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PREDICTION OF THE BEAMS UNDER BENDING LOAD

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ABSTRACT: *Analysis of the beam load must be paid great attention. The external loads determine the resultant force of internal forces in the cross-section. An important task in practice is to determine the properties of the deflection line and the maximum stresses at a crucial point of the beam to assess its functional capability. The magnitude of deflection and stress can be solved both analytically and by numerical simulation. The obtained results allow us to predict critical points of the structure, thus avoiding the accumulation of maximum stresses and deflection, possibly leading to the failure and loss of the structure stability.*

KEYWORDS: Beam, deformation, Deflexion Line, Cross-Section, numerical simulation.

INTRODUCTION

The impact of external forces causes deformation or even destruction of the body (beam). The stresses in the beam arise owing to the resistance of the body to deformation; they are therefore interrelated. The position of the external forces acting on a body gives rise to different types of basic stresses or their combination. The goal of our investigation was bending stress of beams. The beam bending is caused by transverse loading of external forces which causes curvature of the beam. Plane bending typically deforms the body in the plane of the acting external load. The beams that are frequently exposed to bending stress represent the basic elements of any structure. Prediction of the beams bending stress is therefore important in terms of the strength and dimensional stability of a structure.

Load analysis of the beam as the key structural element requires increased attention. External load of beam determines the resulting internal forces in the cross-section. The external load deforms the rectilinear axis of the beam. It is important in practice to determine properties of the deflection line and the maximum stress. Determination of the deflection and stress at a characteristic beam point is important for the assessment of the functional capability of the beam. The magnitude of deflection and stress in the characteristic point of the beam can be solved either analytically or numerically by simulation. The results obtained from the analysis allow us to predict critical points of the structure. Thus, we can avoid the accumulation of the maximum stresses and the deflection possibly leading to the structure failure and loss of stability.

METHODS AND MATERIALS

Several methods can be used to deal with stresses and deformations of the beams under bending load. The choice of a method depends on the investigator as well as the technological utilisation of the structure. Analytical method and modeling using numerical simulation are most often used to predict deformation and stress of structure.

The analytical method is based on the Method of Imaginary Section. In the general section of the beam, the bending moment is equal to the algebraic sum of the moments from the external forces acting along one side of the considered section. The shear force in the general section of the beam is equal to the algebraic sum of the external forces perpendicular to the axis of the beam, acting along one side of the considered section. Magnitude of the shear force can be determined from the bending moment, using the Schwedler–Zhuravsky theorem.

The beam deformation and stress were also analysed by numerical simulation using FEM. Calculation of the maximum stress and the maximum bending moment by modelling using FEM is based on the actual geometric shape and dimensions of the beam under investigation. To create a material model, it is necessary to know the mechanical properties of the material the beam is made of.

Structural elements (beams) are made of structural steel, which is characterized by the following mechanical properties: elasticity modulus of the structural steel E = 2.1 10^5 MPa, its tensile strength R_e = 370 MPa, and Poisson number μ = 0.30.

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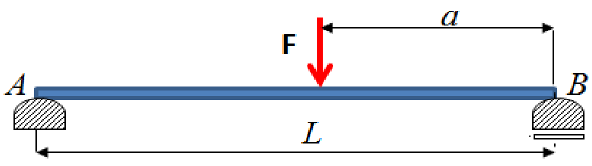
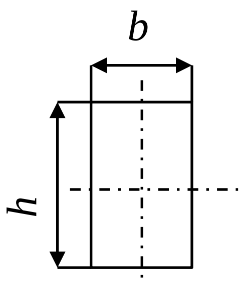
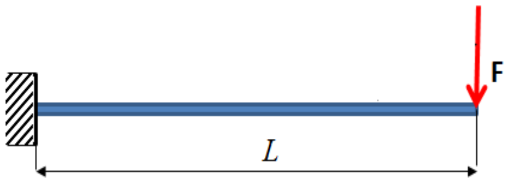
1. Definition and Solution of a Statically Determined Beam under Bending Load

The task was to determine the course of the bending moment and the transverse force of statically determinate beams under external load (see Figure 1). The beam was made of structural steel. The external force F acting perpendicular to the cross-section of the beam causes the maximum normal stress and the maximum deflection at a given point, determining the strength and stability of the beam.



a) A supported beam [1]





b) A clamped beam [1]

Dimensions: L = 1 m, a = 0.5 m, b = 0.02 m, h = 0.04 m,

External load: F = 1 kN

Determine:

Course of the beam bending moment

Course of the beam cross-section force

Maximum beam stress

Maximum beam deflection

Mechanical properties of structure steel:

Modulus of elasticity $E = 2.1.10^5$ MPa,

Poisson number $\mu = 0.30$

Tensile strength R_e = 370 MPa

Figure 1: A beam under bending load

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Analytical Solution of the Beam under Bending Load

The analytical solution of the beam under bending load was performed using the Method of Imaginary Section by calculating the course of the bending moment and the transverse force along the entire beam.

