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ESTIMATION OF ACHIEVEMENT PROBABILITIES IN CORRUPTION DEPENDENT ACHIEVEMENT PROBABILITY MODELS

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ABSTRACT: *This paper proposes, develops and presents an achievement probability model in a population where achievement of some set objective is often dependent and affected by the level of corruption or corrupt practices in the population. It is assumed that in such a population there are three operatives or agencies that form a closed corruption nexus. Methods are provided for the estimation of achievement probabilities of all possible events obtainable in the proposed achievement model in the presence of corrupt practices. Test statistics are also provided for use in testing any desired hypotheses about various achievement probabilities. Some sample data are used to illustrate the proposed method. The results suggest that in some instances some gratification and negotiations between operatives in a corrupt population may be necessary for one to expect high achievement probability in set objective.*

KEYWORDS: Corruption, Probability, Dependent, Model, Conditional, Achievement

INTRODUCTION

In societies or populations where corruption practices are at low levels, meritocracy is often upheld in both public and private affairs. In such populations subjects do not need to cut corners, jump queues, have god-fathers or be cronies to achieve results on set objectives.

However, in populations where corrupt practices are pervasive, endemic and virtually a way of life, rules and regulations are rarely kept and upheld, anything is negotiable, nothing is assured. In such populations the slang “knowing those who know them”, popularity referred to as ‘IM’ in Nigeria, may be the operative phrase and ‘magic wand’ for easily getting anything done and being favoured with any opportunities in the population (Mehmet, 2001). The popular phrase, due to the adverse effect of corruption is that “Corruption is an Elephant in the Room”. A good understanding of the corruption structures and metrics in a society leads to good management of corrupt practices by anti-corruption agencies with the attendant benefits (Bryane, 2013; Joraset et al, 2016; Eiji, 2013).

In such populations any expected favour or even right is invariably preceded or followed by some gift or some form of gratification by the recipient to the expected provider for one to achieve, or secure some position and attain set goal.

The fulfillment or the achievement of some set objective is not necessarily accomplished even when some gratification for the intended purpose has been offered. Hence research interest may be in estimating probabilities of achieving set objectives in the presence of corruption and relative absence of meritocracy in a population.

We here propose and develop a statistical model of corruption in a population and the estimation of achievement or performance probabilities in a corruption dependent economy assuming three mutually cooperating operatives or actors in the system.

The Proposed Method

The act of corruption in a system such as a population is usually committed by at least two persons or actors acting in concert. The first person or actor, usually the big boss often prompts and makes some demand; becoming an expectant receiver of some gratification either in cash, or kind, or both. The second person or actor often a subordinate or executor for the first person gives and sometimes prompts and makes an offer of some gratification again in cash, or kind or both to the first person (boss) who may or may not accept the offer as proposed or made. In some instances, there may also be a third person, perhaps related to the second person, who would be the beneficiary, that is the person who benefits from the opportunities offered by the first person as a result of the gratification given by the second person as a result of the gratification given by the second person and whose interest is in the achievement of the set objective(s).

To develop the proposed model and estimate achievement probabilities in a corruption dependent economy and population assuming the existence of three principal actors in the system or population we may proceed as follows.

Now suppose that the first principal or person X is the owner agency such as population of government parastatals, corporate executives, employers of labour, leaders, individuals, subjects, etc who control the production and distribution of goods and services or who are in management positions in a population. Suppose the second principal or person Y is a population of perhaps subordinates, dependants or agents of population. X such as entrepreneurs, business people, sponsors, the unemployed, candidates, students, etc who need some type of favour from population X. Finally suppose the third principal or person Z is a population of beneficiaries such as the unemployed, candidates, students etc who would benefit from the largesse or opportunities provided by population X and whose interest is in the achievement of some set objective(s).

Note that populations Y and Z may be similar but not necessarily identical populations. Population Z may be only a subset of population Y.

Now suppose a random sample of size n_x agencies or subjects is drawn from population X; a random sample of size n_y of agencies or subjects is drawn from population Y; and a random sample of size n_z of agencies or subjects is similarly drawn from population Z.

Interest is in estimating the probability that a randomly selected subject from population X demands or receives or does not demand or does not receive gratification; the probabilities that a randomly selected subject from population Y offers or gives or does not offer or does not give gratification; and the probability that a randomly selected subject from population Z achieves or does not achieve some set objective.

Now let A and $\bar{A}$ be respectively the events that a randomly selected subject from population X demands or receives and does not demand or does not receives gratification. Let B and $\bar{B}$ be respectively the events that a randomly selected subject from population Y offers or gives and does not offer or does not give gratification. Also let C and $\bar{C}$ be respectively the events that a randomly selected subject from population Z achieves and does not achieve a set objective.

This would yield the following sample space of possible outcomes.

S = (ABC; AB$\bar{C}$; A$\bar{B}$C; A$\bar{B}\bar{C}$; $\bar{A}$BC; $\bar{A}B\bar{C}$; $\bar{A}\bar{B}C$;$\bar{A}\bar{B}\bar{C}$)

In the sequel in all compound events such as those of Eqn. 1, if events A and B are both contained in such compound events, then this would be interpreted to mean that a randomly selected subject from population X prompts, demands or expects some gratification and a randomly selected subject from population Y fails to, that is, does not prompt, offer, or give some gratification. The converse interpretation would hold if events A and B are both contained are both contained in nay compound event.

Now to determine the probability that a randomly selected subject from population X demands, expects or receives some gratification we may let.

$$U_{ix=}\begin{cases} 1, if\ the\ ith\ subject\ randomly\ drawn\ from\ population\ X \\ demands, expects\ or\ receives\ gratification\ either\ in\ kind, cash\ or \\ both\ to\ enable\ a\ transaction\ or\ provide\ goods\ or\ services. \\ 0, otherwise. \end{cases} \quad \ldots \quad (2)$$

for i =1, 2, ⋯,n_x

Let

$$\pi_x^+ = P\ (U_{ix} = 1) \quad \ldots \quad (3)$$

and

$$W_x = \sum_{i=1}^{n_x} U_{ix} \quad \ldots \quad (4)$$

Now the expected value and variance of U_{ix} are respectively

$$E(U_{ix}) = \pi_x^+;\ Var\ (U_{ix}) = \pi_x^+(1 - \pi_x^+) \quad \ldots \quad (5)$$

Similarly, the expected value and variances of W_x are respectively

$$E(W_x) = \sum_{i=1}^{n_x} E(U_{ix}) = n_x.\pi_x^+;\ V_{ar}(W_x) = \sum_{i=1}^{n_x} V_{ar}(U_{ix}) = n_x\pi_x^+\ (1 - \pi_x^+)\ \ldots.(6)$$

Now π_x^+ is the proportion or the probability that a randomly selected subject from Population X demands, expects, receives, or accepts some gratification in kind, cash or both to enable some transaction involving some favour or exchange of goods or services with a second or third party. Its sample estimate is

$$\hat{\pi}_x^+ = \rho_x = \frac{w_x}{n_x} = \frac{f_x^+}{n_x} \quad \ldots \quad (7)$$

Where $f_x^+ = W_x$ is the number of subjects from population X who demand, expect, receive or accept some gratification in exchange for some transactions with a second person or third party. This f_x^+ is the total number of 1's in the frequency distribution of the n_x values of 1's and 0's in U_{ix} for i = 1, 2,. ., n_x.

