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DETERMINATION OF ECCENTRIC ANOMALY FOR KEPLER'S SATELLITE ORBIT USING PERTURBATION-BASED SEEDED SECANT ITERATION SCHEME

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ABSTRACT: *In this paper, the determination of eccentric anomaly (E) for Kepler's satellite orbit using Perturbation-Based Seeded Secant (PBSS) iteration algorithm is presented. The solution is meant for Kepler's orbit with the value of eccentricity (e) in the range $0 \leq e \leq 1$. Such orbits are either circular or elliptical. The demonstration of the applicability of the PBSS iteration is presented using sample numerical examples with different values of mean anomaly (M) and eccentricity (e). The summary of the results of E for $M = 30°$ and e in the range $0.001 \leq e \leq 1$ showed that the convergence cycle (n) increases as e increases. Particularly, n increased from 2 at e = 0.01 to n = 8 at e =1. The implication is that it takes more iterations to arrive at the value of E with the desired accuracy or error performance (which in this case is set to 10^{-12}). Another implication is that a good choice of the initial value of E is essential especially as the value of e increases. As such, effort should be made to develop a means of estimating the initial value of E which will reduce the convergence cycle for higher values of e.*

KEYWORDS: Kepler's Orbit, Eccentric Anomaly, Orbital Motion, Kepler's Equation, Satellite, Seeded Secant, Orbit

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INTRODUCTION

The Kepler's orbital motion equation for eccentric anomaly (E) is a transcendental equation which requires iterative approaches for its solution [1,2,3,4,5,6,7]. In this paper a modified version of the classical secant iteration scheme is developed and applied in the determination of the eccentric anomaly (E) of Kepler's orbit with value of eccentricity (e) in the range 0 ≤ e ≤ 1 [8,9,10,11]. The modified secant presented in this paper is referred to Perturbation-Based Seeded Secant (PBSS) iteration scheme.

Notably, the PBSS uses a single initial guess value of E along with any chosen fraction of the initial guess value of E as the perturbation value to perform the secant iteration. If E_0 is the chosen initial single guess value of E, then the perturbation value, α of 0.01 can be used to obtained the second initial guess root, E_1 where E_1 is (1.01)E_0, that is $\left((1+\ \alpha)\ E_0\right)$. In this way, the user is only required to provide a single initial guess value of E_0 and then the second required root is A variant of E_0. This makes it easier to employ the seeded secant in the solution of transcendental equations.

In this paper, the algorithm for the PBSS iteration is presented. The algorithm is tailored to the solution of Kepler's orbital motion transcendental equation for eccentric anomaly (E) . Some numerical examples are presented and the performance of the algorithm is examined in terms of its convergence cycle for different parameter configurations of the Kepler's transcendental equation for eccentric anomaly.

METHODOLOGY

The eccentric anomaly (E) of Kepler's satellite orbit can be determined from the knowledge of eccentricity (e) and the mean anomaly (M) using the analytical expression [12,13,14,15,16];

$$E\ = M + e(sin\ (E)) \tag{1}$$

There is no closed-form solution to the expression for solving E. As such numerical iteration approach can be used. In this paper, the Perturbation-Based Seeded Secant (PBSS) iteration method is used. The PBSS uses a single initial value of E and a perturbation value (α) of 0.01 (E) to iteratively determine the value of E based on the PBSS algorithm. For the seeded secant, E can be expressed in terms of iteration cycle number, x as follows;

$$E_x\ = M + e(sin\ (E_x)) \tag{2}$$

$$E_{x\alpha} = E_{x(1+\alpha)} + e\left(sin\ (E_{x(1+\alpha)})\right) \tag{3}$$

Hence;

$$E_{x\alpha} = (1+\alpha)E_x = M + e\left(sin\ sin\ \left((1+\alpha)E_x\right)\ \right) \tag{4}$$

In the first cycle, E =M , hence, at x=0,

$$E_0\ = M + e(sin\ (E_0)) = M + e(sin\ (M)) \tag{5}$$

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$$E_{0\alpha} = (1+\alpha)E_0 \ = M + e\left(sin\ sin\ \left((1+\alpha)E_0\right)\right) \tag{6}$$

For $\alpha = 0.01$,

$$E_{0\alpha} = 1.01E_0) = M + e(sin\ sin\ (1.01E_0)\ ) \tag{7}$$

The PBSS algorithm is stated as follows:

Step 1:

Step 1.1: : $E_o = M$

Step 1.2: **α = 0.01**

Step 1.3: Accuracy , $\varepsilon = 10^{-12}$

Step 1.4: Input: Maximum Iterations Cycle, *n*

Step 2: For x = 0 To n Step 1 do:

Step 3:

Step 3.1: Calculate $f(E_x) = M + e(sin\ (E_x))$

Step 3.2: Calculate $f(E_{x\alpha}) = (1+\alpha)E_x = M + e\left(sin\ sin\ \left((1+\alpha)E_x\right)\right)$

Step 4: $E_{x+1} = E_x - f(E_x)\left(\frac{\alpha}{f(E_{x\alpha}) - f(x)}\right)$

Step 5:

Step 5.1: If $|E_{x+1} - E_x| < \varepsilon$ Then

Step 5.1.1: Print E_{x+1}

Step 5.1.2: Go to Step 8;

Step 5.3: EndIf

Step 6: Next x

Step 7: Print "Maximum Iteration Cycle Exceeded"

Step 8 Stop

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RESULTS AND DISCUSSION

Demonstration of the applicability of the PBSS algorithm is presented using sample numerical examples with different values of M and e. The results of E for M = 30° and e =0.01 are given in Table 1, the results of E for M = 30° and e =0.1 are given in Table 2 and the results of E for M = 30° and e = 1 are given in Table 3. The summary of the results of E for M = 30° and e in the range $0.001 \leq e \leq 1$ are given in Table 3. The results show that the convergence cycle number (n) increases as e increases. Particularly, n increased from 2 at e = 0.01 to n = 8 at e =1. The implication is that it takes more iterations to arrive at the value of E with the desired accuracy or error performance (which in this case is set to 10^{-12}).

