British Journal of Computer, Networking and Information Technology
Volume 7, Issue 1, 2024 (pp. 47-57)
www.abjournals.org

VERIFICATION OF THE ANALYTICAL SOLUTION AND NUMERICAL SIMULATION FOR TENSION - PRESSURE

Helena Kravarikova

Faculty of Materials Science and Technology in Trnava, Slovak Republic,

Institute of Applied Informatics, Automation and Mechatronics.

E-mail: helena.kravarikova@stuba.sk, Tel.: +421 906 068 441

Cite this article:

Helena Kravarikova (2024), Verification of the Analytical Solution and Numerical Simulation for Tension - Pressure. British Journal of Computer, Networking and Information Technology 7(1), 47-57. DOI: 10.52589/BJCNIT-QN2K6POF

Manuscript History

Received: 19 Nov 2023

Accepted: 21 Dec 2023

Published: 25 Jan 2024

ABSTRACT: *The proposed constructions have to meet their use in terms of shape, dimensions and above all, strength. Parts of the structure are loaded by external forces. The load on the structure is examined in all parts and directions. Parts of the structure are stressed by a combination of basic types of stress. One of the basic types of stress is pull or pressure. The investigation of the part of the structure under tensile-compression stress is important because of the deformation, stability and strength of the structure as a whole. Analytical solution, classical experiment or modelling FEM can be used to predict the stress of structural parts. Currently, the most preferred solution is FEM numerical simulation. Numerical simulation has a more purposeful use in the solution of structural stability. The article contains the solution, tensile stresses for a steel rod, numerical simulation using the FEM method in the ANSYS program and analytically.*

KEYWORDS: FEM, Modeling, Tension - Pressure, Hooke's law, Tension, Rod

Article DOI: 10.52589/BJCNIT-QN2K6POF
DOI URL: https://doi.org/10.52589/BJCNIT-QN2K6POF

INTRODUCTION

The constructions consist of rods, which, while performing their function, can be stressed by different types of stress or a combination of them. The stress on parts of the structure depends on its load and use. A common phenomenon in constructions is that individual parts are stressed by tension or pressure. Stressing the rods in tension - pressure, after exceeding the yield point, causes permanent - plastic deformations. During plastic deformations, there is a change in the cross-section of the rod. During tensile stress, the cross-section of the rod contracts. By increasing the stress of the structure in the rod up to the strength limit, destruction can occur - breaking of the rod. When designing structures, it is important that its parts are stressed only in the area of elastic deformation. In this way, permanent deformations can be prevented, then the strength limit will not be exceeded, the structure will not be disturbed. Then the structure is stressed in the region of elastic deformation, where Hooke's law applies. It is important to predict the stresses of individual parts of the structure as well as the whole. Analytical solution, experiment or numerical simulation can be used for stress prediction. Currently, numerical simulation is the most preferred because of its more expedient use. In the contribution, the solution of a rod stressed in tension is numerically simulated using the FEM method in the ANSYS program and analytically.

THEORETICAL ASSUMPTIONS FOR THE SOLUTION OF TENSION-PRESSURE

Tension - pressure belongs to the basic stresses of individual parts of the structure or components. Tension - pressure occurs in rods loaded by external forces in the direction of the rod axis. The imaginary section method is used to determine the results of the internal forces acting in the rod. When stressed by tension or pressure, only one internal force acts in the intended cross-section of the rod - normal or axial force $N(x)$ in the rod axis, Figure 1.

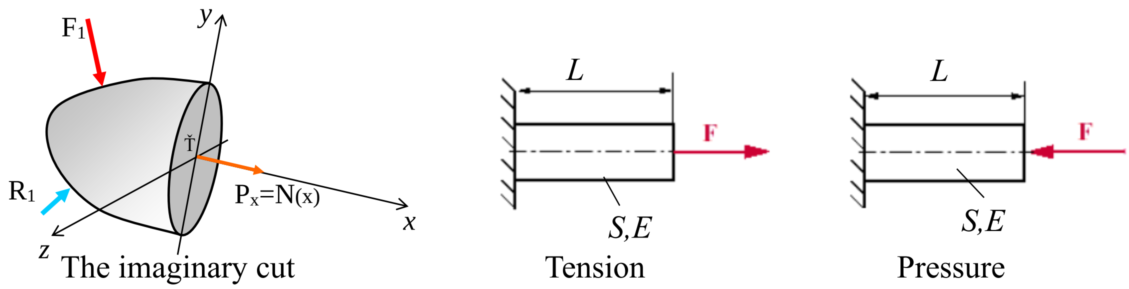


Figure 1: Tension stress – pressure [1, 3]