The sample variance of $\hat{\pi}_x^+$ is from equation 6.

$$V_{ar}(\hat{\pi}_x^+) = \frac{V_{ar}(W_x)}{n_x^2} = \frac{\hat{\pi}_x^+(1-\hat{\pi}_x^+)}{n_x} \quad \ldots \quad (8)$$

A null hypothesis that may often be of interest could be that the proportion π_x^+ of subjects from population X that would demand, expect, receive or accept some form of gratification for a transaction is at least some value π_{x_0}; or symbolically

$$H_0: \pi_x^+ \geq \pi_{x_0} \text{ verses } H_1: \pi_x^+ < \pi_{x_0} \ (o \leq \pi_{x_0} \leq 1) \quad \ldots \quad (9)$$

The null hypothesis H_0 of equation 9 may be tested using the test statistic

$$\chi^2 = \frac{\left(W_x - n_x.\pi_{x_o}\right)^2}{V_{ar}(W_x)} = \frac{n_x(\hat{\pi}_x^+. - \pi_{x_o})^2}{\hat{\pi}_x^+\ (1-\hat{\pi}_x^+)} \quad \ldots. \quad (10)$$

which under H_0 has approximately the Chi-square distribution with 1 degree of freedom for sufficiently large n_x (Agresti, 1996: Sakamoto, 1991).

The null hypothesis H_0 of equation 9 is rejected at the α level of significance if

$$\chi^2 \geq \chi^2_{(1-\alpha,\upsilon)} \quad \ldots \quad (11)$$

Otherwise H_o is accepted.

To obtain the probability that a randomly selected subject from population Y prompts, offers or gives some bribe or gratification, in kind, cash or both in exchange for a favour by a second person or third party we may let

$$U_{iy} \begin{cases} 1, if\ the\ ith\ randomly\ selected\ subject \\ from\ population\ Y\ prompts, offers\ or\ gives \\ some\ gratification\ in\ exchange\ for\ some\ favour \\ O, otherwise \end{cases}$$

for i = 1, 2, . . . n_y

Let

$$\pi_y^+ = P(U_{iy} = 1) \quad \ldots \quad (13)$$

and

$$W_y = \sum_{i=1}^{n_y} U_{iy} \qquad \text{...} \qquad (14)$$

Now

$$E(U_{iy}) = \pi_y^+; \qquad Var\,(U_{iy}) = \pi_y^+(1-\pi_y^+) \qquad \ldots \qquad (15)$$

and

$$E(W_y) = n_y \pi_y^+ ..Var(W_y) \;=\; n_y . \pi_y^+(1-\pi_y^+) \qquad \ldots \qquad (16)$$

Now π_y^+ is the proportion or probability that a randomly selected subject from population Y prompts, offers or gives some form of bribe or gratification in exchange for some favour by a third party. Its sample estimate is

$$\prod_y^+ = \rho_y = \frac{W_y}{n_y} = \frac{f_y^+}{n_y} \qquad \ldots \qquad (17)$$

where $f_y^+ = W_y$ is the total number of subjects from population Y who would prompt, offer or give some form of bribe or gratification to a third party for some favour. Thus f_y^+ is the total number of 1's in the frequency distribution of the n_y values of 1's and Os in U_{iy}; for i = 1, 2 . . . n_y.

The sample variance of $\hat{\pi}_y^+$ is

$$\text{Var}\,(\hat{\pi}_y^+) \;=\; \frac{\text{Var}(W_y)}{n_y^2} = \frac{\hat{\pi}_y^+(1-\hat{\pi}_y^+)}{n_y} \qquad \ldots \qquad (18)$$

A null hypothesis similar to that of Egn. 9 for population X may also be stated and tested for population Y; specifically that the proportion of subjects from population Y who would prompt, offer or give some bribe or gratifictioon for a favour is at least some value π_{y_o}

OR

$$H_o\!: \pi_y^+ \;\geq\; \pi_{y_o} \; versus \; H_1\!: \pi_y^+ < \pi_{y_o} \left(0 \leq \pi_{y_o} \leq 1\right) \qquad \text{...} \qquad (19)$$

The null hypothesis of equation 19 is tested using the test statistic

$$\chi^2 = \frac{\left(W_y - n_y . \pi_{y_0}\right)^2}{Var(W_y)} = \frac{n_y\left(\hat{\pi}_y^+ - \pi_{y_o}\right)^2}{\hat{\pi}_y^+(1-\hat{\pi}_y^+)} \qquad \text{...} \qquad (20)$$

Which under H_0 has approximately the Chi-square distribution with 1 degree of freedom for sufficiently large n_y (Agresti, 1996: Sakamoto, 1991).

The null hypothesis H_0 of equation 19 is rejected at the α level of significance if equation 11 is satisfied, otherwise H_0 is accepted.

Now to determine the probability that a randomly selected subject from population Z achieves, or expects to achieve a set objective, we may let

$$U_{iz}\begin{cases}1, if\ the\ ith\ randomly\ selected\ subject\ from\ population\ Z\\ achieves\ or\ expects\ to\ achieve\ a\ set\ objective\\ 0, otherwise\end{cases} \quad \ldots \quad (21)$$

For i = 1, 2, … ,n_z

$$\pi_z^+ = P(U_{iz} = 1) \quad \ldots \quad (22)$$

and

$$W_z = \sum_{i=1}^{n_z} U_{iz} \quad \ldots \quad (23)$$

Now

$$E(U_{iz}) = \pi_z^+;\ \ Var\ (U_{iz}) = \ \pi_z^+\ (1 - \pi_z^+) \quad \ldots \quad (24)$$

and

$$E(W_z) = \ n_z\ .\pi_z^+;\ \ Var\ (W_z) = \ n_z\ .\pi_z^+\ (1 - \pi_z^+) \quad \ldots \quad (25)$$

Note that π_z^+ is the proportion or the probability that a randomly selected subject from population Z achieves or expects to achieve a set objective.