Another implication is that a good choice of the initial value is essential especially as the value of e increases. As such, effort should be made to develop a means of estimating the initial value of E which will reduce the convergence cycle for higher values of e.

Table 1 The results of E for M = 30° and e =0.01

Cycle	Ex	Exα	f(Ex)	f(Exα)	M (Radian)
0	0.523599	0.524122	-5.00000E-03	-4.48094E-03	0.523598776
1	0.528642	0.529171	5.71775E-08	5.24136E-04	M (Degree)
2	0.528642	0.529171	7.69385E-14	5.24078E-04	30
3	0.528642	0.529171	0.00000E+00	5.24078E-04	e
4	0.528642	0.529171	0.00000E+00	5.24078E-04	0.01
5	0.528642	0.529171	0.00000E+00	5.24078E-04	E (Radian)
6	0.528642	0.529171	0.00000E+00	5.24078E-04	0.528642391
7	0.528642	0.529171	0.00000E+00	5.24078E-04	E (Degree)
8	0.528642	0.529171	0.00000E+00	5.24078E-04	30.28897786
9	0.528642	0.529171	0.00000E+00	5.24078E-04	Convergence Cycle
10	0.528642	0.529171	0.00000E+00	5.24078E-04	2
11	0.528642	0.529171	0.00000E+00	5.24078E-04	Error At Convergence (radian)
12	0.528642	0.529171	0.00000E+00	5.24078E-04	7.69385E-14

Table 2 The results of E for M = 30° and e =0.1

Cycle	Ex	Exα	f(Ex)	f(Exα)	M (Radian)
0	0.523599	0.524122	-5.00000E-02	-4.95217E-02	0.523598776
1	0.578339	0.578917	7.65431E-05	6.06462E-04	M (Degree)
2	0.578255	0.578833	1.51158E-09	5.29842E-04	30
3	0.578255	0.578833	2.60902E-14	5.29840E-04	e
4	0.578255	0.578833	0.00000E+00	5.29840E-04	0.1
5	0.578255	0.578833	0.00000E+00	5.29840E-04	E (Radian)
6	0.578255	0.578833	0.00000E+00	5.29840E-04	0.578255134
7	0.578255	0.578833	0.00000E+00	5.29840E-04	E (Degree)

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8	0.578255	0.578833	0.00000E+00	5.29840E-04	33.13157869
9	0.578255	0.578833	0.00000E+00	5.29840E-04	Convergence Cycle
10	0.578255	0.578833	0.00000E+00	5.29840E-04	3
11	0.578255	0.578833	0.00000E+00	5.29840E-04	Error At Convergence (radian)
12	0.578255	0.578833	0.00000E+00	5.29840E-04	2.60902E-14

Table 3 The results of E for M = 30° and e =1

Cycle	Ex	Exα	f(Ex)	f(Exα)	M (Radian)
0	0.523599	0.524122	-5.00000E-01	-4.99930E-01	0.523598776
1	4.252006	4.256258	4.62429E+00	4.63042E+00	M (Degree)
2	1.046005	1.047051	-3.43022E-01	-3.42500E-01	30
3	1.732846	1.734579	2.22348E-01	2.24362E-01	e
4	1.541528	1.54307	1.83579E-02	1.98555E-02	1
5	1.522632	1.524155	1.92980E-04	1.64346E-03	E (Radian)
6	1.52243	1.523952	1.74547E-07	1.45016E-03	1.52242932
7	1.522429	1.523952	1.39363E-10	1.44998E-03	E (Degree)
8	1.522429	1.523952	1.11466E-13	1.44998E-03	87.22877464
9	1.522429	1.523952	0.00000E+00	1.44998E-03	Convergence Cycle
10	1.522429	1.523952	0.00000E+00	1.44998E-03	8
11	1.522429	1.523952	0.00000E+00	1.44998E-03	Error At Convergence (radian)
12	1.522429	1.523952	0.00000E+00	1.44998E-03	1.11466E-13

Table 4 Summary of the results of E for M = 30° and 0.001 ≤ e ≤1

S/N	**M°**	**e**	**E**	**Convergence Cycle**
1	**30**	**0.001**	**30.02867272**	**1**
2	**30**	**0.005**	**30.14386194**	**2**
3	**30**	**0.01**	30.28897786	**2**
4	**30**	**0.05**	31.49670777	**3**
5	**30**	**0.1**	33.13157869	**3**
6	**30**	**0.5**	52.82708717	**5**
7	**30**	**1**	87.22877464	**8**

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CONCLUSION

Computation of the eccentric anomaly (E) of Kepler's satellite orbit using perturbation-based seeded secant algorithm is presented. The solution is for the orbit with the value of eccentricity (e) in the range $0 \leq e \leq 1$. Such orbits are either circular or elliptical. The perturbation-based seeded secant algorithm is presented along with the sample numerical example used to demonstrate the applicability of the perturbation-based seeded secant method. The results show that the convergence number increases as the value of e increases. This means that a good choice of the initial value of eccentric anomaly (E) is needed to reduce the convergence cycle in those cases when the value of e is high.

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