Application of the Imaginary Section Method

The application of analytical method with analytical calculation and graphical representation, using the Imaginary Section Method of a supported beam is illustrated in Figure 2; Figure 3 shows a clamped beam. The imaginary section method (ISM) determines the course of the bending moment and the transverse force along the entire length of the beam loaded by an external force.

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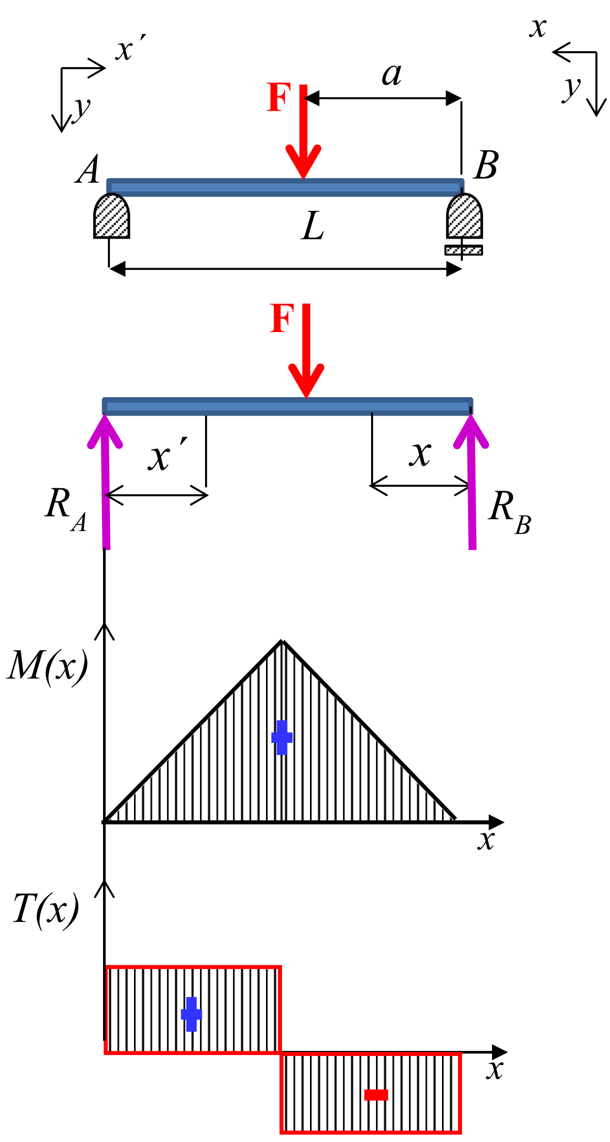
GRAPHICAL REPRESENTATION OF THE SOLUTION



CONDITIONS OF ANALYTICAL SOLUTION

Static conditions of equilibrium:

$\sum F_i(x) = 0$

$R_A + R_B - F = 0 \rightarrow R_A = F - R_B = F/2$

$\sum M_i(x) = 0$

$R_B \cdot L - F.L/2 = 0 \quad \rightarrow R_B = F/2$

Method of Imaginary Section:

$x \epsilon \langle 0, L/2 \rangle \qquad M(x) = R_B x = \frac{F}{2} x$

$$T(x) = -\frac{dM(x)}{dx} = -R_B = -\frac{F}{2}$$

$x' \epsilon \langle 0, L/2 \rangle \qquad M(x') = R_A x' = \frac{F}{2} x'$

$$T(x') = \frac{dM(x')}{dx'} = R_A = \frac{F}{2}$$

Course of bending moment:

$x = 0 \quad M(x) = 0 \qquad x = L/2 \; M(x) = F.L/4$

$x' = 0 \quad M(x') = 0 \qquad x' = L/2 \; M(x') = F.L/4$

Course of transverse force:

$x = 0 \quad T(x) = -F/2 \qquad x = L/2 \quad T(x) = -F/2$

$x' = 0 \quad T(x') = F/2 \qquad x' = L/2 \quad T(x' = F/2$

Figure 2: Analytical solution of a supported beam under bending load

Strength Check

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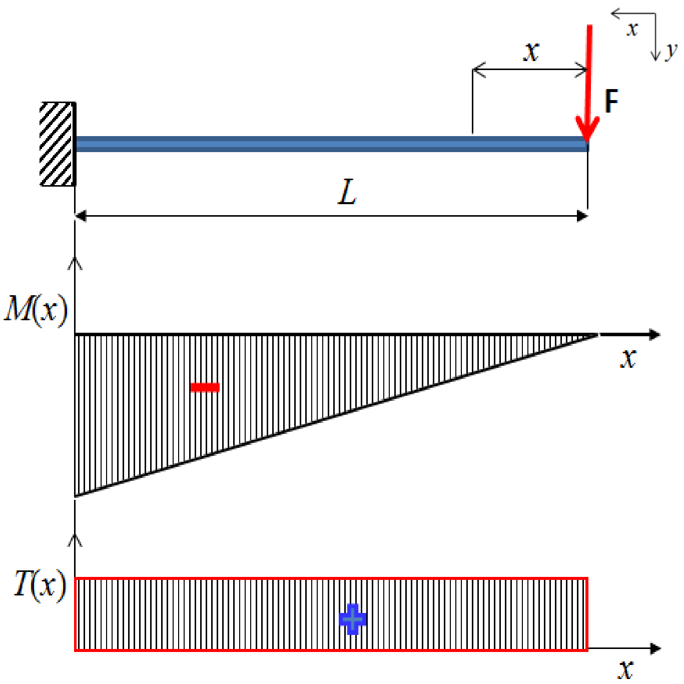
When checking the strength, the safety conditions of the stressed beam must be met. The safety of a beam under bending force is determined by the method of validity of the allowable stress. The method of allowable stress applies to the maximum normal stresses, the maximum shear stresses and the reference stresses defined by the relations given in the paper [4].

GRAPHICAL REPRESENTATION OF SOLUTION	CONDITIONS OF ANALYTICAL SOLUTION



Method of Imaginary Section:

$x \epsilon \langle 0, L \rangle M(x) = -F.x$

$T(x) = -\frac{dM(x)}{dx} = F$

Course of bending moment:

$x = 0 \quad M(x) = 0$

$x = L \quad M(x) = -F.L$

Course of transverse force:

$x = 0 \quad T(x) = F$

$x = L \quad T(x) = F$

Figure 3: Analytical solution of a clamped beam under bending load

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Maximum Deflexion

Check of strength of the beams under bending load can be determined by the maximum allowable stresses as well as the maximum deformation. The rectilinear axis of beam is deformed under an external load. The deformed axis is called a bending (deflection) line. In practice, determination of the bending line properties is a frequent and important task, particularly determination of the deflection or the beam rotation at its characteristic point. Determination of the beam deflection or rotation at the characteristic point serves to assess the functional capability of the beam as the key element of the structure.

Deformation of a beam under bending load can be calculated by means of [1,2]:

- Differential equation of the beam bending line,
- Mohr's method,
- Catigliano's first theorem.

Load of the beam suggests that the beam's maximum deflection will be at the point of the maximum load of external force. Therefore, in the case of our beams under bending load, the maximum deflection of the beam was calculated by using the Catigliano's first theorem [1, 2, 6, 7].

$$w_{\max} = \frac{\partial A}{\partial F} = \frac{1}{2EJ}\left[\int_0^{L/2} 2M(x)\frac{\partial M(x)}{\partial F}dx + \int_0^{L/2} 2M(x')\frac{\partial M(x')}{\partial F}dx'\right] = \quad [\mathrm{m}] \qquad (1)$$

$$= \frac{FL^3}{48E\,J} = \frac{F.L^3}{48E\dfrac{b.h^3}{12}}$$

Numerical Simulation of the Beam under Bending Load

The direct beams under bending load were investigated by numerical simulation using the FEM in the ANSYS program. Numerical simulation is based on the real conditions in the analytical solution of both, a supported beam and the clamped one under the external force causing the beam to bend, as shown in Figure 1.