The basic law of elasticity, strength and plasticity is Hooke's law. Hooke's law expresses the relationship between the applied stress and the relative extension when the body is stressed by tension or when the body is stressed by compression by a proportional shortening. The tensile test diagram is shown in Figure 2.

Article DOI: 10.52589/BJCNIT-QN2K6POF
DOI URL: https://doi.org/10.52589/BJCNIT-QN2K6POF

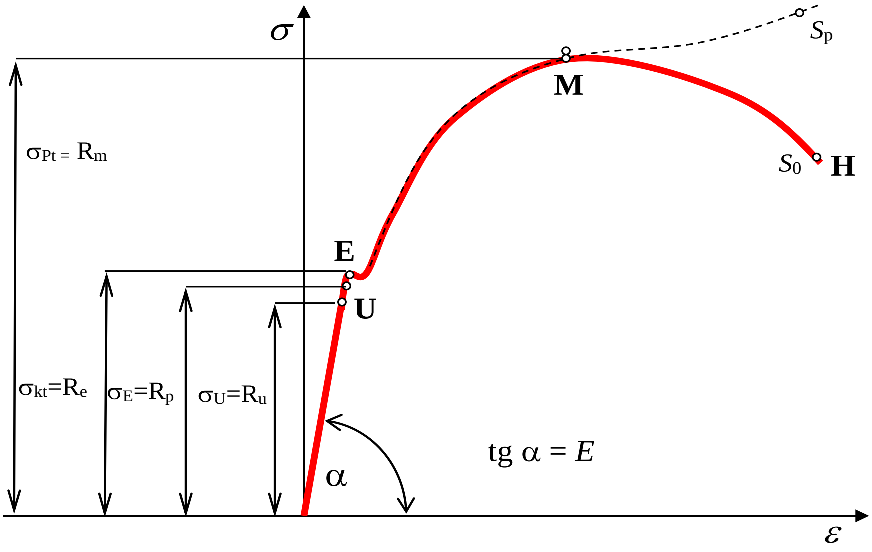


Figure 2: Diagram of the tensile test [3]

Mathematical expression of the basic law of elasticity and strength - Hooke's law [1, 3, 4]:

$$\varepsilon = \frac{\sigma}{E} \qquad [-] \qquad (1)$$

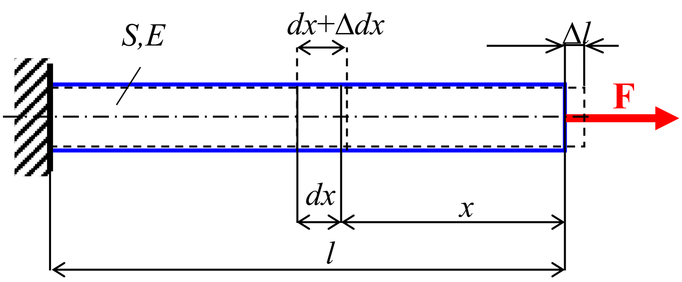
Where:

σ - normal stress,

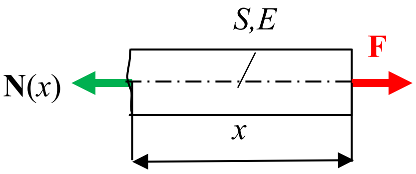
E – modulus of elasticity in the tension of the material (for steel it is approx. 2.10^5 MPa),

ε - proportional lengthening/shortening under stress.

For the analytical solution, to determine the results of the internal forces acting in the rod, the imaginary section method is used, Figure 3. The method of two or more close imaginary cuts is used when the external force load varies along the length of the member. That is, when the load is not constant over the entire length of the rod.



