-Correlation Function (ACF) graph illustrates the autocorrelation in the daily return data for lags ranging from 0 to 10. The vertical lines represent the strength of the autocorrelation at each lag. If a line falls within the two horizontal dotted lines (representing the 95% confidence interval), it indicates that the autocorrelation is statistically insignificant, assuming the null hypothesis that the autocorrelation is zero. From the ACF graph, it is evident that autocorrelation becomes statistically insignificant starting from lag 2 onwards. This suggests that the daily return is significantly correlated only with its immediate past (lag 1), indicating short-term dependencies and no significant autocorrelation beyond the first lag. This finding further supports the Markov property of the process.

Since we have now verified the daily return of the share price is stationary, ergodic and significantly correlated with its immediate lag, we can now use the time estimates to check if there is a drift in the share price.

- Increments mean: 0.072
- Increments Variance: 50.482

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The estimate of the ensemble mean is 0.072, which is approximately 0. We can further hypothesize that the ensemble mean is zero. This hypothesis will be tested using the Z-test, based on the assumption that the sample mean follows a normal distribution due to the Central Limit Theorem (CLT). Because the sample size is large, we rely on the CLT to approximate the distribution of the sample mean to be approximately normal, with the population mean (ensemble mean) assumed to be zero.

Z-test

- Null Hypothesis (H_0): The ensemble mean is 0.
- Alternative Hypothesis (H_1): The ensemble mean is not 0 (two-tailed test).
- Sample Size (n): 3713
- Z-statistic: 0.617
- Z-critical value (95% confidence): ±1.96
- P-value: 0.538

Test result:

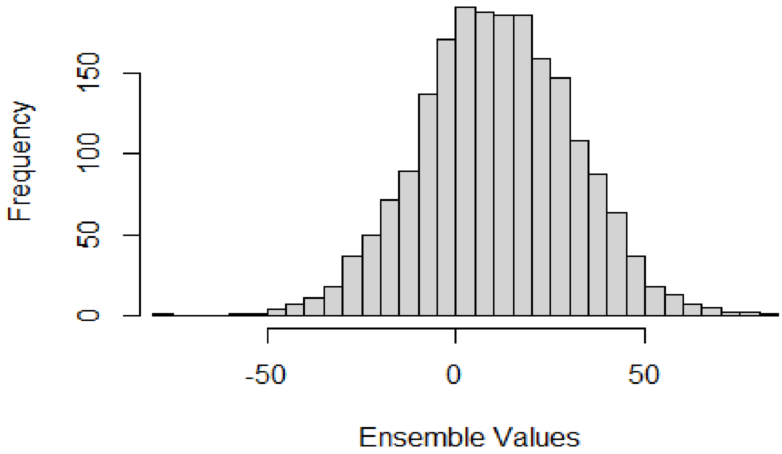
The Z-statistic (0.617) lies within the critical range of ±1.96, and the P-value (0.538) is greater than the significance level of 0.05. Therefore, we fail to reject the null hypothesis.

DISCUSSION

Interestingly, the unconditional distribution of the state-based daily return of the share price reached its stationary distribution after 12 trading days. This means that$P(X_t = i_t)$ remains constant for all $t \geq 12$ and $i \in \mathbf{S}$. Another observation is the minimal difference between the probabilities of a positive return ($P_{pos} = 0.312$) and a negative return ($P_{neg} = 0.300$). Additionally, the 38.8% probability of a zero return indicates a low level of market activity, which aligns with the low liquidity typically observed in the Nairobi Securities Exchange, further emphasizing its status as an emerging market.

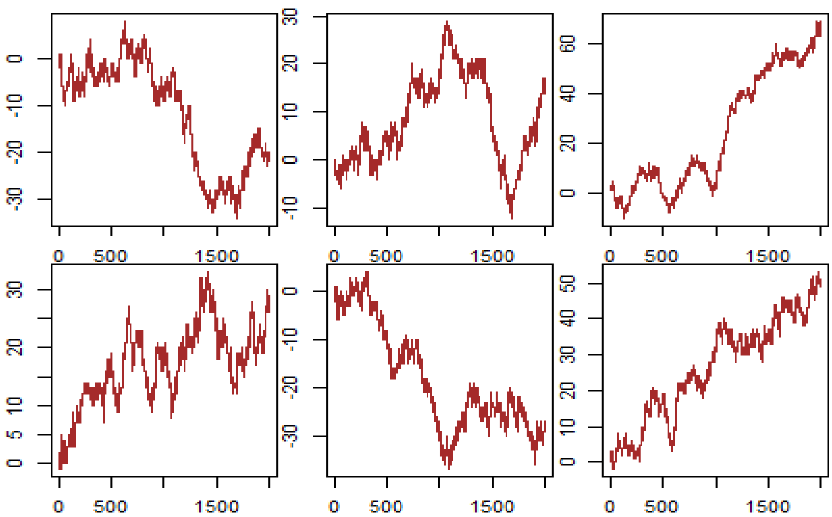
Below is the distribution of values obtained at the end of 2,000 simulated paths, each spanning 2,000 trading days from today. The simulation assumes that the share price today is KSH 0, and the magnitude of positive and negative returns is constant at KSH 1. The algorithm used to simulate a path is derived from Jubilee-code, a customized code base available on my GitHub (here is the link). It begins by using the general initial probability distribution to determine the initial state of JHL's daily return, then computes all subsequent states based on the conditional distribution corresponding to the rows of the transition matrix. Finally, it calculates the cumulative sequence of the return vector. This approach effectively accounts for the dependencies in the transitions.