Its sample estimate is

$$\hat{\pi}_z^+ = \ \rho_z = \frac{W_z}{n_z} = \frac{f_z^+}{n_z} \quad \ldots \quad (26)$$

Where $f_z^+ = W_z$ is the number of subjects from population Z who achieve or expect to achieve set objective.

In other words

f_z^+ is the total number of 1's in the frequency distribution of the n_z values of 1's and 0's in U_{iz}, for i = 1,2, . . . ,n_z. The sample estimate of the variance of $\hat{\pi}_z^+$ is

$$Var(\hat{\pi}_z^+) = \frac{Var(W_z)}{n_z^2} = \frac{\hat{\pi}_z^+(1 - \hat{\pi}_z^+)}{n_z} \quad \ldots \quad (27)$$

Again, a null hypothesis that may be of interest could be that a randomly selected subject from population Z has a low probability of achieving set objective or

$$H_o: \pi_z^+ \le \pi_{z_o}\ versus\ H_1: \pi_z^+ > \pi_{z_o}\ \left(0 \le \pi_{z_o} \le 1\right) \quad \ldots \quad (28)$$

The null hypothesis of Eqn. 28 is tested using the test-statistic.

$$\chi^2 = \frac{\left(W_z - n_z\ .\pi_{z_o}\right)^2}{Var(W_z)} = n_z\frac{n_z\left(\hat{\pi}_z^+ - \pi_{z_o}\right)^2}{\hat{\pi}_z^+(1 - \hat{\pi}_z^+)} \quad \ldots \quad (29)$$

The null hypothesis Ho of Eqn. 28 is rejected at the α level of significance if equation 11 is satisfied, otherwise H_0 is accepted.

Having obtained sample estimates of these probabilities one may then also wish to obtain sample estimates of P(B/A), the conditional probability that a randomly selected subject from population Y prompts, offers, or gives some bribe or gratification to a randomly selected subject from population X who has prompted, demanded or asked for gratification; P(C/A), the conditional probability that a randomly selected subject from population Z achieves or expects to achieve a set objective given that some bribe or gratification has been given and accepted by a randomly selected subject from population X; and P(C/B), the conditional probability that a randomly selected subject from population Z achieves, or expects to achieve a set objective given that some bribe or gratification has been given and accepted by a randomly selected subject from population Y has offered or given some gratification to a third party:

Now to estimate the conditional probability P(B/A), suppose that of the n_x subjects from population. X studied, $n_{y.x}$ subjects would prompt, demand, receive or accept some bribe or gratification for any favour or transaction with a third party or subject from population Y.

Now note from above and in particular from equation 7 that $n_{y.x} = f_x^+$, the number of subjects from population X who prompt, demand, receive or accept some bribe or gratification in exchange for some favour.

Hence to obtain the conditional probability P(B/A) we may let

$$U_{iy.x} = \begin{cases} 1, if\, the\; ith\; subject\; from\; population\; Y\; prompts, offers\; or\; gives \\ some\; bribe\; or\; gratification\; given\; that\; such\; a\; demand\; has\; been \\ made\; or\; received\; by\; a\; randomly\; selected\; subject\; from \\ population\; space\; X\; in\; exchange\; for\; some\; favour. \\ O, otherwise \end{cases} \quad \ldots (30)$$

$$for\; i = 1.2 \ldots, n_{y.x} = f_z^+$$

Let

$$\pi_{y.x}^+ = P(U_{iy.x} = 1) \quad \ldots \quad (31)$$

and

$$W_{y.x} = \sum_{i=1}^{n_{y.x}} U_{iy.x} \quad \ldots \quad (32)$$

Note that

$$E(U_{iy.x}) = \pi_{y.x}^+; \;\; Var(U_{iy.\,x}) = \pi_{y.x}^+(1 - \pi_{y.x}^+) \quad \ldots \quad (33)$$

And

$$E(W_{y.x}) = n_{y.x}\pi_{y.x}^+; \; V_{ar}(W_{y.x}) = n_{y.x}\, \pi_{y.x}^+(1 - \pi_{y.x}^+) \quad \ldots \quad (34)$$

Now $\pi^{+}_{y.x}$ is the proportion or conditional probability that a randomly selected subject from population Y prompts, offers or gives some form of gratification given that such gratification has been demanded, accepted or received by a randomly selected subject from population X. Its sample estimate is

$$\hat{\pi}^{+}_{y.x} = \rho_{y.x} = \frac{W_{y.x}}{n_{y.x}} = \frac{f^{+}_{y.x}}{n_{y.x}} \quad \ldots \quad (35)$$

Where $f^{+}_{y.x} = W_{y.x}$ is the total number of subjects from population Y who prompt, offer or give some bribe or gratification following such demand by subjects from population X to affect some favour. This $f^{+}_{y.x}$ is the total number of 1's in the frequency distribution of the nyx values of 1's and 0's in $U_{iy.x}$,

$$for\ i = 1,2; \ldots n_{y.x} = f^{+}_{x}.$$

The sample estimate of the variance of $\hat{\pi}^{+}_{y.x}$ is

$$V_{ar}\left(\hat{\pi}^{+}_{y.x}\right) = \frac{Var\left(W_{y.x}\right)}{n^{2}_{y.x}} = \frac{\hat{\pi}^{+}_{y.x}(1-\hat{\pi}^{+}_{y.x})}{n_{y.x}}$$

If interest is in testing the null hypothesis that the proportion of subjects in a population who would prompt, offer, or pay gratification for an opportunity when demanded is at least some value, $\pi^{+}_{y.x_0}$, then one may test this null hypothesis using the test statistic

$$\chi^2 = \frac{n_{y.x}\left(\hat{\pi}^{+}_{y.x} - \pi_{y.x_0}\right)^2}{\hat{\pi}^{+}_{y.x}(1-\hat{\pi}^{+}_{y.x})} \quad \ldots \quad (37)$$

The null hypothesis is rejected at the α level of significance if equation 11 is satisfied, otherwise H_0 is accepted.