Article DOI: 10.52589/BJCNIT-QN2K6POF
DOI URL: https://doi.org/10.52589/BJCNIT-QN2K6POF

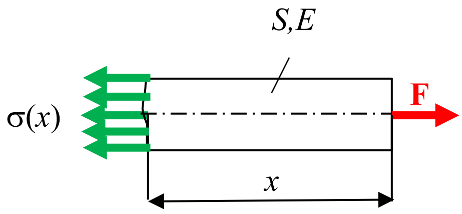


Normal or axial force $N(x)$:

$x \in < 0, l >$

$$\Sigma F_i(x) = 0 \quad [N] \qquad (2)$$

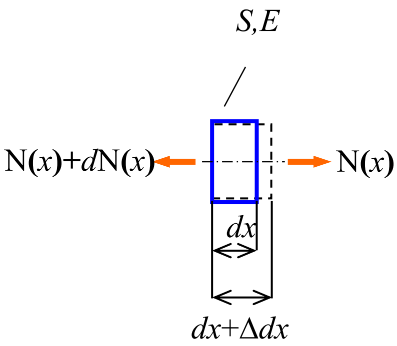
$$N(x) = F \quad [N] \qquad (3)$$



Normal stress $\sigma(x)$:

$x \in < 0, l >$

$$\sigma(x) = \frac{N(x)}{S} \quad [Pa] \qquad (4)$$



Deformation of a rod element

of infinitesimally small length dx:

$$\Delta dx = \varepsilon . dx \quad [m] \qquad (5)$$

Figure 3: The rod stressed in tension with a constant cross-section *S* and loaded by a constant force *F* [1, 2]

During tensile or compressive stress, only one internal force acts in the imaginary section of the rod, the normal - axial force $N(x)$ in the axis of the rod, Figure 3. The normal - axial force $N(x)$ in the rod axis is equal to the algebraic sum of all external forces acting in the rod axis of the imaginary section and is determined from the static equilibrium conditions.

The normal stress $\sigma(x)$ expresses the internal forces acting on a unit of cross-sectional area, Figure 3 [1, 2, 3, 4]:

$$\sigma(x) = \frac{N(x)}{S} \quad [Pa] \qquad (4)$$

Article DOI: 10.52589/BJCNIT-QN2K6POF
DOI URL: https://doi.org/10.52589/BJCNIT-QN2K6POF

In the analytical solution of stressed rods for tension - compression, it is possible to use the validity of the law of superposition for:

- axial forces from individual rod loads,

- stresses from individual rod loads,

- rod extension from individual rod loads,

- tension energy in parts of the rod.

Assuming the validity of Hooke's law by tension energy, we understand the energy that is equal to the work of external forces, performed in the deformation of the body. By tension energy, we mean the energy that accumulates in an initially unloaded body from all the internal forces acting on the infinitesimal elements of the rod. The external and internal forces must be in static equilibrium.

The deformation work of external forces is defined by the relation [1, 2, 3]:

$$W = \int_S F.ds \qquad [J] \qquad (6)$$

Using equation (6), we determine the stress energy for normal and shear stresses. The tension energy of a rod stressed by simple tension or pressure we calculate [1, 2, 3, 4]:

$$A = \frac{1}{2E}\int_{(l)} \frac{N^2(x)}{S}\, dx \qquad [J] \qquad (7)$$

Tension energy is always positive, regardless of whether the rod is stressed in tension or compression. In the case of a statically determined stressing of the rod for tension - pressure, reshaping - deformation of the rod takes place, which is manifested by its lengthening or shortening.

The total extension or shortening of the rod is obtained by adding up the elementary deformations of the rod along its entire length [1, 2, 3]:

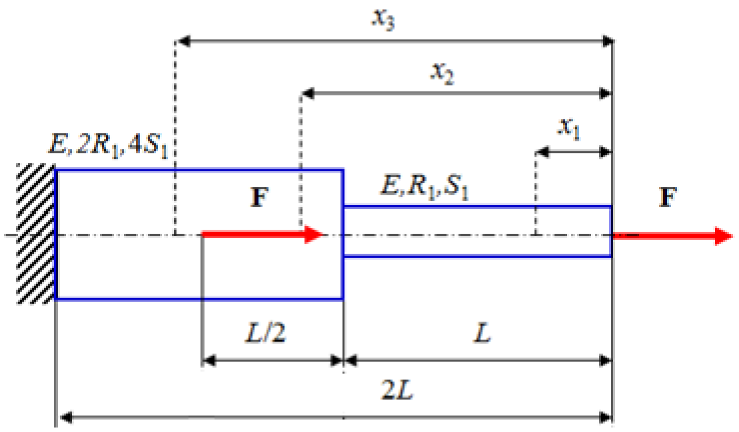
$$\Delta\, l = \int_{(l)} \Delta\, dx = \int_{(l)} \varepsilon\, dx = \int_{(l)} \frac{\sigma(x)}{E}\, dx = \int_{(l)} \frac{N(x)}{E.S}\, dx \qquad [m] \qquad (8)$$

Article DOI: 10.52589/BJCNIT-QN2K6POF
DOI URL: https://doi.org/10.52589/BJCNIT-QN2K6POF

SOLVING THE PROBLEM OF THE ROD TENSION STRESS

Defining the task

A structural steel rod was chosen to solve the prediction. The rod is clamped on one side and is loaded by an external force F at the free end. According to the attachment of the rod, we solve the task as statically determined. Along the length of the rod, the cross-section of the rod and the external force load F change. For the calculation, the geometric shape and dimensions of the rod are given together with the external load in Figure 4: L = 0,3 m, R_1 = 5 mm, $R_2 = 2R_1$, S_1 = 0,785.10^{-4} m^2, $S_2 = 4S_1$, F = 20000 N. Elasticity modulus of the structural steel E = 2.1 10^5 MPa and Poisson number μ = 0.30.



The data:

L = 0,3 m,

R_1 = 5 mm,

$R_2 = 2R_1$,

S_1 = 0,785.10^{-4} m^2,

$S_2 = 4S_1$,

E = 2,1.10^5 MPa,

F = 20000 N

Figure 4: Shape and dimensions of a rod under tensile stress

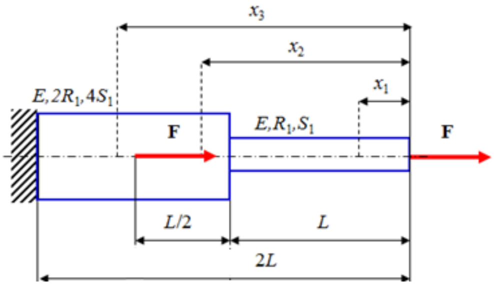
Analytical solution

Analytical solution is a method of imaginary section according to the shape and load of the rod based on the validity of Hooke's law. The results of the analytical solution are shown in Table no. 1. Graphical representation of the results of the analytical solution are in Figure 5. In Figure 5*a)* is a geometrical shape with dimensions and loading of the rod together with the notional sections indicated. Figure 5*b*) graphically shows the course of the axial force $N(x)$ calculated analytically along the entire length of the rod. The largest value of the axial force $N(x)$ is in the imaginary section x_3, where the cross-section of the rod is $4S_1$. In this imaginary section, the magnitude of the axial force changes only with the change in the load on the member. In Figure 5*c)* is a graphical representation of the normal stress $\sigma(x)$ along the entire length of the rod. The largest normal stress $\sigma(x)$ is in the imaginary cross-section x_1, where the cross-section of the rod is S_1. The magnitude of the normal stress varies with the cross-section and load of the member.