The geometric model corresponded to the dimensions and shape of the supported beam (Fig. 1a) and the clamped beam (Fig. 1b). Numerical simulation for a given beam type and its load allows for a simplified way, i.e., solving a 3D problem in 1D, using a beam type element. The beam load can be modelled using one-dimensional elements in the plane, which simplifies the solution. In this particular case, a linear element of the BEAM 188 type was used. Such type of element can only be used for slender beams of various cross-sectional profiles, provided that the deformation of the cross-section is sufficiently small. The material model comprised mechanical properties of structural steel [3, 5].

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Results of Analytical Solution

The analytical calculation of a straight beam under bending load provided the values of the required quantities for allowable stress and the maximum deformation of the beam required for each structural element. Results from the analytical solution of the supported beam are given in Table 1, and those of the clamped beam in Table 2.

Table 1: Results of Analytical Solution of the Supported Beam

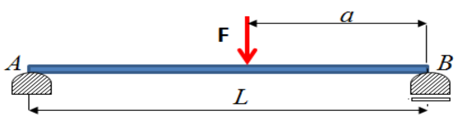
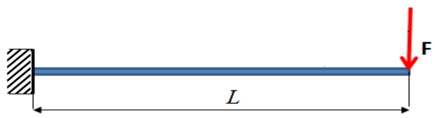
RESULTS OF ANALYTICAL SOLUTION						
Load	***Internal forces***		***Stress***			***Deformation***
External force [N]	Max. bending moment [Nm]	Max. transverse force [N]	Max. normal [MPa]	Max. shear [MPa]	Max. comparison [MPa]	Max. deflexion [m]
1 000	250	|500|	46.875	0.938	46.903	$0.93.10^{-3}$

Table 2: Results of Analytical Solution of the Clamped Beam

RESULTS OF ANALYTICAL SOLUTION						
Load	***Internal forces***		***Stress***			***Deformation***
External force [N]	Max. bending moment [Nm]	Max. transverse force [N]	Max. normal [MPa]	Max. shear [MPa]	Max. comparison [MPa]	Max. deflexion [m]
1000	|1000|	1000	187.5	1.875	187.53	0.01488

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RESULTS OF NUMERICAL SIMULATION

Figure 4 shows the graphical outputs of the results from numerical simulation of a supported beam under bending load, along with the geometric model and the generated net. Result of the stress analysis is the stress distribution along the length of the entire beam. The maximum normal stress of the beam occurs in the upper fibres of the cross-section at the point of the maximum deflection. The maximum stress value according to von Mises is 46.7 MPa. The calculated maximum course from the numerical analysis attained the value of 0.93510^{-3} m at the place of the external load application. The solution showed the course of a bending line.