Now to estimate the conditional probability P(C/A) that on the average a randomly selected subject from population Z achieves or expects to achieve set objective given the opportunity provided a randomly selected subject from population X after receiving some gratification for the same purpose, we again note that the number of sample subjects from population Y who achieve or expect to achieve set objective given relevant opportunities provided by subjects from population X after receiving some gratification is $n_{z.x} = f^{+}_{x}$

Thus, to estimate the conditional probability P(C/A) we may let

$$U_{iz.x}\begin{cases} 1, if\ the\ ith\ subject\ from\ population\ Z\ achieves\ or\ expects\ to\ achieve \\ a\ set\ objective\ given\ opportunity\ offered\ by\ a\ randomly\ selected \\ subject\ from\ population\ X\ after\ receiving\ some\ gratification. \\ 0, otherwise \end{cases} \quad \ldots (38)$$

$$for\ i = 1, 2, \ldots, n_{z.x} = f^{+}_{x}$$

Let

$$\pi^{+}_{z.x} = P(U_{iz.x} = 1) \quad \ldots \quad (39)$$

and

$$W_{z.x} = \sum_{i=1}^{n_{z.x}} U_{iz.x} \quad \ldots \quad (40)$$

Again, note that:

$$E(U_{iz.x}) = \pi^{+}_{z.x};\ \mathrm{Var}\ (\mathrm{U}_{\mathrm{iz}}\,.x) = \pi^{+}_{z.x}(1 - \pi^{+}_{z.x}) \quad \ldots \quad (41)$$

and

$$E(W_{z.x}) = \mathrm{n}_{z.x}.\,\pi^{+}_{z.x};\ \ \mathrm{Var}\ (\mathrm{W}_{z.x}) = \mathrm{n}_{z.x}\,.\,\pi^{+}_{z.x}(1 - \pi^{+}_{z.x}) \quad \ldots \quad (42)$$

As above $\pi^{+}_{z.x}$ is the proportion or the probability that on the average a randomly selected subject from population Z achieves, or expects to achieve set objectives given that some gratification has been given to and received by a randomly selected subject from population X in exchange for the opportunity offered.

Its sample estimate is

$$\widehat{\Pi}^{+}_{z.x} = \rho_{z.x} = \frac{W_{z.x}}{n_{z.x}} = \frac{f^{+}_{z.x}}{n_{z.x}} \quad \ldots \quad (43)$$

Where $f^{+}_{z.x} = W_{z.x}$ is the total number of 1's in the frequency distribution of the $n_{z.x}$ values of 1's and 0's in $U_{iz.x}$, for i=1, 2, . . . ,$n_{z.x} = f^{+}_{x}$.

The sample variance of $\widehat{\Pi}^{+}_{z.x}$ is similarly estimated as in equation 36.

Any desired null hypothesis may be similarly tested for $\Pi^{+}_{z.x}$ using a test statistic similar to Eqn. 37.

For the conditional probability P(C/B), that a randomly selected subjected from population Z achieves or expects to achieve set objective as a beneficiary that is following a randomly selected subject, from population Y for the purpose, we note that the number of sample subjects from population Y who prompt, offer or give some form of gratification to a third party for some favour or opportunity for population Z is obtained from above as $n_{z.y} = f^{+}_{y}$.

Hence to obtain a sample estimate of the conditional probability P(C/B) we may let

$$U_{iz.y} \begin{cases} 1, \mathit{if\ the\ ith\ randomly\ selected\ subject\ from\ population\ Z} \\ \mathit{achieves\ or\ expects\ to\ achieve\ a\ set\ objective\ following\ an} \\ \mathit{offer, that\ is\ given\ the\ opportunity, provided\ by\ a\ randomly} \\ \mathit{selecting\ subject\ from\ population\ Y\ as\ a\ result\ of\ some\ earlier} \\ \ \mathit{gratification\ given\ to\ a\ third\ party.} \\ O, \mathit{otherwise} \end{cases}$$

$$for\quad i = 1, 2, \ldots . n_{z.y} = f^{+}_{y}$$

Let

$$\pi^{+}_{z.x} = P(U_{iz.y} = 1) \quad \ldots \quad (45)$$

and

$$W_{z.y} = \sum_{i=1}^{n_{z.y}} U_{iz.y} \quad \ldots \quad (46)$$

Now

$$E(U_{iz.y}) = \pi^{+}_{z.y}; \quad Var(U_{iz.y}) = \pi^{+}_{z.y}(1 - \pi^{+}_{z.y}) \quad \ldots \quad (47)$$

and

$$E(W_{z.y}) = n_{z.y}.\pi^{+}_{z.y}; \quad V_{ar}(W_{z.y}) = n_{z.y}.\pi^{+}_{z.y}(1 - \pi^{+}_{z.y}) \quad \ldots \quad (48)$$

Here $\pi^{+}_{z.y}$ is the proportion or probability that a randomly selected subject from population Z on the average achieves or expects to achieve set objective given that the subject is a beneficiary of an opportunity provided by a randomly selected subject from population Y who has given some gratification to a third party for the same purpose.

It is easily shown, following the above approaches that the sample estimate of $\pi^{+}_{z.y}$ is

$$\hat{\pi}^{+}_{z.y} = \rho_{z.y} = \frac{W_{z.y}}{n_{z.y}} = \frac{f^{+}_{z.y}}{n_{z.y}} \quad \ldots \quad (49)$$

Where $f^{+}_{z.y} = W_{z.y}$ is the total number of 1's in the frequency distribution of the $n_{z.y}$ values of 1's and 0's in $U_{iz.y}, i = 1, 2, \ldots., n_{z.y} = f^{+}_{y}$

The sample variance of $\hat{\pi}^{+}_{z.y}$ is

$$V_{ar}(\hat{\pi}^{+}_{z.y}) = \frac{V_{ar}(W_{z.y})}{n^2{}_{z.y}} = \frac{\hat{\pi}^{+}_{z.y}(1 - \hat{\pi}^{+}_{z.y})}{n_{z.y}} \quad \ldots \quad (50)$$

Any desired hypothesis may be stated and tested for $\hat{\pi}^{+}_{z.y}$ in the usual way.

Interest may also be in estimating P(C/AB), the conditional probability that a randomly selected subject such as an applicant or a student from population Z achieves or expects to achieve set objective such as securing a job or passing an examination given that a randomly selected subject such as a candidates sponsor from population Y has prompted, offered or given a randomly selected subject such as an employer of labour or a teacher from population X has demanded, received or accepted some gratification to provide some needed opportunity.