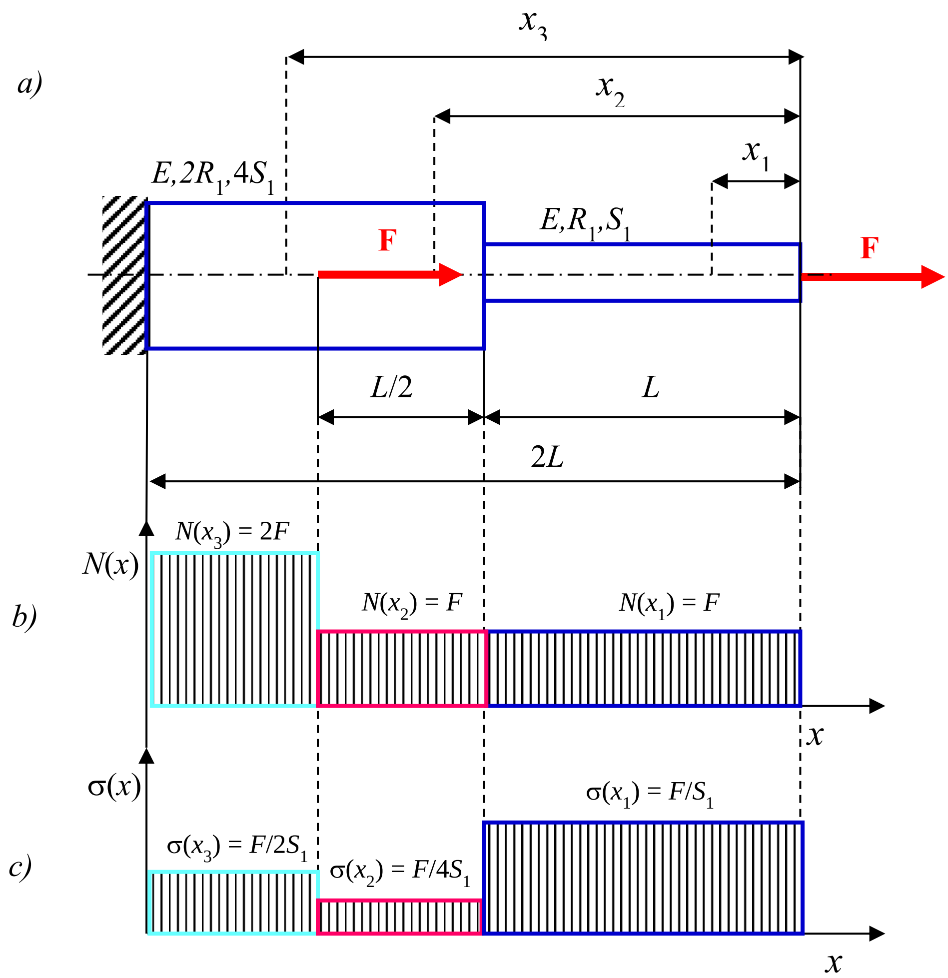
Article DOI: 10.52589/BJCNIT-QN2K6POF
DOI URL: https://doi.org/10.52589/BJCNIT-QN2K6POF

Table 1: The results of the analytical solution

RESULTS OF ANALYTICAL SOLUTION



Cross section	Imaginary cut	Force $N(x)$ [N]	Max. stress [MPa]	Max. extension [m]
S_1	$x_1 \in \langle 0, L \rangle$	20 000	254,78	$0{,}364.10^{-3}$
$4S_1$	$x_2 \in \langle L, \frac{3}{2}L \rangle$	20 000	63,695	$0{,}455.10^{-4}$
$4S_1$	$x_3 \in \langle \frac{3}{2}L, 2L \rangle$	40 000	127,39	$0{,}91.10^{-4}$



Article DOI: 10.52589/BJCNIT-QN2K6POF
DOI URL: https://doi.org/10.52589/BJCNIT-QN2K6POF

Figure 5: Graphical representation of the analytical solution

a) Dimensions of the rod with external load

b) Graphical course of the axial force $N(x)$

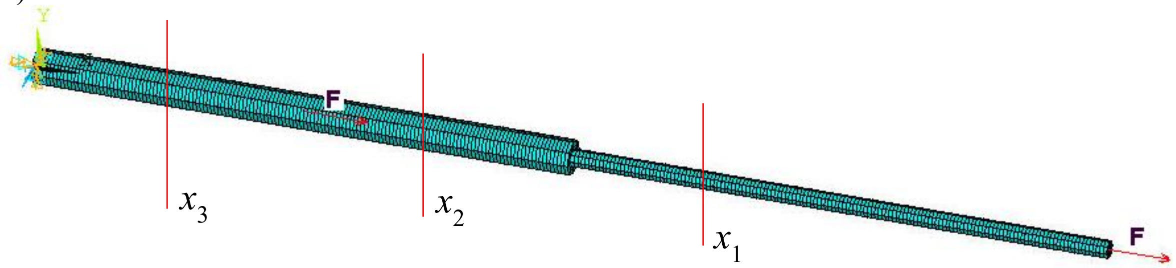
c) Graphical course of normal stress $\sigma(x)$