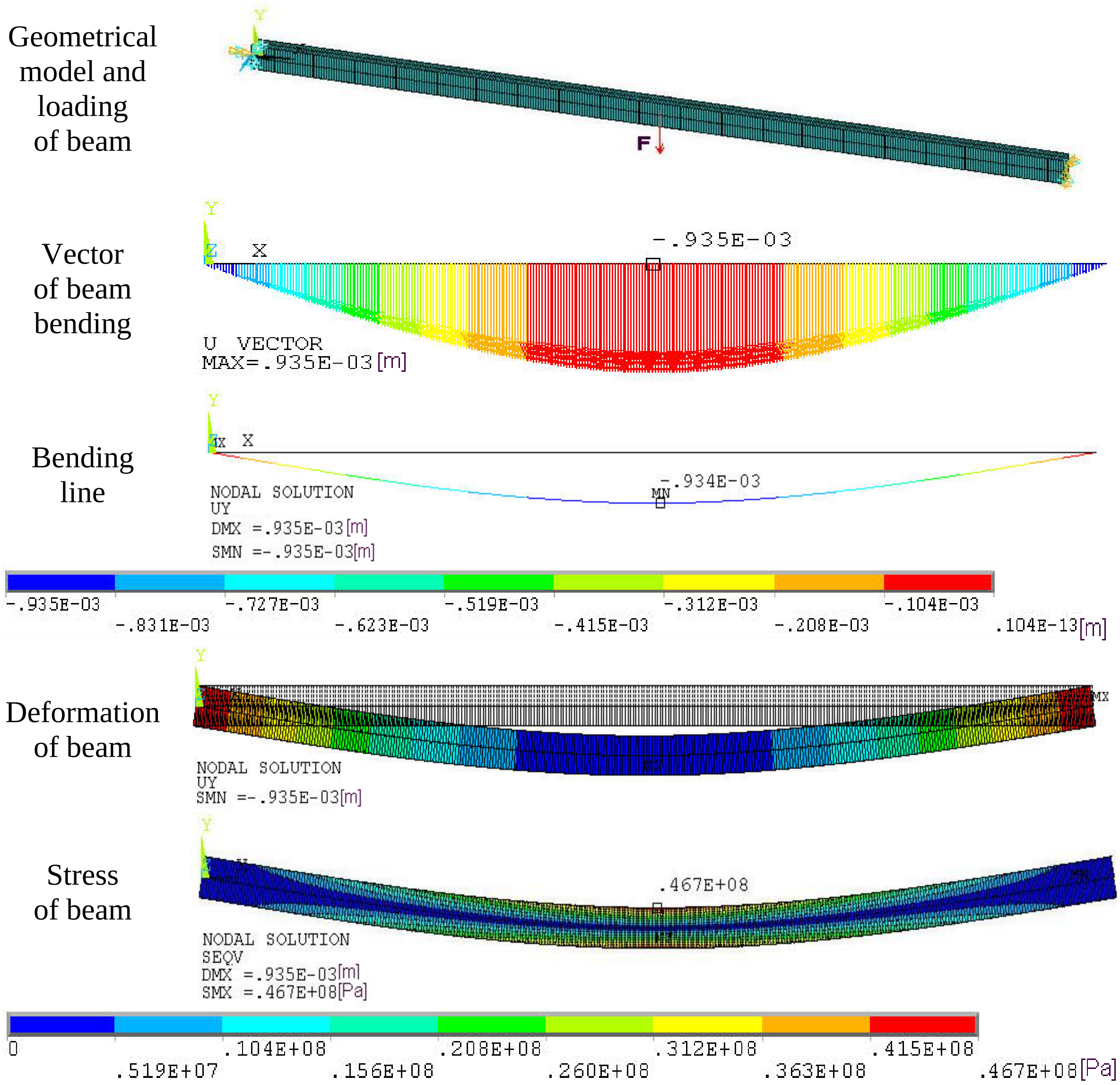

Figure 4: Graphical output from numerical simulation of supported beam under bending load

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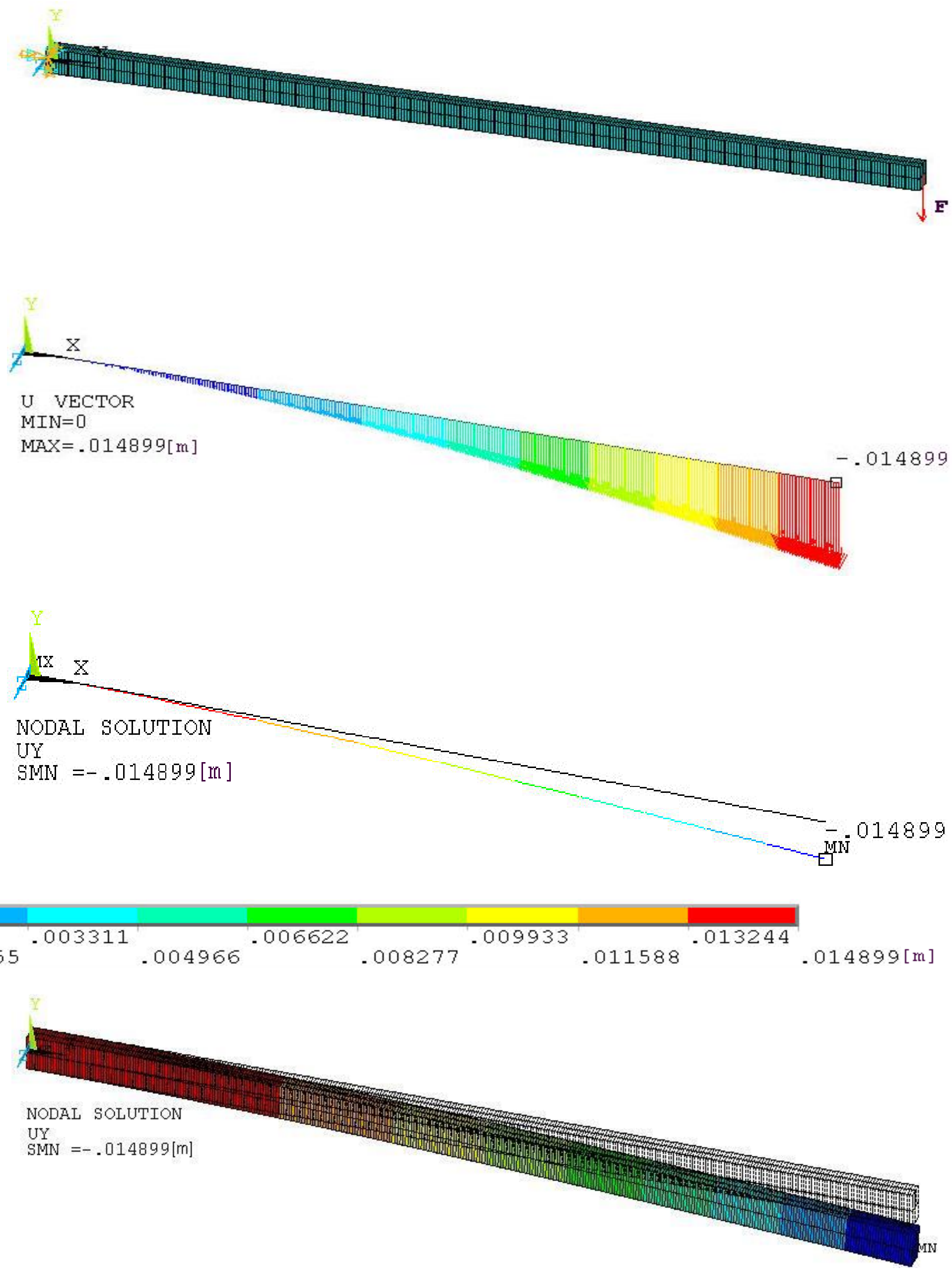
Figure 5 shows the graphical outputs of the results from numerical simulation of a clamped beam under bending loads along with a geometric model and a generated net. The result of the stress analysis is the stress distribution along the entire length of the beam. The maximum normal stress of the beam occurs in the upper fibres of the cross-section at the point of the beam clamping. The maximum stress value according to von Mises is 187 MPa. The maximum course calculated from the numerical analysis reached the value of 0.014899 m at the place of the external load application. The solution showed the course of the bending line.