To do this note from above that the number of sample subjects from population Y who prompt, offer or give some gratification after such a demand, that is after being so prompted or asked by a randomly selected subject from population X from whom a randomly selected subject from population Z would, under the circumstance, benefit is $n_{z.xy} = f^{+}_{y.x}$

Hence to estimate the conditional probability P(C/AB), we may let.

$$U_{iz.xy} \begin{cases} 1, if\ the\ ith\ randomly\ selected\ subject\ from\ population\ Z\ achieves\ or \\ expects\ to\ achieve\ a\ set\ objective\ given\ that\ a\ randomly\ selected \\ subject\ from\ population\ Y has\ prompted, offered, given\ and\ a \\ \ \ randomly\ selected\ subject\ from\ population\ X\ has\ received\ or \\ demadned\ some\ gratification\ to\ provide\ some\ needed\ opportunity. \\ 0, otherwise \end{cases} \quad ..(51)$$

$$for \quad i = 1, 2, \ldots . n_{z.xy} = f^{+}_{z.xy}$$

Let

$$\pi^{+}_{z.xy} = \mathrm{P}(\mathrm{U}_{\mathrm{iz.xy}} = 1) \quad \ldots \quad (52)$$

and

$$W_{z.xy} = \sum_{i=1}^{n_{z.xy}} U_{iz.xy} \quad \ldots \quad (53)$$

Now, following the above procedures we have that;

$$E(U_{iz.xy}) = \pi^{+}_{z.xy}\ ; Var\ (U_{iz.xy}) = \pi^{+}_{z.xy}\ (1 - \pi^{+}_{z.xy}) \quad \ldots \quad (54)$$

and

$$E(W_{z.xy}) = n_{z.xy} \,.\, \pi^{+}_{z.xy}\ ; V_{ar}(W_{z.xy}) = n_{z.xy}.\pi^{+}_{z.xy}(1 - \pi^{+}_{z.xy}) \quad \ldots \quad (55)$$

Now $\Pi^{+}_{z.xy}$ is the proportion or the probability that a randomly selected subject from population Z achieves or expects to achieve set objective given that a randomly selected subject from population X has demanded or received gratification offered or given by a randomly selected subject from population Y to facilitate achievement of objective.

Its sample estimate is:

$$\hat{\pi}^{+}_{z.xy} = \rho_{z.xy} = \frac{W_{z.xy}}{n_{z.xy}} = \frac{f^{+}_{z.xy}}{n_{z.xy}} \quad \ldots \quad (56)$$

where $f^{+}_{z.xy} = W_{z.xy}$ is the total number of randomly selected subjects from population Z who achieve or expect to achieve set objective given that randomly selected subjects from population Y have prompted, offered or given, and randomly selected subjects from population X have demanded or received some gratification to facilitate achievement of set objective. In other words $f^{+}_{z.xy}$ is the total number of 1's in the frequency distribution of the $n_{z.xy}$ values of 1's and 0's in $U_{iz.xy}, for\ i = 1, 2, \ldots, n_{z.xy} = f^{+}_{y.x}$

The sample variance of $\hat{\pi}^{+}_{z.xy}$ is

$$V_{ar}(\hat{\pi}^{+}_{z.xy}) = \frac{V_{ar}(W_{z.xy})}{n^{2}_{z.xy}} = \frac{\hat{\pi}^{+}_{z.xy}\ (1 - \hat{\pi}^{+}_{z.xy})}{n^{2}_{z.xy}} \quad \ldots \quad (57)$$

A null hypothesis that may be of interest could be that the proportion of subjects from population Z who would achieve or expect to achieve set objective given that subjects from population X have demanded or received gratification offered or given by subjects from population Y to facilitate achievement of set objective is at least some value θ_0, or the null hypothesis;

$$Ho: \pi^{+}_{z.xy} \geq \theta_o \; versus \; H_1: \pi^{+}_{z.xy} < \theta_o (0 \leq \theta_o \leq 1) \quad \ldots \quad (58)$$

The null hypothesis H_0 of equation 58 is tested using the test statistic

$$\chi^2 = \frac{(W_{y.xy} - n_{z.xy} \, . \, \theta_o)^2}{V_{ar}(W_{z.xy})} = \frac{n_{z.xy} \, (\hat{\pi}^{+}_{z.x} - \theta_o)^2}{\hat{\pi}^{+}_{z.xy} \, (1 - \hat{\pi}^{+}_{z.xy})} \quad \ldots \quad (59)$$

The null hypothesis H_0 of equation 58 is rejected at the α level of significance if equation 11 is satisfied, otherwise H_o is accepted.

From the above results we obtain summary estimates of required probabilities as

$$\hat{P}(A) = \hat{\pi}^{+}_{x} = \rho_x; \; \hat{P}(B) = \hat{\pi}^{+}_{y} = \rho_y; \; \hat{P}(C) = \hat{\pi}^{+}_{z} = \rho_z \quad \ldots \quad (60)$$

and

$$\hat{P}(B/A) = \hat{\pi}^{+}_{y.x} = \rho_{x.y}; \; \hat{P}(C/A) = \hat{\pi}^{+}_{z.x} = \rho_{z.x}; \; \hat{P}(C/B) = \hat{\pi}^{+}_{z.y} = \rho_{z.y} \; ;$$

$$\hat{P}(C/AB) = \hat{\pi}^{+}_{z.xy} = \rho_{z.xy} \quad \ldots \quad (61)$$

With the sample estimates of equation 60-61 of achievement probabilities one may then proceed to estimate the probabilities of other possible outcomes in a corruption dependent achievement probability model including the probabilities of the events in equation 1.

For example, one may wish to estimate the probability of the event, (ABC), that is the probability that a randomly selected subject from population Z achieves or expects to achieve a set objective and that some gratification has been demanded or received by a randomly selected subject from population X and offered or given by a randomly selected subject from population Y. This probability is estimated as

$\hat{P}(ABC) = \hat{P}(C/AB) \, . \, \hat{P}(B/A) \, . \, \hat{P}(A)$ (Meyer, 1974)

Which when evaluated using the sample estimate of 60 – 61 becomes

$$\hat{P}(ABC) = \rho_{z.xy} \, . \, \rho_{y.x} \, . \, \rho_x \quad \ldots \quad (62)$$

The probability of the event $(C/\bar{A}\bar{B})$, the conditional probability that a randomly selected subject achieves or expects to achieve set objective given that no gratification is prompted, demanded or accepted and none is offered or given for the same purpose.