Numerical simulation

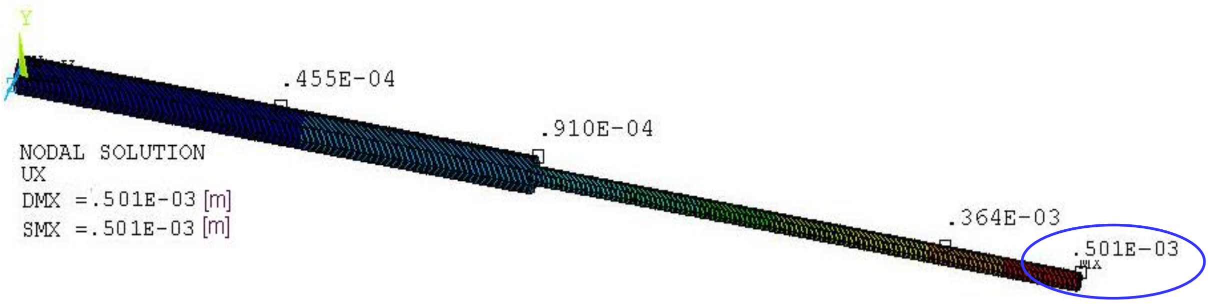
The rod tensile load were is investigated by numerical simulation using the FEM in the ANSYS program. Numerical simulation is based on the real conditions the analytical solution the clamped rod one under the external force causing to the rod tensile and deformation in form elongation rod. The solution by numerical simulation using the FEM in the ANSYS program was identical in shape and size to the analytical solution for the rod. For a given rod type and its load, numerical simulation using the FEM in the ANSYS program allows for a simplified way creating geometric model with element type selection solving a 3D problem in 1D type element. The material model comprised mechanical properties of structural steel [5, 6].

The results of the numerical simulation are shown graphically in Figure 6 and listed in Table 2. In Figure 6*a*) is a created geometric model with generated mesh and external load. The geometric model was created based on the actual dimensions of the member together with the indicated imaginary sections, which were used in the analytical solution. Figure 6*b*) graphically displays the calculation of the extension by numerical simulation. On the graphic display, extensions are marked in individual intended cuts together with the overall extension of the rod. The largest extension is in the rod with cross-section S_1 in the imaginary section x_1 with a value of $0.364.10^{-3}$ m. The total extension of the rod has a value of $0.501.10^{-3}$ m. Figure 6*c*) graphically shows the course of the normal stress calculated by numerical simulation. On the graphic display, the normal stresses in the individual imaginary sections together are marked. The stress reaches a maximum value of $+0.255.10^{9}$ Pa, along the entire length of the rod with cross-section S_1, in the imaginary section x_1.

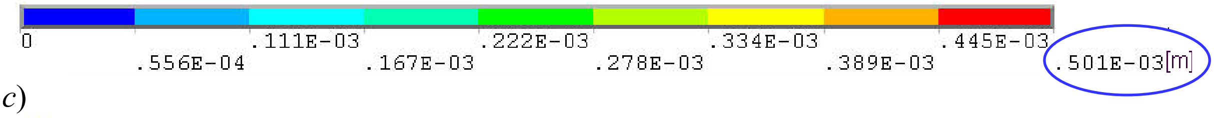
a)



b)



Article DOI: 10.52589/BJCNIT-QN2K6POF
DOI URL: https://doi.org/10.52589/BJCNIT-QN2K6POF



c)