Geometrical model and loading of beam

Vector of beam bending

Bending line

Deformation of beam



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Stress

of beam

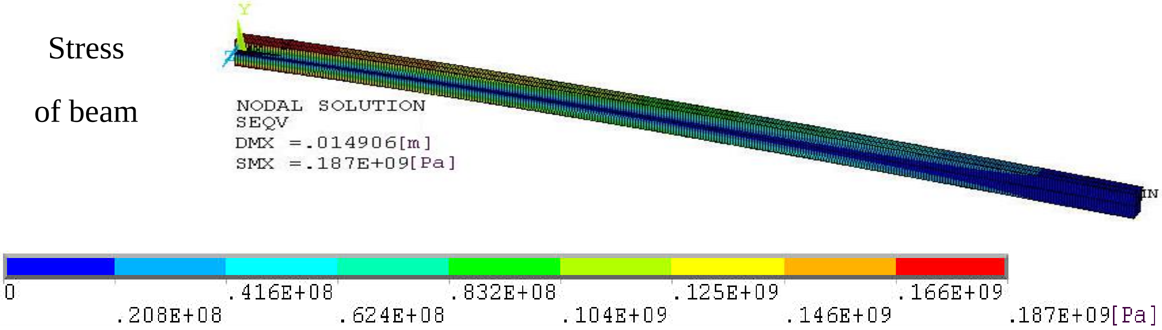
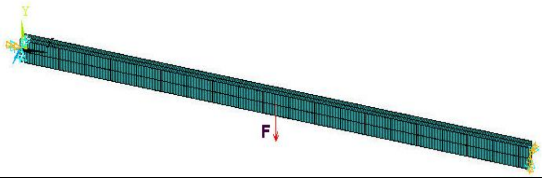


Figure 5: Graphical output from numerical simulation of a clamped beam under bending load

The summary of the results from numerical simulation of stress distribution and deformation of the supported beam under bending stress is given in Table 3.