This probability is estimated as

$$\hat{P}(C/\bar{A}\bar{B}) = \frac{\hat{P}(\bar{B}C) - \hat{P}(AC) + \hat{P}(ABC)}{\hat{P}(\bar{B}/\bar{A}) \, . \, \hat{P}(\bar{A})}$$

which using the sample estimates of equations 60-61 after further simplification becomes

$$\hat{P}(C/\bar{A}\bar{B}) = \frac{\rho_z - \rho_{z.y}.\rho_y - \rho_{z.x}.\rho_x + \rho_{z.x.y}\cdot\rho_{y.x}\cdot\rho_x}{1-\rho_y-(1-\rho_{y.x}).\rho_x} \quad \ldots \quad (63)$$

The probability that no randomly selected subject from population Z achieves or expects to achieve set objective, no subject from population X prompts, demands or receives and no subject from population Y prompts, offers or gives some gratification to facilitate achievement of set objective is the probability of the event $(\bar{A}\bar{B}\bar{C})$, which is estimated as;

$$\hat{P}(\bar{A}\bar{B}\bar{C}) = 1 - \big(\hat{P}(A) + \hat{P}(B) + \hat{p}(C) - \hat{P}(B/A).\hat{P}(A) - \hat{p}(C/A).\hat{P}(A) - \hat{P}(C/B).\hat{P}(B) + \hat{P}(ABC)\big)$$

or

Using equations 60-61

$$\hat{P}(\bar{A}\bar{B}\bar{C}) = 1 - \big(\rho_x + \rho_y + \rho_z - \rho_{y.x}\cdot\rho_x - \rho_{z.x}\cdot\rho_x - \rho_{z.y}\cdot\rho_y + \rho_{z.x.y} + \rho_{y.x}\cdot\rho_x\big) \quad \ldots \quad (64)$$

The probabilities of other possible outcomes including the outcomes in the sample space S of equation 1 are similarly estimated. The results are shown in Table 1.

Table 1: Sample Estimate of Some Achievement Probabilities in Corruption Dependent Achievement Probability Model

S/No.	Event *(Outcome)*	Achievement Probability
1.	ABC	$\rho_{z.x.y}\cdot\rho_{y.x}\cdot\rho_x$
2.	$AB\bar{C}$	$\rho_x\cdot\rho_{y.x} - \rho_{z.x.y}\cdot\rho_{y.x}\cdot\rho_x$
3.	$A\bar{B}C$	$\rho_x\cdot\rho_{z.x} - \rho_{z.x.y}\cdot\rho_{y.x}\cdot\rho_x$
4.	$A\bar{B}\bar{C}$	$\rho_x - \rho_x.\rho_{z.x} - \rho_x.\rho_{y.x} + \rho_{z.x.y}\cdot\rho_{y.x}\cdot\rho_x$
5.	ĀBC	$\rho_y.\rho_{z.y} - \rho_z.xy.\rho_y.x.\rho_x$
6.	$\bar{A}B\bar{C}$	$\rho_y - \rho_y.\rho_{z.y} - \rho_x.\rho_{y.x} + \rho_{z.xy}\cdot\rho_{yx}\cdot\rho_x$
7.	$\bar{A}\bar{B}C$	$\rho_z - \rho_x.\rho_{z.x} - \rho_y.\rho_{z.y} + \rho_{z.xy}\cdot\rho_{y.x}\cdot\rho_x$
8.	$\bar{A}\bar{B}\bar{C}$	1-$(\rho_x + \rho_y + \rho_z - \rho_x.\rho_{y.x} - \rho_x\cdot\rho_{z.x} - p_y.\rho_{z.y} + \rho_{z.xy}\cdot\rho_{y.x}\cdot\rho_x)$
9.	$B/\bar{A}$	$\frac{\rho_y - \rho_{y.x}\cdot\rho_x}{1-\rho_x}$
10.	$C/\bar{A}$	$\frac{\rho_z - \rho_{z.x}\cdot\rho_x}{1-\rho_x}$
11.	$C/\bar{B}$	$\frac{\rho_z - \rho_{z.y}\cdot\rho_y}{1-p_y}$
12.	$C/\bar{A}B$	$\frac{\rho_{z.y}\cdot\rho_y - \rho_{z.xy}\cdot\rho_{y.x}\cdot\rho_x}{\rho_y - \rho_{y.x}\cdot\rho_x}$
13.	$C/\bar{A}\bar{B}$	$\frac{\rho_z - \rho_{zy}\cdot\rho_y - \rho_{z.x}\cdot\rho_x + \rho_{z.xy}\cdot\rho_{y.x}\cdot\rho_x}{1-\rho_y(1-\rho_{y.x})\cdot\rho_x}$

Illustrative Example

An educationist is interested in studying the problems of examination racketeering in a certain University. She believes that there is often no free lunch. She therefore collects a random sample of 40 course instructors (X) from the University and asked them whether they would accept or have ever accepted some gift or present from students. She also collected a random sample of 50 students from the same university and asked them whether (1) they (Y) would offer or have ever given some gift or present to course instructors (2) they (Z) passed or expected to have passed own most problematic course taken last semester and to state the course code.

The possible responses by course instructors are: No (N): Yes (Y); what for, Never (Wn); yes why not, if offered (Wy); Depends (D). The possible responses by students to question 1 are: yes, sometimes (Ys); No. what for (Nw): Yes, to get by (Yb); No (N): and to Question 2: Yes (y); No (N), Not even after my request (Nr): Yes, thank God (YG).

Each student also stated own most problematic course code during the semester, which the investigator, that is the educationist, matched with corresponding course instructors for use in subsequent analyses.

The responses by the course instructors and students to the questions are presented in table 2.