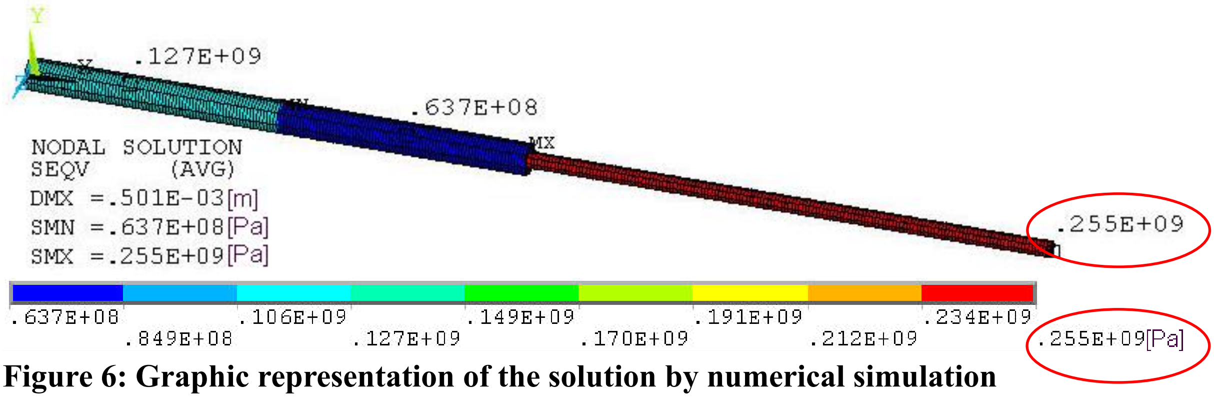
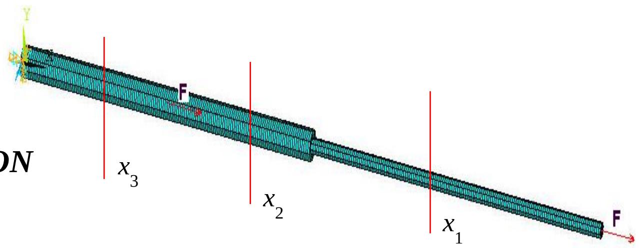


Figure 6: Graphic representation of the solution by numerical simulation
a) Geometric model with generated mesh and external load
b) The course of rod extension
c) Course of stress in the rod

Table 2: The results of the numerical simulation

RESULTS OF NUMERICAL SIMULATION



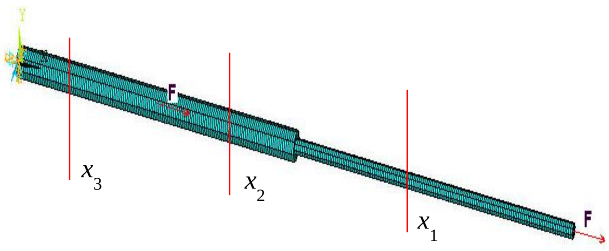
Cross section	Imaginary cut	Force $N(x)$ [N]	Max. stress [MPa]	Max. extension [m]
S_1	$x_1 \in \langle 0, L \rangle$	20 000	255	$0{,}364.10^{-3}$
$4S_1$	$x_2 \in \langle L, \frac{3}{2}L \rangle$	20 000	63,7	$0{,}455.10^{-4}$
$4S_1$	$x_3 \in \langle \frac{3}{2}L, 2L \rangle$	40 000	127	$0{,}91.10^{-4}$

Article DOI: 10.52589/BJCNIT-QN2K6POF
DOI URL: https://doi.org/10.52589/BJCNIT-QN2K6POF

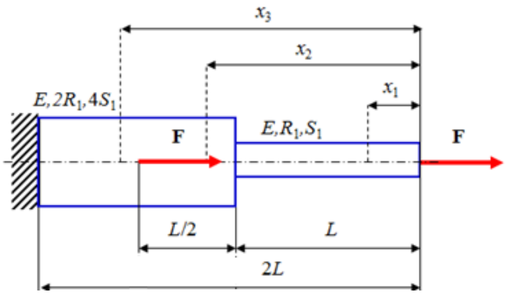
ANALYSIS OF RESULTS AND DISKUSION

In Table no. 3 are the results from the analytical solution and the numerical simulation by FEM solution in the ANSYS program file.