Table 3: Results of Numerical Simulation of the Supported Beam

RESULTS OF NUMERICAL SIMULATION					
Loading	***Internal forces***		***Stress***		***Deformation***
External force [N]	Max. bending moment [Nm]	Max. shear force [N]	Max. normal [MPa]	Max. comparison [MPa]	Max. deflexion [m]
1 000	250	|500|	46.7	46.7	$0.935.10^{-3}$

The summary of the results from numerical simulation of the stress distribution and deformation of the clamped beam under bending stress is given in Table 4.

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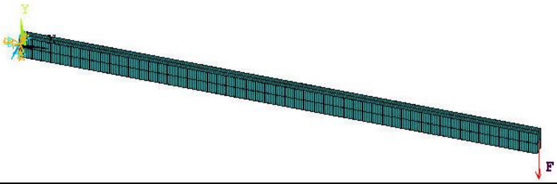
Table 4: Results of Numerical Simulation of the Clamped Beam

RESULTS OF NUMERICAL SIMULATION					
Loading	***Internal forces***		***Stress***		***Deformation***
External force [N]	Max. bending moment [Nm]	Max. shear force [N]	Max. normal [MPa]	Max. comparison [MPa]	Max. deflexion [m]
1 000	|1000|	1000	187	187	0.014899

Evaluation of Results

Evaluation of the results from analytical solution and numerical simulation of the supported beam under bending load is given in Table 5.

Table 5: Evaluation of Results of the Supported Beam

Results	***Analytical solution***		***Numerical simulation***		***Aberration of numerical simulation***	
External force [N]	Max. stress [MPa]	Max. deflexion [m]	Max. stress [MPa]	Max. deflexion [m]	Max. stress [MPa]	Max. deflexion [m]
1 000	46.903	$0.93.10^{-3}$	46.7	$0.935.10^{-3}$	-0.203	$+0.005.10^{-3}$

Figure 6 illustrates the size and course of the stresses along the cross-section of the supported beam under bending load, calculated analytically and via numerical simulation.

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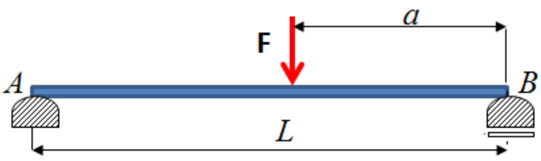
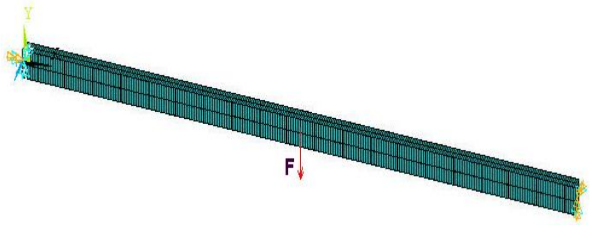
GRAPHICAL REPRESENTATION OF THE ANALYTICAL SOLUTION

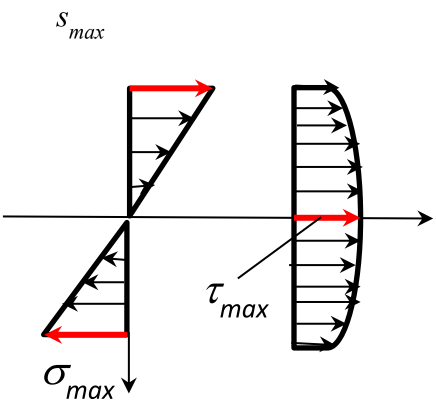
GRAPHICAL REPRESENTATION OF THE NUMERICAL SIMULATION



a) Supported beam

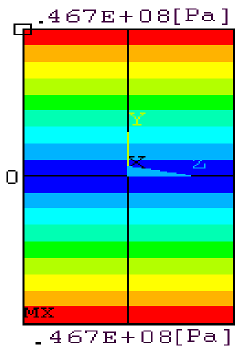


b) Geometrical model and generated net



c) Stress of beam along cross-section

σ_{max}= 46.875 MPa



d) Stress of beam along cross-section

σ_{max}= 46.7 MPa

Figure 6: Size and curves of stresses along cross-section of a supported beam under bending load

Evaluation of the results of analytical solution and numerical simulation of the supported beam under bending stress is illustrated in Table 6.