Table 2: Sample Data on Response to Questions on Possible Exchange of Gift between Course Instructor and Student in a University

S/N	Response	S/N	Response	S/N	Response	S/N	Response	S/N	Response	S/N	Response
1.	D	21.	Y	1.	Nw	26.	Nw	1.	YG	26.	YG
2.	Wn	22.	Y	2.	Ys	27.	Nw	2.	N	27.	N
3.	Wy	23.	Wn	3.	Yb	28.	Yb	3.	N	28.	YG
4.	D	24.	N	4.	N	29.	Nw	4.	YG	29.	Y
5.	Y	25.	Y	5.	N	30.	Ys	5.	Nr	30.	Y
6.	Wy	26.	N	6.	Yb	31.	N	6.	Y	31.	N
7.	N	27.	N	7.	Nw	32.	Yb	7.	Y	32.	N
8.	Wn	28.	N	8.	Yb	33.	Nw	8.	N	33.	Y
9.	Y	29.	D	9.	Yb	34.	Ys	9.	Nr	34.	Nr
10.	Y	30.	Y	10.	Ys	35.	Ys	10.	Nr	35.	N
11.	D	31.	Wn	11.	Ys	36.	N	11.	N	36.	Y
12.	N	32.	N	12.	Nw	37.	N	12.	N	37.	N
13.	Wn	33.	Wy	13.	Yb	38.	N	13.	N	38.	Nr
14.	Y	34.	N	14.	Yb	39.	Yb	14.	Y	39.	Nr
15.	N	35.	Wn	15.	Ys	40.	N	15.	N	40.	Y
16.	Wn	36.	Wy	16.	Ys	41.	Ys	16.	Y	41.	Nr
17.	N	37.	N	17.	Ys	42.	N	17.	Nr	42.	Y
18.	Wn	38.	Wn	18.	Yb	43.	Nw	18.	YG	43.	Nr
19.	Wy	39.	Y	19.	Nw	44.	N	19.	Nr	44.	Y
20.	Y	40.	Y	20.	N	45.	Nw	20.	Y	45.	Y
Nj		$Nj = 40\ (n_x)$		21.	Yb	46.	Ys	21.	Nr	46.	Nr
f_j^+		$f_j^+ = 16(f_j^+)$		22.	Nw	47.	Ys	22.	Y	47.	Y
Pj		$Pj = 0.40(p_x)$		23.	Nw	48.	Yb	23.	Y	48.	Y

			24.	Ys	49.	Nw	24.	YG	49.	Nr
			25.	Nw	50.	N	25.	N	50.	Nr
					$N_J = 50(n_y)$				$50(N_Z)$	
					$f_j^+ = 25(f_j^+)$				$23(f_y^+)$	
					$p_j = 0.50(p_y)$				$o.46(p_z)$	

The educationist subsequently matched the responses by students on course codes with identified corresponding course instructors who are found to be only five in number, namely course instructor numbers; 1(response D); 12 (response N); 20 (response Y); 33 (response Wy) and 35 (response Wn). The results are shown in table 3.

Table 3: Response by Students Matched with Responses by Course Instructors

S/No	Y	Z	S/No	X	S/No	Y	Z	S/No	X
1	NW	YG	1	D	26	NW	YG	1	D
2	YS	N	20	Y	27	NW	N	20	Y
3	YB	N	20	Y	28	YB	YG	1	D
4	N	YG	20	Y	29	NW	Y	20	Y
5	N	NR	12	N	30	YS	Y	20	Y
6	YB	Y	33	WY	31	N	N	20	Y
7	NW	Y	1	D	32	YB	N	20	Y
8	YB	N	12	N	33	NW	Y	20	Y
9	YB	NR	20	Y	34	YS	NR	20	Y
10	YS	NR	33	WY	35	YS	N	12	N
11	YS	N	1	D	36	N	Y	12	N
12	NW	N	20	Y	37	N	N	1	D
13	YB	N	33	WY	38	N	NR	1	D
14	YB	N	35	WN	39	YB	NR	12	N
15	YS	N	35	WN	40	N	Y	33	WY
16	YS	Y	20	Y	41	YS	NR	35	WN
17	YS	NR	33	WY	42	N	Y	35	WN
18	YB	YG	35	WN	43	NW	NR	33	WY
19	NW	NR	35	WN	44	N	Y	12	N
20	N	Y	1	D	45	NW	Y	12	N
21	YB	NR	12	N	46	YS	NR	1	D
22	NW	Y	33	WY	47	YS	Y	1	D
23	NW	Y	35	WN	48	YB	Y	1	D
24	YS	YG	1	D	49	NW	NR	35	WN
25	NW	N	1	D	50	N	NR	33	WY

To analyze the data of Table 2 using the proposed method we note that the number of course instructors from the sampled population X who would demand, accept or receive or who do accept some gift from the sampled population of students Y is $f_x^+ = 16$. Hence from Equation 7 we have that

$$\hat{\pi}_x^+ = \rho_x = \frac{f_x^+}{n_x} = \frac{16}{40} = 0.40$$

The number of students from population Y who would offer or who do give some gift to course instructor is $f_y^+ = 25$.

Hence the corresponding sample proportion of student who are likely to offer or give some present to course instructor is $\hat{\pi}_y^+ = \rho_y = \frac{25}{50} = 0.50$

Similarly, the number of students from the population of sampled students Z who pass or expected to pass or do well in problematic course taken last semester is

$$\hat{\pi}_x^+ = \rho_z = \frac{23}{50} = 0.46$$

Now from Table 3, out of the five course instructors identified by the researcher educationist as having taught courses during the semester there are 13 (with a yes) and 8 (with a Wy), that is $n_{y.x}$ = 13+8=21 instructor-gift events or contact persons who are definite about the possibility of demanding accepting or receiving some present or gift from students.

Also, from Table 3 it is seen that of the 21 students who would give or do give some gift to course instructor, 6 (with a Ys) and 5 (with a YB) or a total of $\left(f_{y.x}^+ = 11\right)$ students actually do so result in a sample proportion of

$$\hat{\pi}_{y.x}^+ = \rho_{y.x} = \frac{f_{y.x}^+}{n_{y.x}} = \frac{11}{21} = 0.524$$

Similarly, of the 21 students from whom any of the five course instructors would probably demand or have received some present or gift, only $f_{z.x}^+ = 8$ students actually pass problematic course during the semester, giving a sample proportion of such students as

$$\hat{\pi}_{z.x}^+ = \rho_{z.x} = \frac{8}{21} = 0.381$$

Note that the number of sampled students from population Z who hope or expect to pass problematic course given that they have offered, that is after offering some gift, or with the belief or expectation that such gift would be of some help is $n_{z.y} = 25$. However, the number of sampled students from population Z who are seen to have actually passed problematic course after offering some gift is $f_{z.y}^+ = 10$. Hence the sample proportion of students who actually passed problematic course given that such students have offered some present or gift for the same purpose is

$$\hat{\pi}_{z.y}^+ = \rho_{z.y} = \frac{f_{z.y}^+}{n_{z.y}} = \frac{10}{25} = 0.40$$

Now the number of sampled students from population Z who expected to pass problematic course given that some course instructors from population X have demanded or received some gratification or gift from these students is $n_{z.xy} = 11$. However, the total number of sampled students from population Z who actually passed problematic course after giving some gratification or gift to some course instructor from population X as requested, demanded or expected is $f_{z.xy}^{+} = 3$.