Article DOI: 10.52589/AJMSS-LPIDHOVK
DOI URL: https://doi.org/10.52589/AJMSS-LPIDHOVK



Distribution of Ensemble Values After Simulation.

The ensemble of values from the simulated paths exhibits a symmetrical bell-shaped structure, indicating that it follows a normal distribution. The simulated data has a mean of 11.717 and a standard deviation of 20.20082. Assuming that the constant magnitude of change holds—though this assumption would not strictly hold in real-world scenarios—there is a 70.9% chance of an investor benefiting from capital gains and a 29.1% chance of incurring capital losses.



Six simulated paths.

When a time series is stationary, it exhibits cyclical variation around its ensemble mean. Due to weak autocorrelation and long-term independence inherent in stationary time series, it may resemble an independent and identically distributed (iid) process, especially over extended

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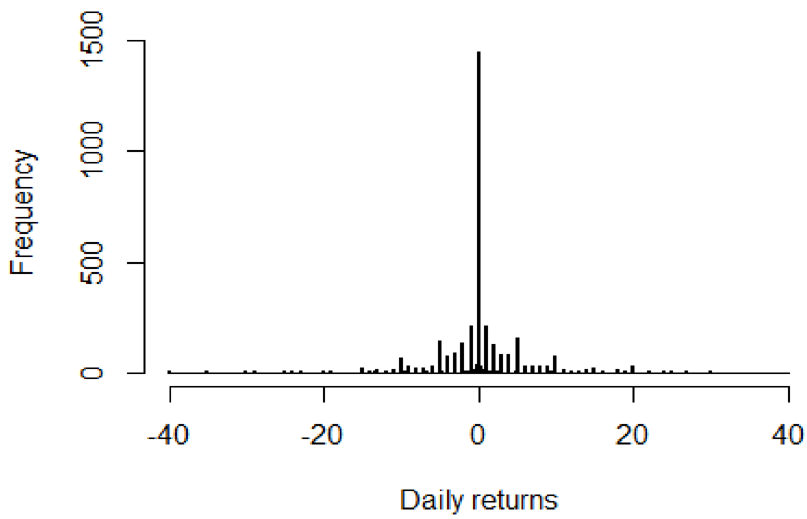
time periods. This results in a high symmetric concentration of values around the mean. Given that the ensemble mean of JHL's daily return is zero and the process is stationary, we can conclude that there is no drift in the share price. Future daily returns are expected to evolve symmetrically around zero, with fluctuations primarily driven by random noise. Over time, these fluctuations will average out to zero, in accordance with the Central Limit Theorem (CLT).

Given this, we should not expect any significant directional movement—whether positive or negative—in Jubilee Holdings Limited's share price in the long run. The daily returns will remain concentrated around their mean, which is zero. However, it is important to note that while a stochastic process with no drift typically exhibits no sustained directional trend, it can occasionally enter a state where directional trends are observed. These trends, however, are generally temporary.

Below is the observed distribution of the daily returns which should help you verify the behavior around the mean, inferred to be 0.



Distribution of the observed daily returns.

CONCLUSION

This paper investigates the long-term behavior of Jubilee Holdings Limited's share price using a Markov model to perform a state-based analysis of its daily returns and leveraging the concepts of ergodicity and stationarity to evaluate the magnitude of these returns. The state space comprises three states: Positive, Negative, and Zero. The Markov model results reveal long-term probabilities of 31.2% for a positive return, 30.0% for a negative return, and 38.8% for a zero return.

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The drift analysis indicates that the ensemble mean of the daily returns is zero, a hypothesis validated by the Z-test. This finding confirms the absence of drift in the share price, meaning there is no inherent tendency for the price to trend upward or downward. Consequently, the share price is expected to remain stable over the long term, with no significant directional movement.

These findings suggest that Jubilee Holdings Limited's share price may not be particularly attractive for investors seeking capital gains, as there is no statistical evidence for a potential sustained upward price movement. However, it is important to note that the emerging status of the Kenya Securities Market could mean that share price behavior may not fully reflect the company's performance.

It is also worth emphasizing that this analysis pertains strictly to capital gains. Despite the lack of significant price movement, Jubilee Holdings Limited has demonstrated consistent dividend payments over the past few years, which may make it a valuable investment for income-focused investors.

This text assumes that the market conditions reflected in the data remain constant. This does not imply that the market is fixed, as the data also capture patterns of change that persist during the studied period. These variations are considered acceptable. However, any unusual events or deviations from the conditions observed may contradict the findings presented here.

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