Table 3: The results from the analytical solution and the numerical simulation



Geometric model of numerical simulation



Geometric model of analytical solution

RESULTS			***Analytical solution***		***Numerical simulation***		***Aberration of numerical simulation***	
Cross section	**Imaginary cut**	**Force $N(x)$ [N]**	**Max. stress [MPa]**	**Max. extension [m]**	**Max. stress [MPa]**	**Max. extension [m]**	**Max. stress [MPa]**	**Max. extension [m]**
S_1	$x_1 \in \langle 0, L \rangle$	**20 000**	**254,78**	**0,364.10^{-3}**	**255**	**0,364.10^{-3}**	**+0,22**	**0**
$4S_1$	$x_2 \in \langle L, \frac{3}{2}L \rangle$	**20 000**	**63,695**	**0,455.10^{-4}**	**63,7**	**0,455.10^{-4}**	**+0,005**	**0**
$4S_1$	$x_3 \in \langle \frac{3}{2}L, 2L \rangle$	**40 000**	**127,39**	**0,91.10^{-4}**	**127**	**0,91.10^{-4}**	**-0,39**	**0**

By comparing the results of both solutions, there was a deviation from the analytical solution when calculating the stress using the numerical method. In the imaginary section x_1 with the cross-section of the rod S_1, the deviation in the numerical simulation has a value of +0.00086% from the analytical solution. In the imaginary section x_2 with the cross-section of the rod $4S_1$, when rounding the results from the analytical solution, the deviation is 0%. In the imaginary section x_3 with the cross-section of the rod $4S_1$, a deviation of -0.003% from the analytical solution arose during the numerical simulation.

The results obtained by numerical simulation are identical to the results of the analytical solution for the calculation of axial force and extension. There is a very small deviation when calculating stresses by numerical simulation. The difference in the calculation is almost negligible.

Article DOI: 10.52589/BJCNIT-QN2K6POF
DOI URL: https://doi.org/10.52589/BJCNIT-QN2K6POF

CONCLUSION

The calculated results by numerical simulation using FEM in the ANSYS program are identical to the results of the analytical solution for the calculation of the axial force and extension of the rod.

In this case, we can state that when calculating in both ways, there is no difference in the calculated values of deformation, which characterizes the extension of the rod, even when calculating the values of internal - axial forces.

There is a very small deviation from the analytical solution when calculating stresses by numerical simulation. In the 1st imaginary section, the deviation during calculation by numerical simulation is +0.22 MPa, which is expressed as a percentage of 0.00086%. In the 2nd imaginary section, there is no difference in the calculations, which is expressed as a percentage of 0.000%. In the 3rd imaginary section, the deviation in the calculation by numerical simulation is -0.39 MPa, which is expressed as a percentage of 0.003%. The biggest deviation when calculating the stress by numerical simulation from the analytical solution is in the 3rd imaginary section. In conclusion, we can state that the deviations in the calculations by numerical simulation using FEM in the ANSYS program and the analytical solution are almost negligible.

This indicates that the prediction of the maximum stresses and deformations of individual structural elements under tension-compression stress is very important in terms of strength and dimensional stability of the entire structure.

The prediction of stresses and deformations using numerical simulation is currently more practical, because the difference in the calculation of the maximum values is negligible, almost zero. Solving tasks by numerical simulation using FEM is more flexible in terms of changing the material, placing the rod, loading the rod as well as changing the shape of the cross section of the rod in order to improve the stiffness (strength) of the structure.

REFERENCES

[1] Trebuňa, F., Šimčák, F., Jurica, V.: Pružnosť a pevnosť I., *(Flexibility and Strength I)* ISBN 80-7099-477-0 Vienala, Košice, 2000.
[2] Z. Murčinkova, J.: *Pružnosť a pevnosť I.*Teória a príklady, *(Flexibility and Strength I, Theory and Examples)*ISBN 978-80-553-1567-6, TUKE Prešov, 2013
[3] http://uamt.fei.stuba.sk/vyucbove_materialy/mechanika/index.html
[4] SADD, M. H.: Elasticity, Theory, Applications, and Numerics 2. Elsevier Edt., USA 2009.
[5] V. Kutiš: *Modelovanie multifyzikálnych procesov a dejov (Modelling multi physical processes and actions)*, ISBN 978-80-227-4232-0, ES STU, Bratislava, 2014,
[6] V. Kutiš, J. Murín, J. Paulech, V. Goga: *Metóda konečných prvkov v Mechatronike I.*, Úvod do programu ANSYS (F*inite Elements Method in Mechatronics I.*, Introduction into the ANSYS program), ISBN 978-80-227-4129-3, ES STU, Bratislava, 2014.

Article DOI: 10.52589/BJCNIT-QN2K6POF
DOI URL: https://doi.org/10.52589/BJCNIT-QN2K6POF