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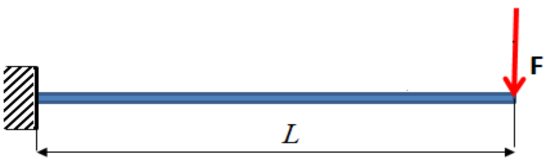
Table 6: Evaluation of Results the Clamped Beam

Results	***Analytical solution***		***Numerical simulation***		***Aberration of numerical simulation***	
External Force [N]	Max. stress [MPa]	Max. deflexion [m]	Max. stress [MPa]	Max. deflexion [m]	Max. stress [MPa]	Max. deflexion [m]
1 000	187.53	0.01488	187	0.014899	-0.53	$+0.19\ 10^{-4}$

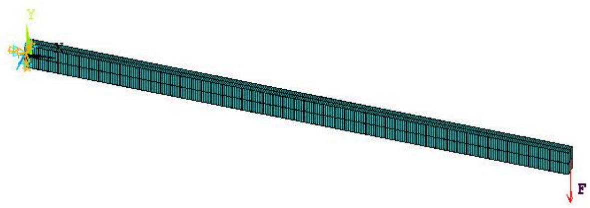
Figure 7 illustrates the size and curve of the stresses along the cross-section of the clamped beam under bending load, calculated analytically and via numerical simulation.

GRAPHICAL REPRESENTATION OF THE ANALYTICAL SOLUTION

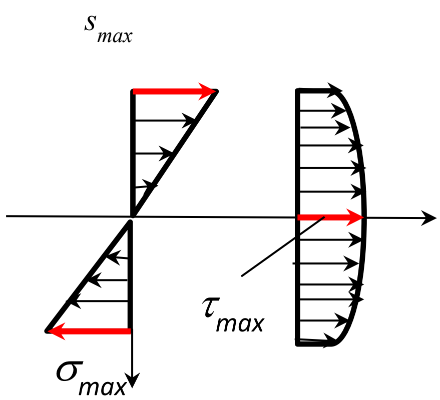
GRAPHICAL REPRESENTATION OF THE NUMERICAL SIMULATION



a) Clamped beam

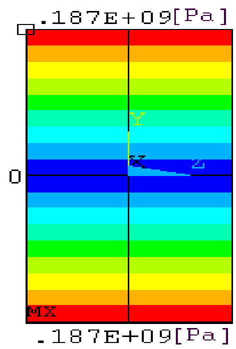


b) Geometrical model and generated net



c) Stress of beam along cross-section

σ_{max}= 187.5 MPa



d) Stress of beam along cross-section

σ_{max}= 187 MPa

Figure 7: Size and curves of stresses along cross-section of a clamped beam under bend

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CONCLUSION

Results of the analytical solution and the numerical simulation of direct supported and embedded beams under bending stress, illustrated in Figure 1, are almost (approximately) identical. Aberration of the calculated values from both analyses for the maximum stress and strain is evident in the results of numerical simulation.

As for the supported beam, the maximum value of von Mises stress calculated by numerical method was by 0.203 MPa lower, and the maximum deflection at the point of action of the concentrated external load was by 0.005 10^{-3} m higher. For the clamped beam, the maximum von Mises stress value calculated by the numerical method was by 0.53 MPa lower, and the maximum deflection at the point of application of the concentrated external load was by 0.19 10^{-4} m higher.

The given differences in the calculation of the maximum stresses are due to the fact that the analytical calculation considered the maximum value of the shear stress acting along the height of the cross-section. The calculated value of the maximum shear stress is negligible in case of the beams under bending load, compared to normal stress, as shown in Tables 1 and 2. The beam's strength and its deformation are affected by the maximum normal stress; shear stress can be therefore neglected in most cases. These differences in calculation of the maximum deflections are due to the fact that, in numerical simulation, the size and shape of the net as well as the type of the selected element significantly influence the result. In both cases, the calculated value of the maximum deflection was therefore higher. Aberration of the calculated maximum deflections in numerical simulation is very small or even negligible.

Our analysis suggests that the magnitude of the maximum stress and its deformation are significantly influenced by the beam embedding. Tables 5 and 6 show that the maximum stress value for a clamped beam was 4 times greater compared to that for a supported beam. The maximum deflection for a clamped beam was 16 times greater than that for a supported beam. This indicates that prediction of the maximum stresses and deformations of individual structural elements under bending stress is very important in terms of the strength and dimensional stability of the entire structure. Prediction of stresses and deformations via numerical simulation is currently more practical, as the difference in calculation of the maximum values is negligible; the solution is more flexible in terms of the material change and the beam embedding as well as the changes in the shape of the beam's cross-section, while regarding the aim to improve the rigidity (strength) of the structure.

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