Hence the proportion or the probability that a randomly selected student from population Z passed problem course during the semester given that the student had offered to some randomly selected course instructor from population X some gratification or gift as demanded, requested or expected by the instructor is estimated using Eqn. 55 as

$$\hat{\pi}_{\mathrm{z.x.y}}^{+} = \rho_{z.xy} = \frac{f_{z.xy}^{+}}{n_{z.xy}} = \frac{3}{11} = 0.273$$

The summary of the estimated sample probabilities is then, from Eqns. 60-61

$$\hat{P}(A) = \hat{\pi}_{\mathrm{x}}^{+} = \rho_{x} = 0.40;\ \hat{\mathrm{P}}(\mathrm{B}) = \hat{\pi}_{\mathrm{x}}^{+} = \rho_{\mathrm{y}} = 0.50;\ \hat{\mathrm{P}}(\mathrm{C}) = \hat{\pi}_{\mathrm{x}}^{+} = \rho_{\mathrm{z}} = 0.46 \quad \ldots \quad (65)$$

and

$$\hat{P}(B/A) = \hat{\pi}_{\mathrm{y.x}}^{+} = \rho_{\mathrm{y.x}} = 0.524;\ \hat{\mathrm{P}}(\mathrm{C/A}) = \hat{\pi}_{\mathrm{z.x}}^{+} = \rho_{\mathrm{z}.x} = 0.381;$$

$$\hat{\mathrm{P}}(\mathrm{C/B}) = \hat{\pi}_{\mathrm{z.y}}^{+} = \rho_{\mathrm{z.y}} = 0.40;\ \hat{\mathrm{P}}(\mathrm{C/AB}) = \hat{\pi}_{\mathrm{z.xy}}^{+} = \rho_{\mathrm{z}.xy} = 0.273 \quad \ldots \quad (66)$$

Using the sample estimates of equations 65-66 one may now estimate achievement probabilities of all other possible events including the events in table 1.

For example, the probability of the event ABC is estimated using equations 65-66 in equation 62 as

$$\hat{P}(ABC) = \rho_{z.xy}.\rho_{y.x}.\rho_{x} = (0.273)(0.524)(0.40) = 0.057$$

The probability of the event $(C/\bar{A}\bar{B})$ is estimated from eqns. 62 and 65-66 as

$$\hat{P}(C/\bar{A}\bar{B}) = \frac{\rho_{z} - \rho_{z.y}.\rho_{y} - \rho_{z.x}.\rho_{x} + \rho_{z.xy}.\rho_{y.x}.\rho_{x}}{1 - \rho_{y} - (1 - \rho_{y.x}).\rho_{x}}$$

$$= \frac{0.46 - (0.40)(0.50) - (0.381)(0.4) + (0.273)(0.524)(0.4)}{1 - 0.50 - (1 - 0.0.524).(0.40)} = 0.532$$

The probability of the event $\bar{A}\bar{B}\bar{C}$ is estimated from equation 64 using equations 65-66 as

$$\hat{P}(\bar{A}\bar{B}\bar{C}) = 1 - (0.40 + 0.50 + 0.46 - (0.524)(0.40) - (0.381)(0.40) - (0.40)(0.50) + 0.057)$$

$$= 1 - 0.855 = 0.145$$

The sample estimates of achievement probabilities of events in Table 1 are similarly obtained. The results are shown in table 4

Table 4: Sample Estimates of Achievement Probabilities of Events in Table 1

S/No.	**Event** *(Outcome)*	**Achievement Probability**
1.	ABC	0.057
2.	$AB\bar{C}$	0.152
3.	$A\bar{B}C$	0.095
4.	$A\bar{B}\bar{C}$	0.095
5.	$\bar{A}BC$	0.143
6.	$\bar{A}B\bar{C}$	0.148
7.	$\bar{A}\bar{B}C$	0.165
8.	$\bar{A}\bar{B}\bar{C}$	0.145
9.	$B/\bar{A}$	0.484
10.	$C/\bar{A}$	0.513
11.	$C/\bar{B}$	0.520
12.	$C/\bar{A}B$	0.492
13.	$C/\bar{A}\bar{B}$	0.532

It is seen from table 4 that the estimated probability is only about 5.7 percent that a randomly selected student who offers or gives some gift or gratification to some course instructor as demanded or requested achieves expected objective, that is passes some problematic course during the semester. The probability is about 14.5 percent that students who are not requested to, do not offer some gratification or gift to course instructors and do not pass problematic course during the semester.

The proportion of students who would probably offer some gratification to course instructor without being requested or prompted is 0.484; while the proportion of students who would likely pass problematic course given that course instructor does not demand or receive some gift is 51.3 percent.

The probability that a randomly selected student passed problematic course assuming the student fails to offer some gratification is estimated to be about 52.0 percent.

However, if students offer some gratification without being asked or requested, probably suggesting a lack of appreciation by course instructor, then the probability is slightly lower (49.2 percent) that such students will pass some problematic course. The probability of achieving set objective, that is passing some problematic course is 53.2 percent if a randomly selected student does not offer or give and course instructor does not request or receive some gratification or gift.

The probability that a randomly selected student gives some gratification even when such gratification is not prompted or expected by course instructor and passes problematic course is estimated as:

$$\hat{P}(BC/\bar{A}) = 1 - \rho_{y.x} - \rho_{z.x} + \rho_{z.xy}.\rho_{y.x}$$

$$= 1 - 0.524 - 0.381 + (0.273)(0.524) = 0.238 \; or \; only \; 23.8 \; percent$$

A result that would suggest that course instructor perhaps expects some form of gratification or negotiation from problem students before any offer could be made. Other achievement probabilities as desired may be similarly estimated.

SUMMARY AND CONCLUSION

This paper has proposed, developed and presented an achievement probability model that may be applicable to a population in which achievement of set objective is dependent and perhaps hampered by the level of corrupt practices prevalent in the population. Estimates of achievement probabilities in such populations are provided, assuming a nexus of three principal actors or operative agencies, in a corruption dependent achievement model namely: the ‘owner agency’, the ‘service provider agency’ and the ‘beneficiary agency’.

Test statistics are provided for use in testing any desired hypotheses about various achievement probabilities. Some sample data are used to illustrate the proposed method and the results suggest that in some instances some gratification and negotiations between the operatives may be necessary for the achievement of set objectives.

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